Vector Angular Momentum

and Frames of Reference

First - some clarification

The Bad News

- There is NO SUCH THING as velocity, torque, energy, angular momentum etc.
- There is only velocity relative to a reference frame, torque about an axis, energy compared to a zero point etc.

The Good News

- All the laws of physics apply regardless of which reference frame, zero point or axis we choose.
- And while your choice of reference frame won't affect your answer, a clever choice can make your calculation much easier.

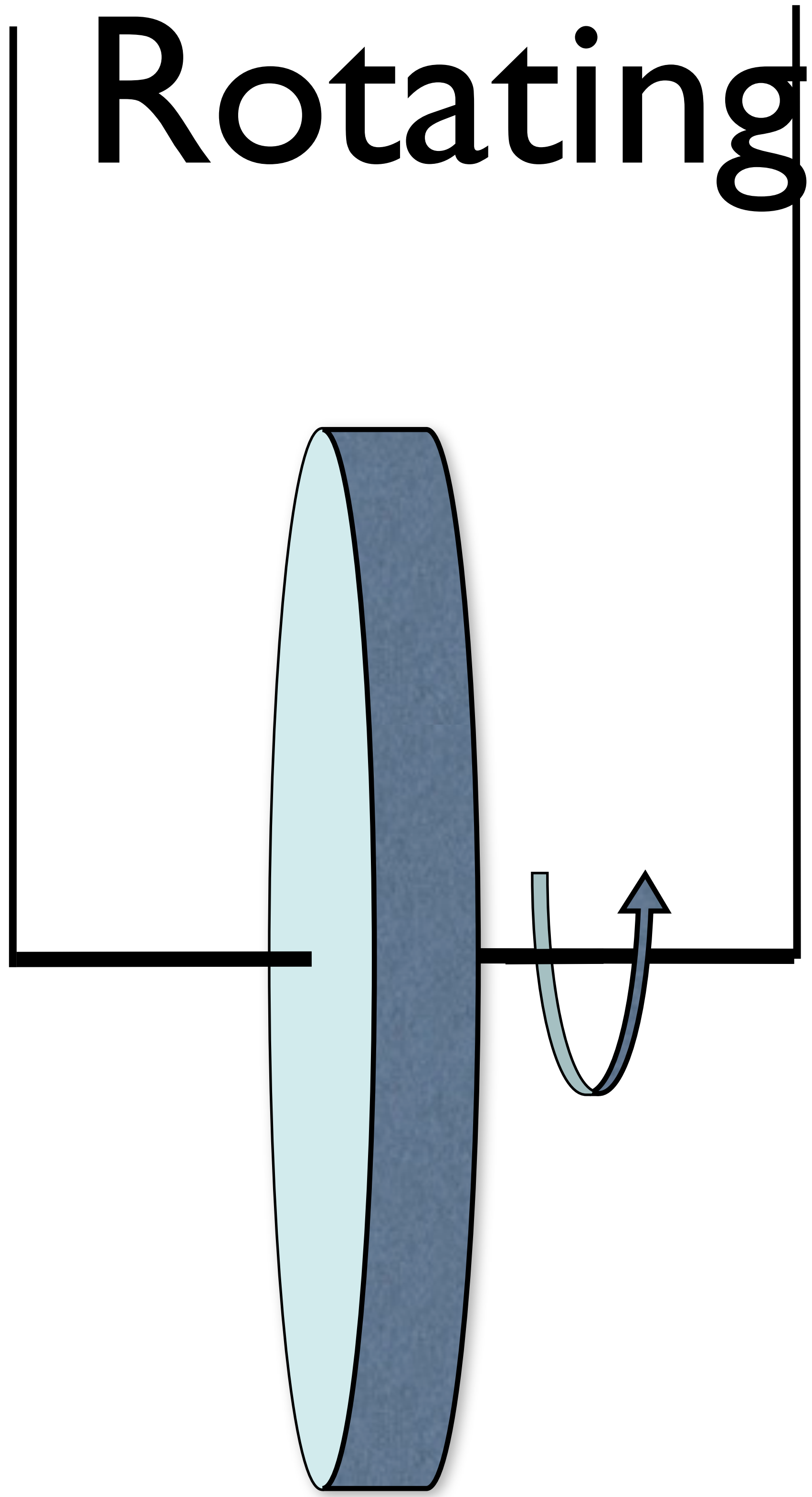
Angular Momentum

- What's the angular momentum of my fist?
- It all depends on where you choose your axis.
- None about itself, something about my head, more about Pluto.
- All are correct. But some are more useful that others...

Weird motion of the bike wheel

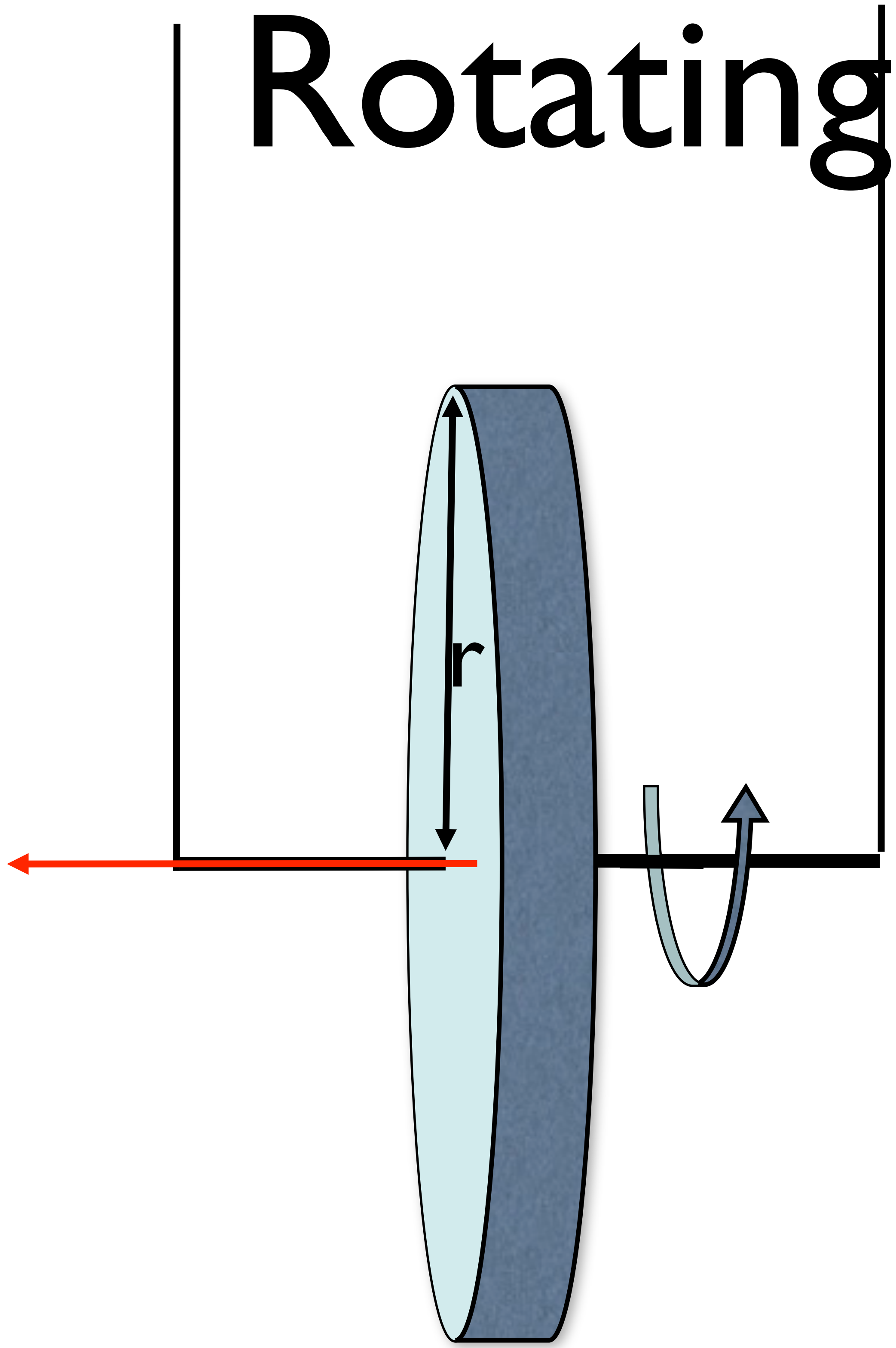
- When you try to turn it one way, it moves off at right-angles!
- Why?

Rotating wheel



- Supported by two cables.
- In which direction does its angular momentum point?

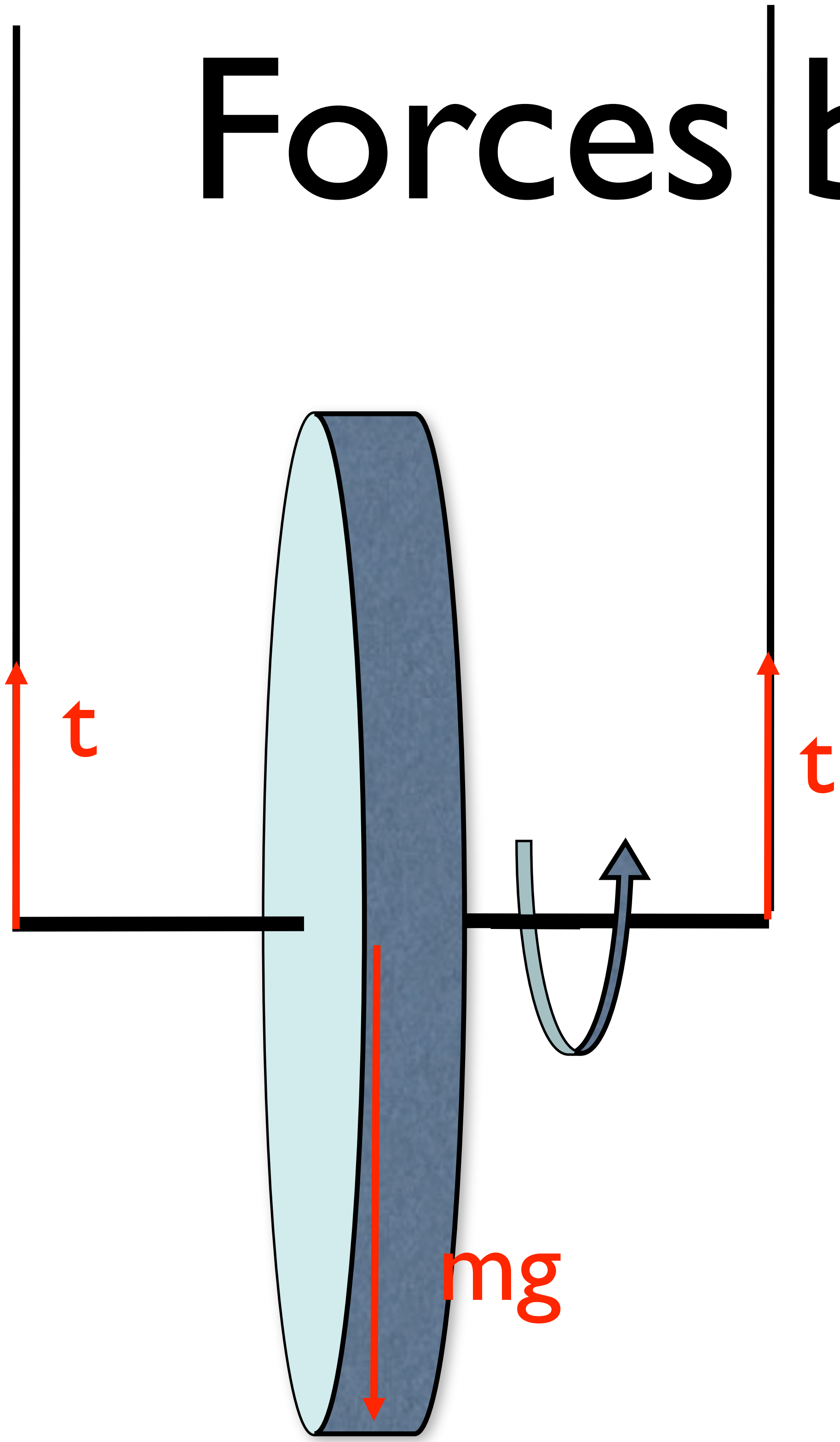
Rotating wheel



- By the right-hand rule, the angular momentum points out to the left.

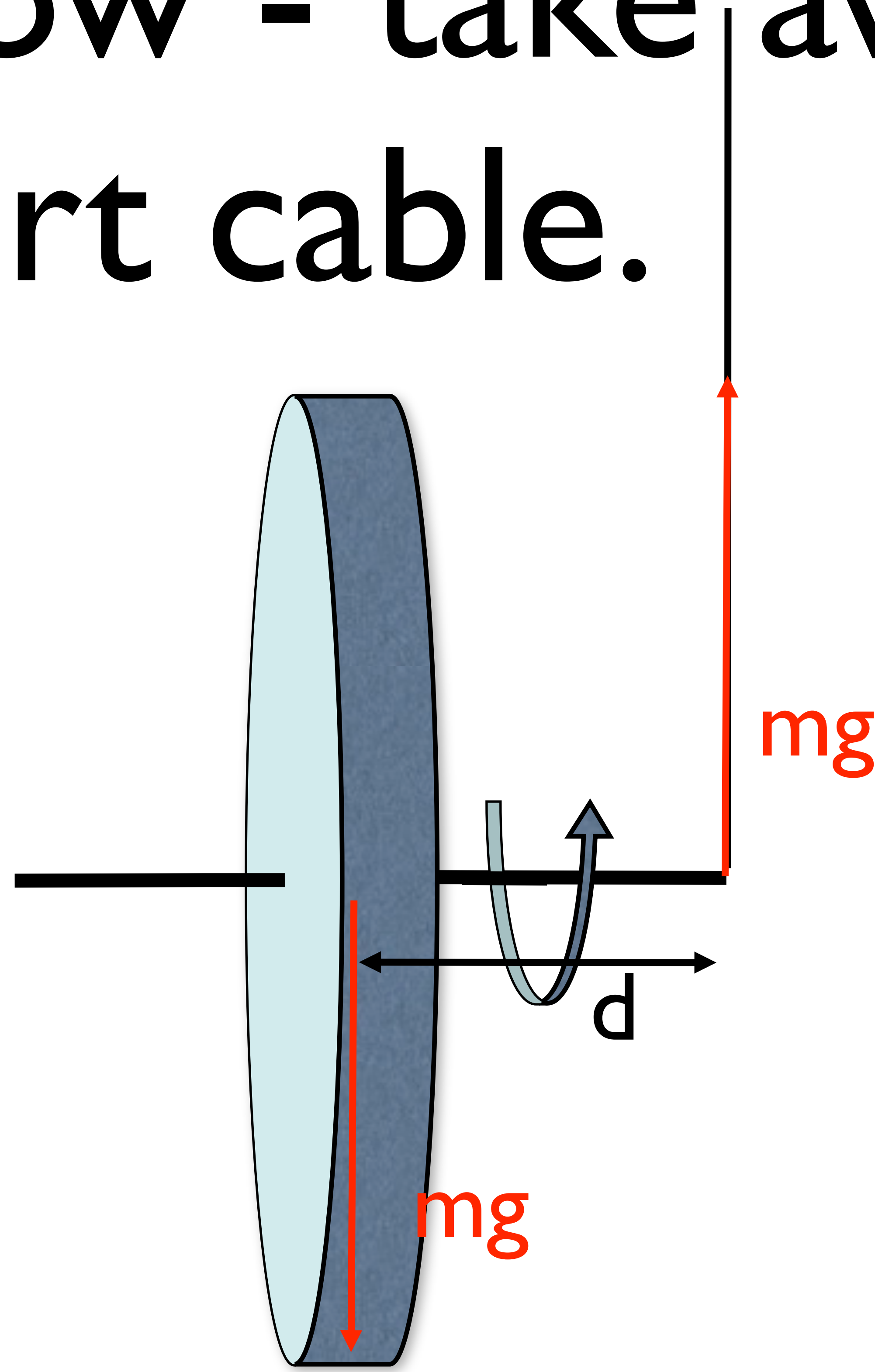
$$L = I\omega = \tfrac{1}{2} m r^2 \omega$$

Forces balance



- So there must be equal tension forces in each cable, equal to mg/2.
- To balance gravity

But now - take away one support cable.



- A torque is now applied to the spinning wheel.
- Let's chose the cable as our axis.
- The magnitude of the torque is dmg, but what is its direction?

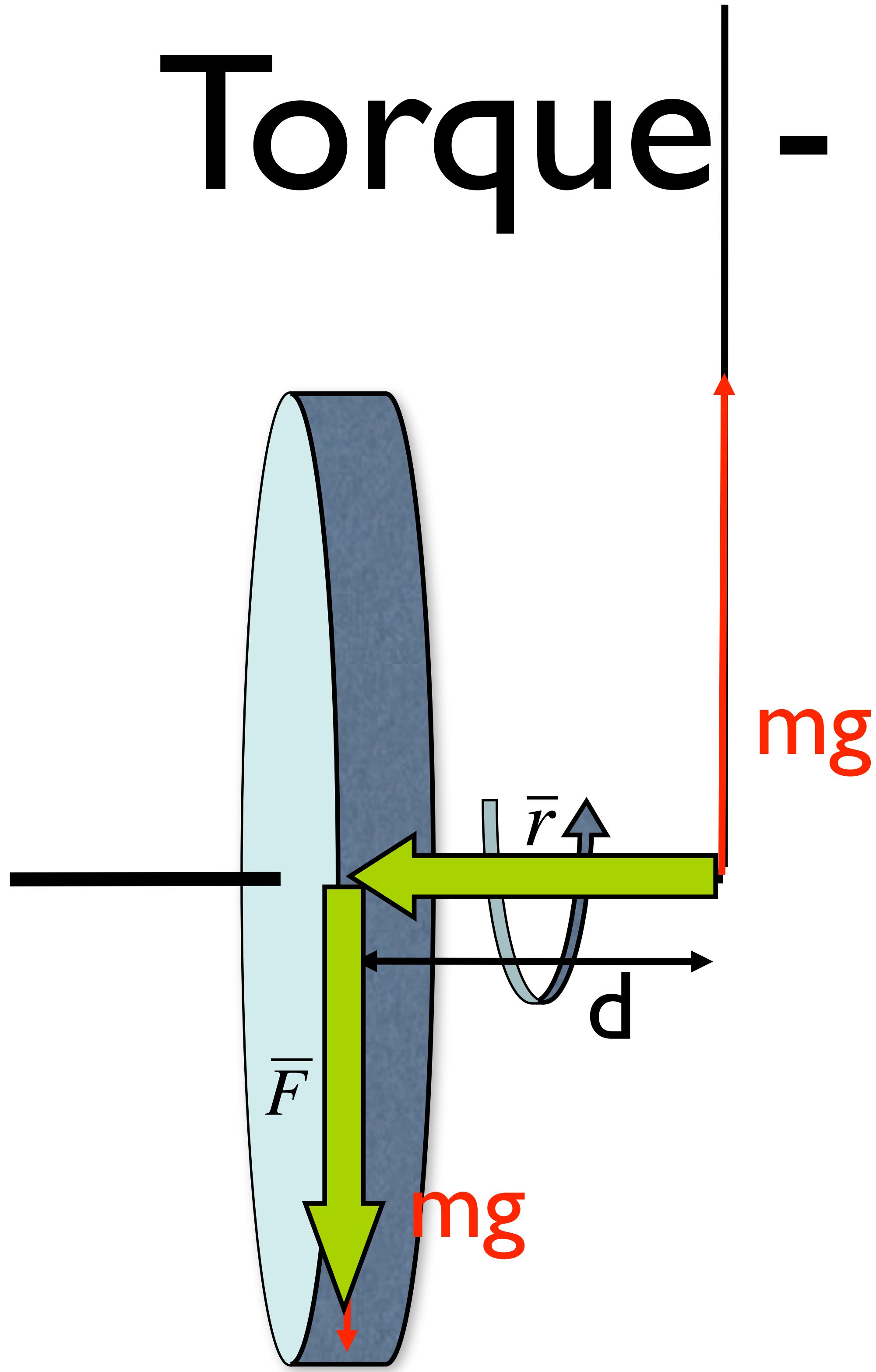
Clicker Question

- In what direction is the torque?

remember -

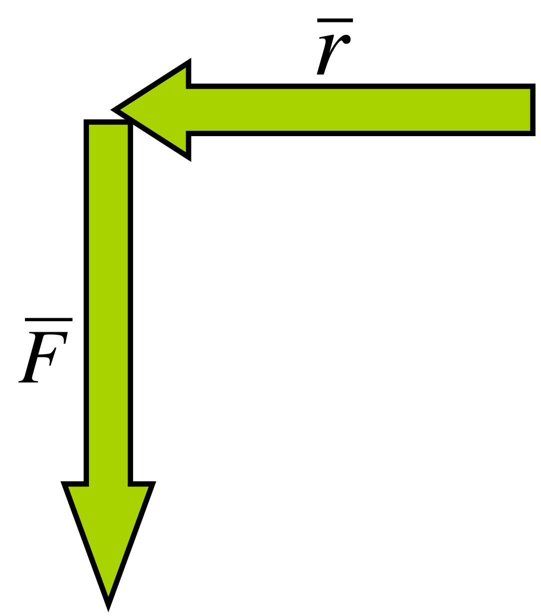
$$\bar{\tau} \equiv \bar{r} \times \bar{F}_{net}$$

Torque - $r \times f$



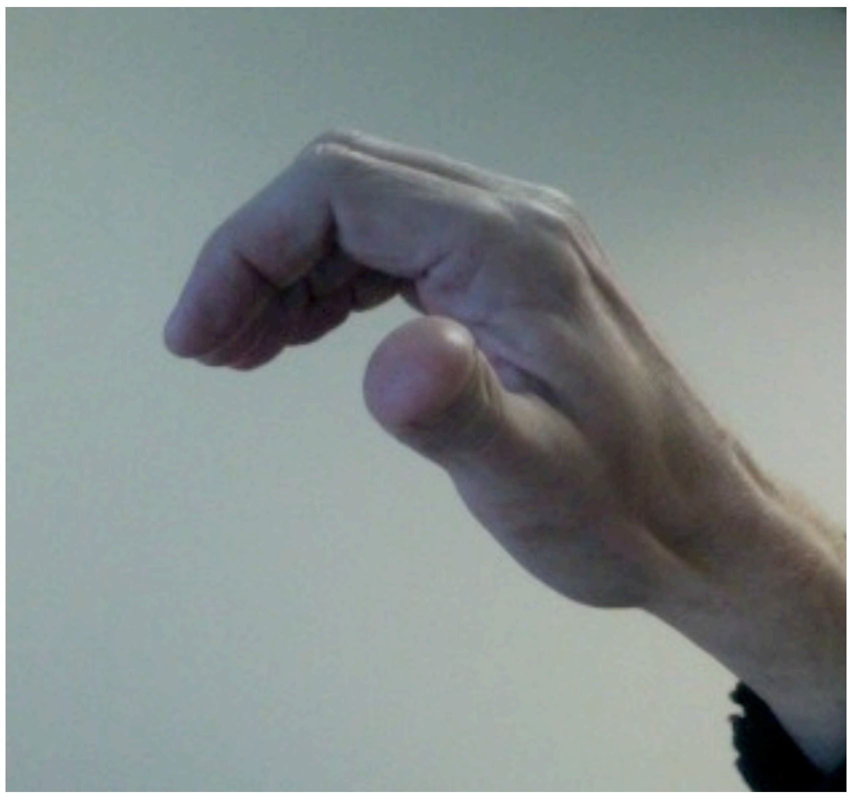
- In this case *r* is the vector from the hinge to the centre of mass of the wheel, and F is the gravitational force on the wheel (a vector pointing downwards of magnitude mg).

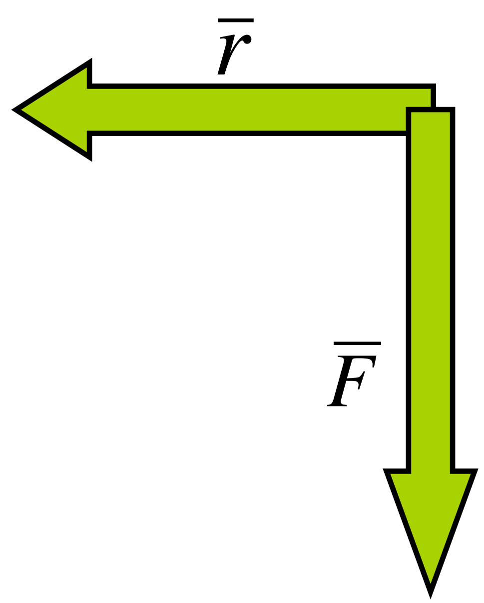
Look at the two vectors



- To take the cross product, slide them until they start in the same place.

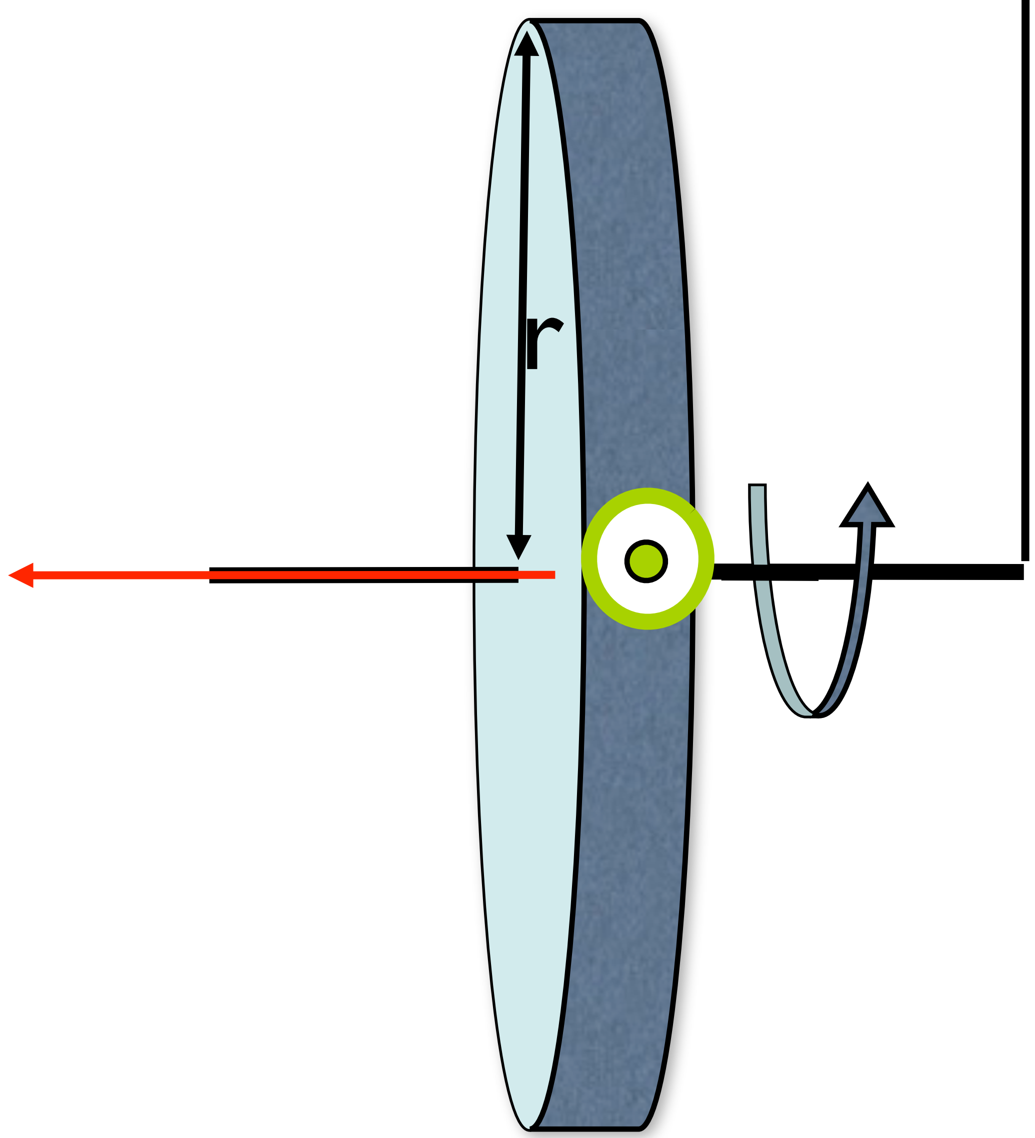
Look at the two vectors





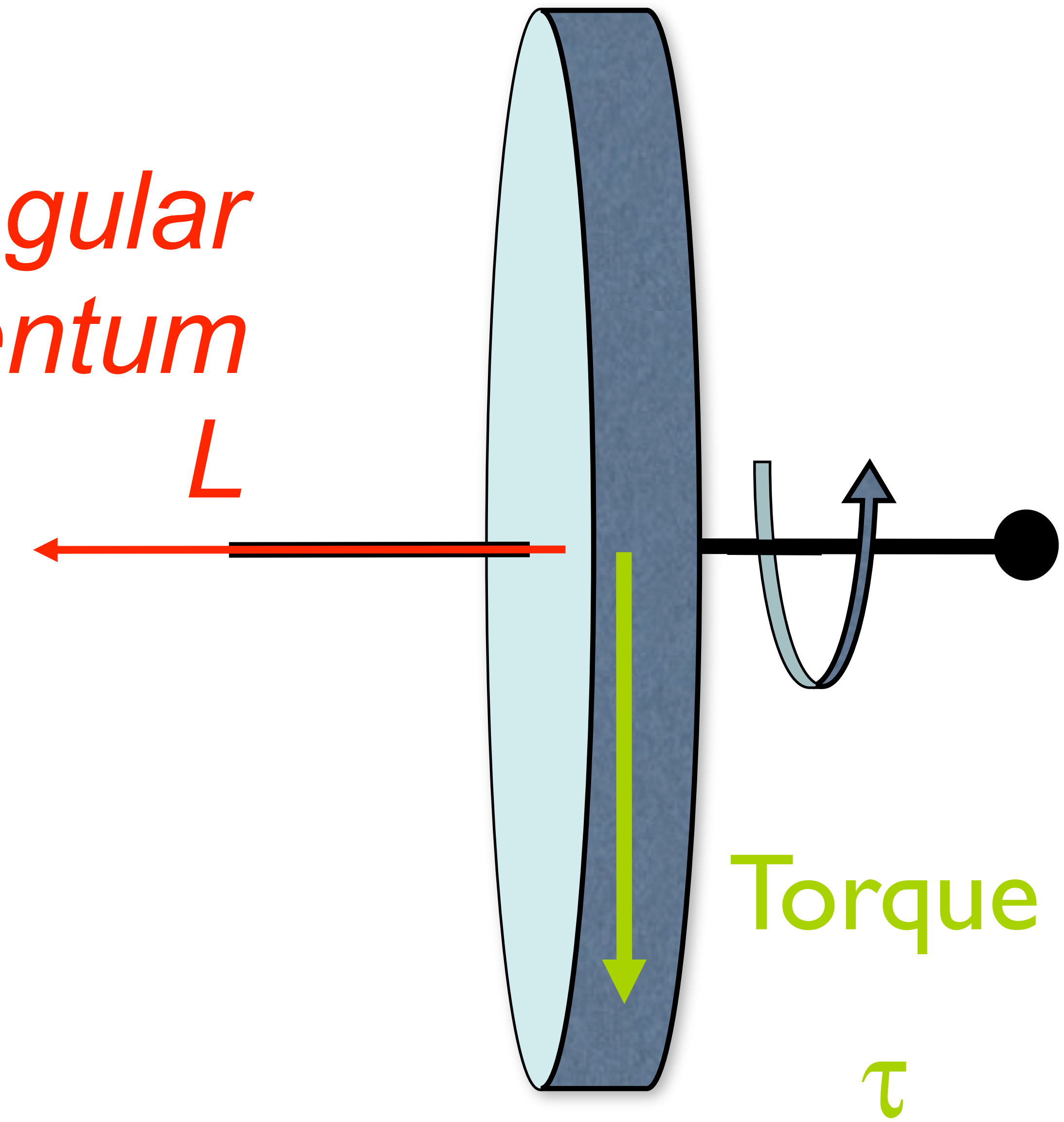
- The magnitude of the cross product is the magnitude of the first (d) times the magnitude of the second (mg) times sin(θ), which is 1 in this case.
- The direction comes from the right-hand rule.
- Start the four fingers at the first vector (r) then rotate to the second (F). This can only be done in the orientation shown - so the torque points out of the page towards us.

Add torque times time to L



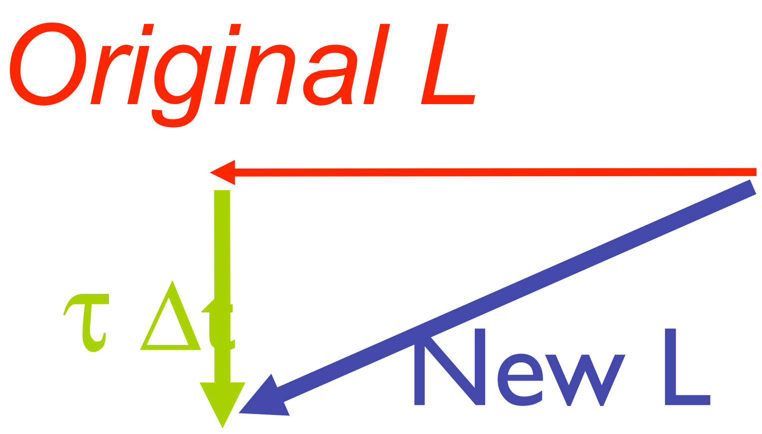
- So the angular momentum vector points left, and the torque points out of the page towards us.

Look down from above



- And remember that torque is rate of change of momentum.
- So to work out what the angular momentum will be in a short while Δt, add $\tau\Delta t$ to L,

Vector addition



- Do it the usual way.
- And we see that the axis of rotation rotates anticlockwise (as seen from above.

So this explains it

- This explains the weird behaviour of the spinning bike wheel.
- Or indeed anything spinning.
- When you try to rotate it perpendicular to the spin axis, it always moves off at right angles to both the push and the spin axis.

Riding a Bike…

- Always seems a very implausible activity…
- Why doesn't everyone just fall off?
- And why is a bike more stable when it is moving - preferably fast?

In which direction is the angular momentum of my front wheel?

- To my right? Left? Forwards? Backwards? Up? Down?

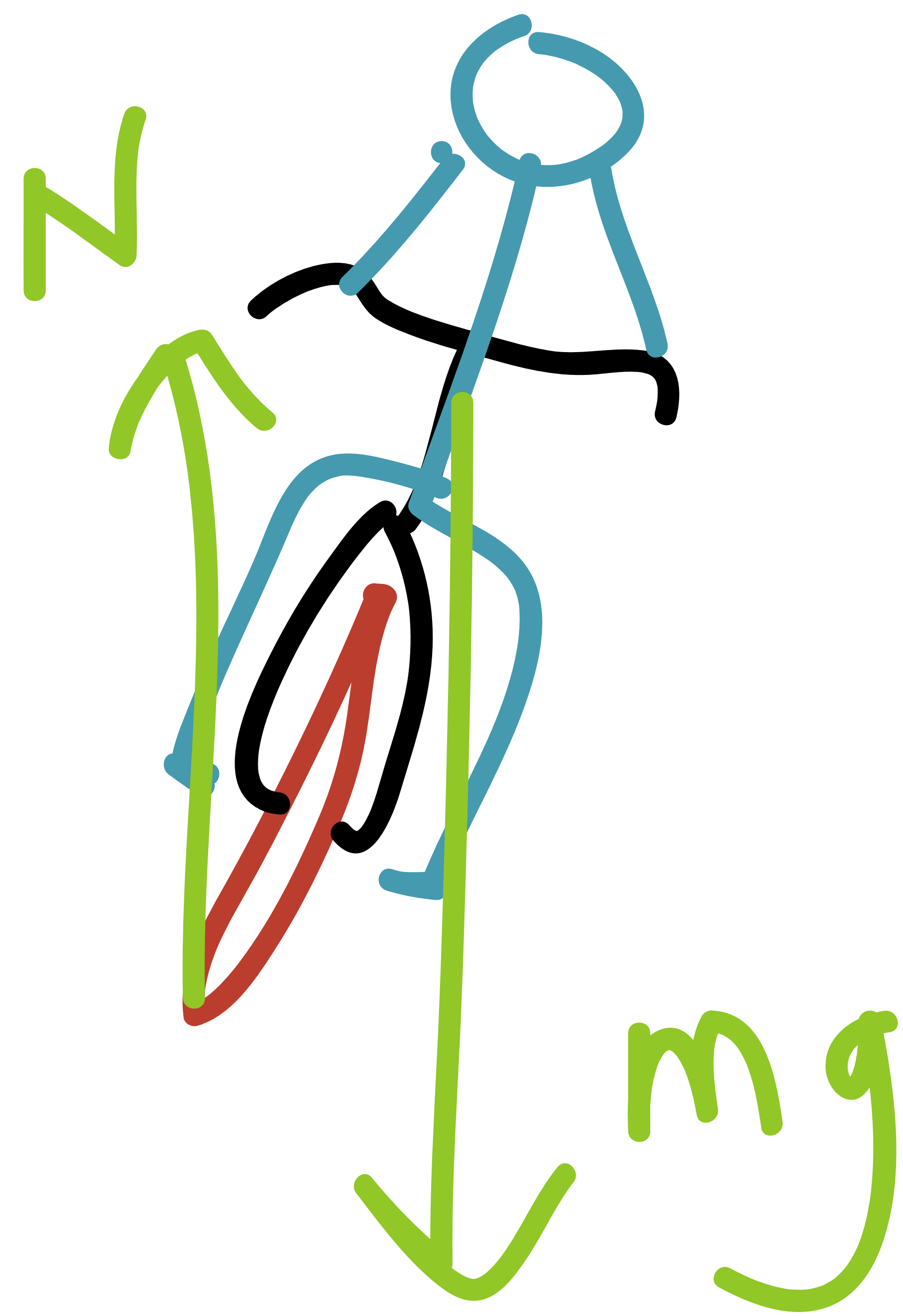
Think about the angular momentum of the front wheel

- It's a vector pointing to your left as you cycle along.

Now - imagine I'm falling off to the right...

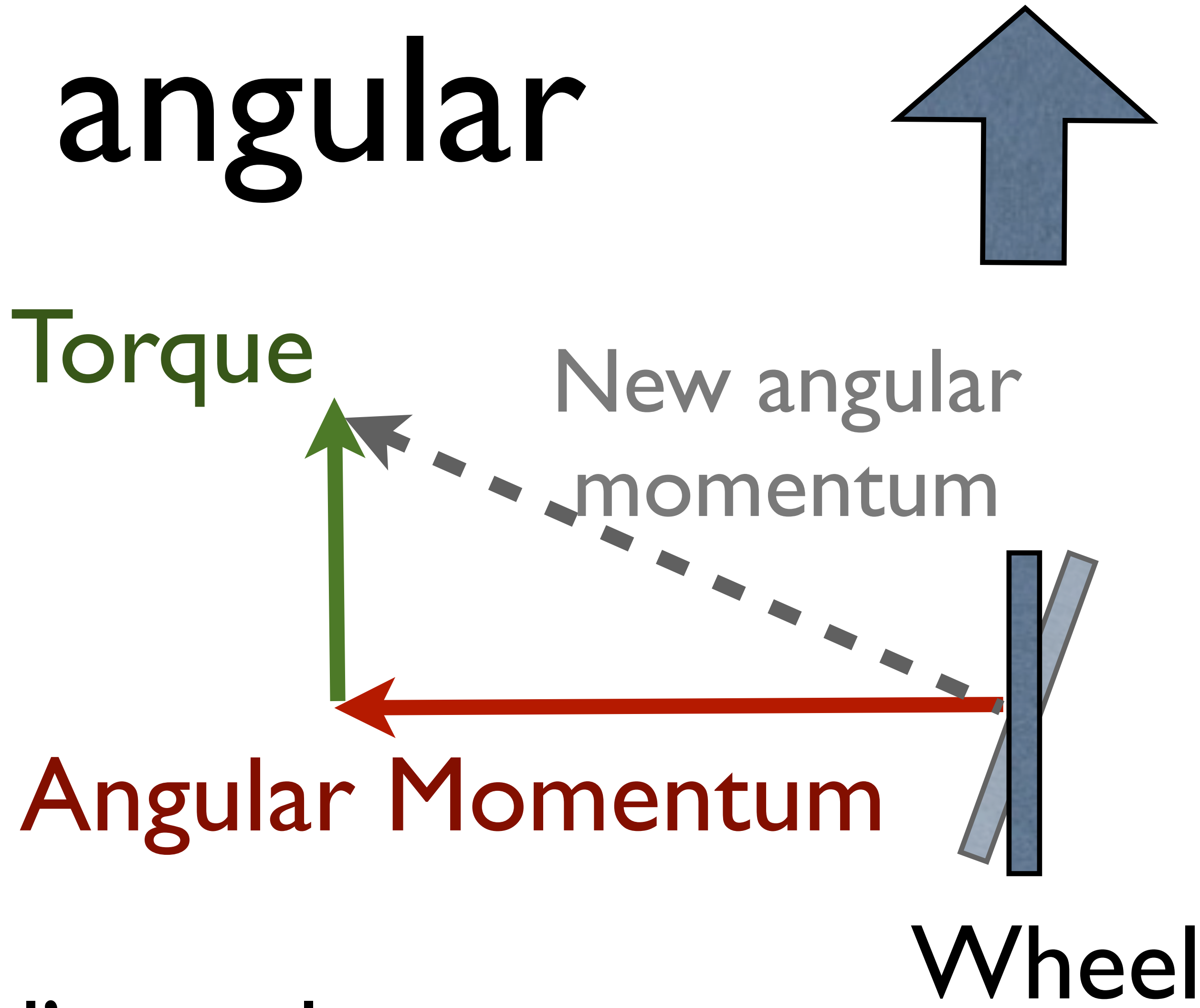
- In what direction is the torque applied by my weight and the normal force?

Use the right-hand rule



- Torque is forward.

Add torque to angular momentum



- Add this to the wheel's angular momentum - and it rotates the wheel to the right.
- Thus turning into the direction of fall and preventing the fall…
- Nice theory - not clear it works…

Angular momentum conserved.

- And it is a vector.
- So a spinning object tends to keep its axis in the same direction regardless of how it moves, unless a torque is applied to it.
- This means it can act as a sort of compass.
- Set a wheel spinning and it will hold its direction.

Gyrocompass - uses rotation to hold a direction

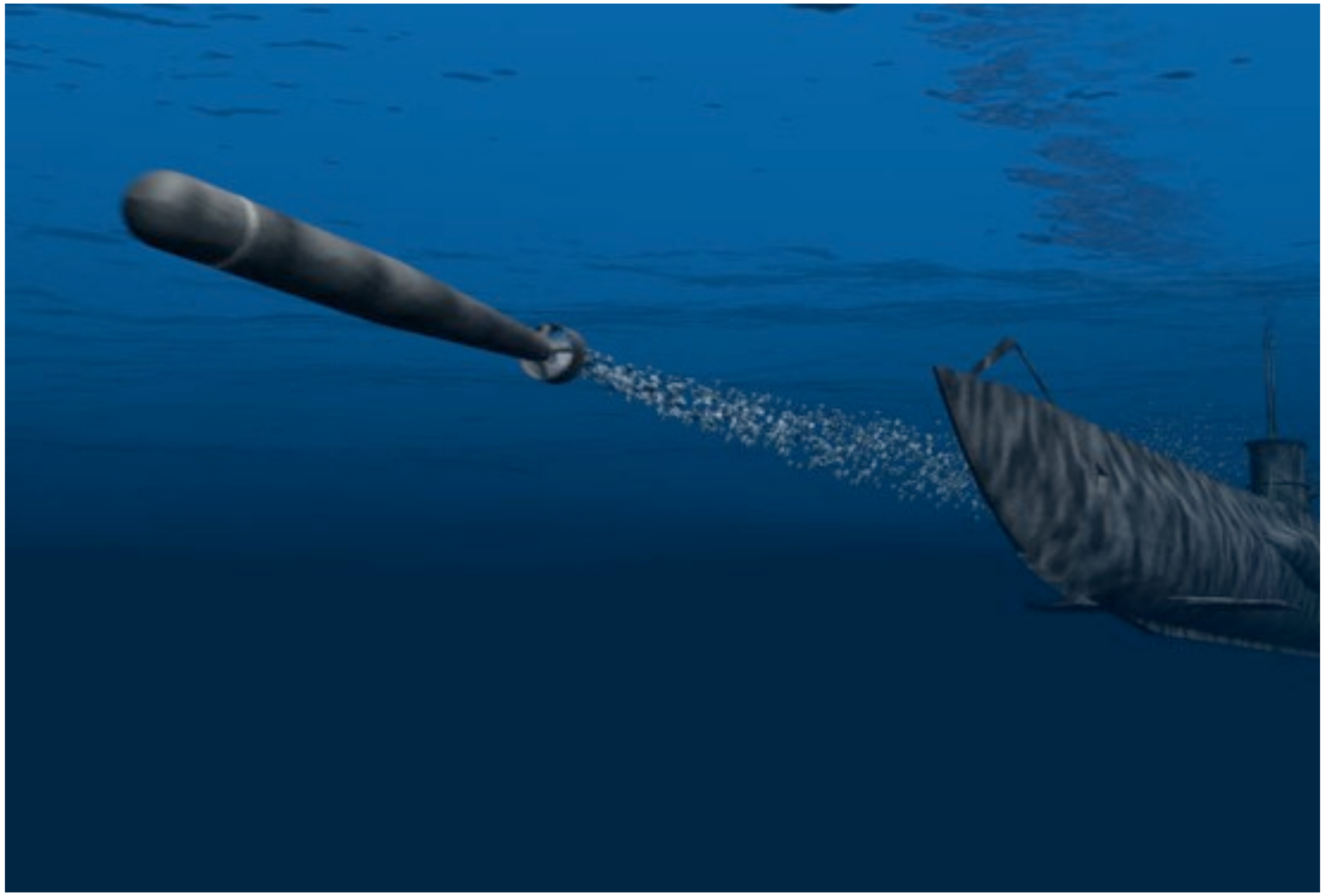
- More accurate than magnetic compasses.
- Not affected by metal in ships, and can work in space away from Earth's magnetic field.

Gyroscopic gun-sight

- Used to better orientate guns or bomb aiming systems.
- Used, for example, in battleship fire-control systems to keep track of horizontal despite rough seas.
- Very high precision was required - even in World War I, battles such as Jutland were conducted at ranges of over 20 km.

But what really decided both world wars…

- Torpedoes
- Which cannot work without gyroscopic navigation to keep them on track.



Used to orientate rockets and satellites.

- The Hubble Space Telescope uses six gyroscopes to determine exactly where it is pointing, to allow sharp images.

Can be used to change orientation

- By adjusting the speed or orientation of spinning wheels, you can rotate an object such as a space-probe.
- How? Overall angular momentum is conserved - so if you change the angular momentum of these wheels one way, an equal and opposite change is made to the rest of your object.

International space station has four "Control moment gyroscopes"

If it didn't have these…

- The very small torques applied to the space station by air drag would continuously build up over time, until it was spinning so fast it would fly to pieces.

Inertial Navigation Systems

- Combine gyroscopes with accelerometers to keep continuous track of what direction you are pointing and how you have moved.
- Very big military technology due to applications in ballistic missiles - this is the only form of navigation that cannot be jammed.

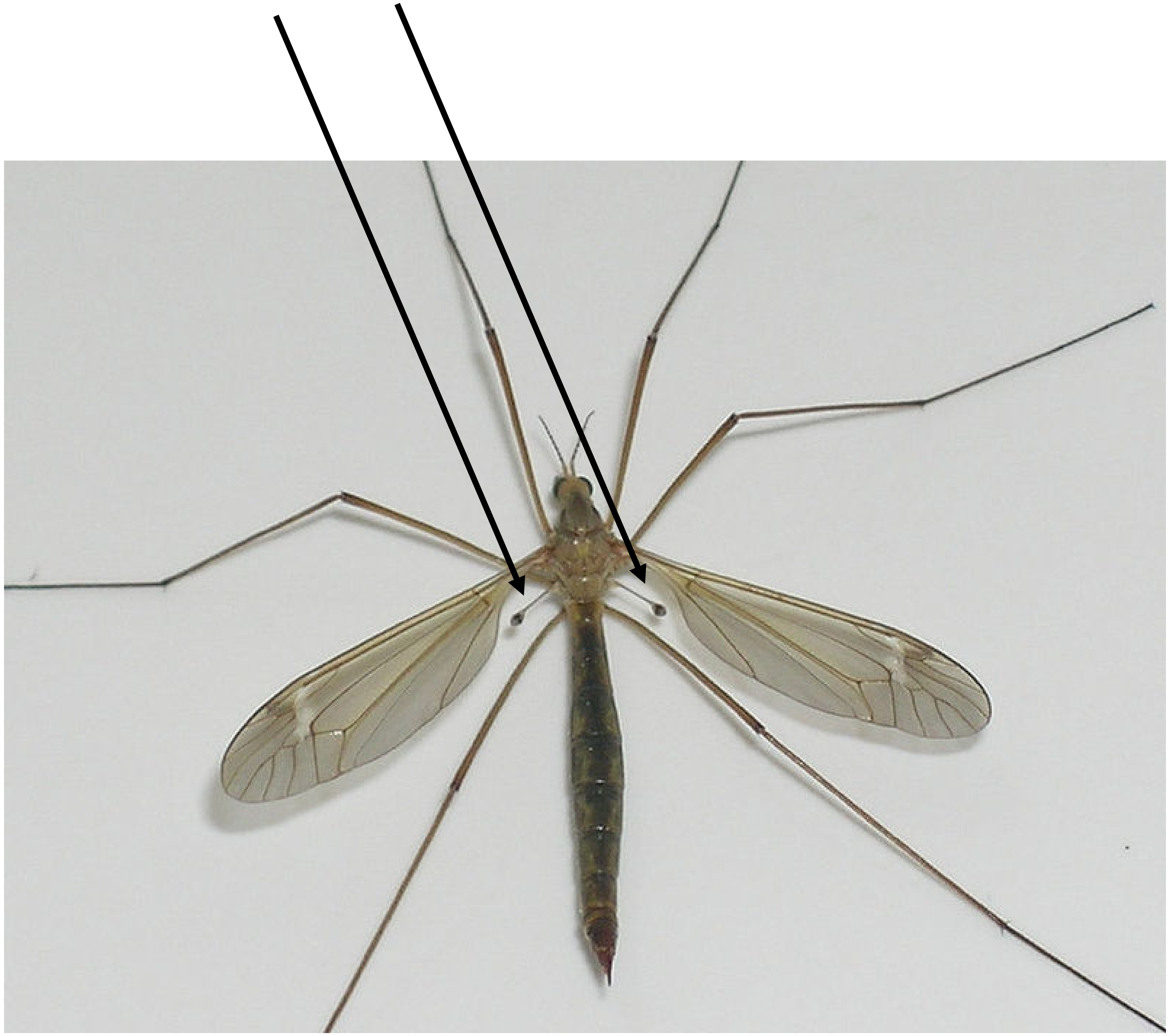

ICBM
Delivery in 30 minutes or less, or the next one's free.

State of the art - MEMS vibrating structure gyroscopes

- MEMS = micro-electro-mechanical System.
- Basically a machine etched into Silicon using the same techniques as chip manufacture.
- Not as accurate as rotating gyroscopes but can be made very small and light (etched onto the surface of a silicon chip, for example).
- Don't actually use angular momentum at all.
- The physical principle is very simple: a vibrating object tends to keep vibrating in the same plane as its support is rotated.
- Used in some game controllers.

Halteres

- Some insects use the same principle to help them navigate in the dark.
- Vibrating weights on little antennae…

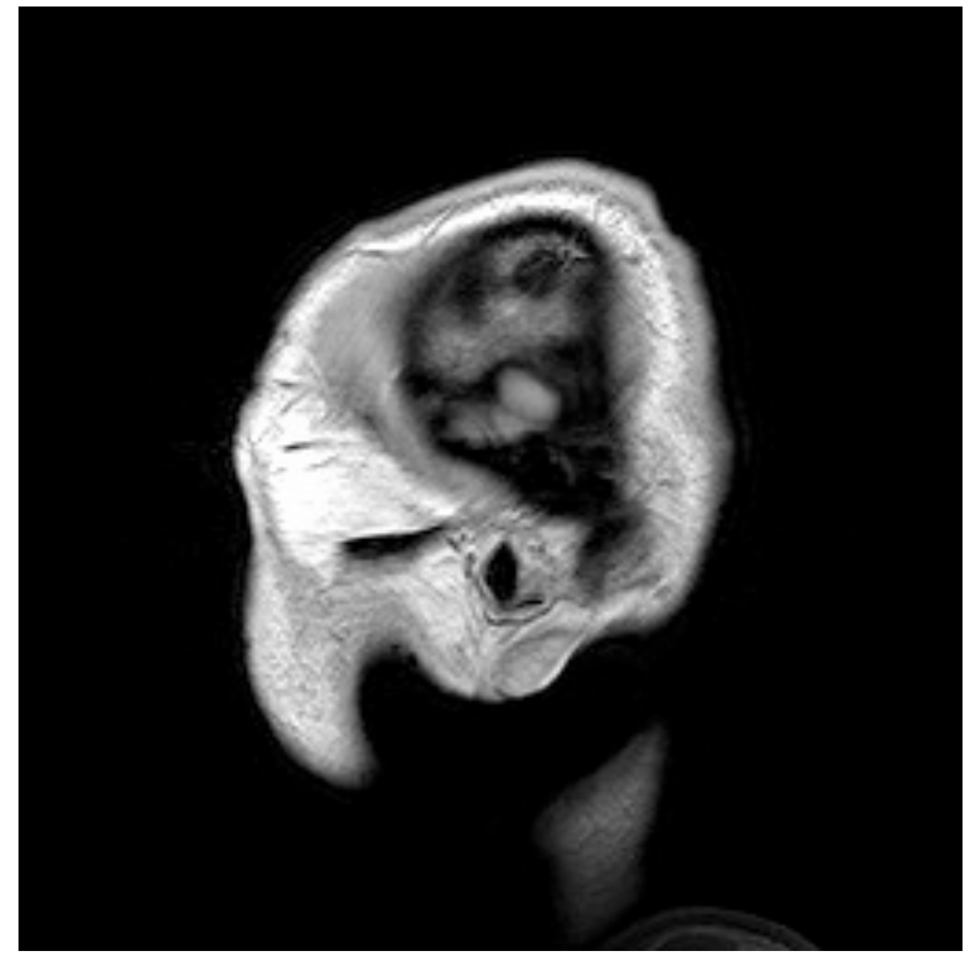


Precession VPython simulation

This is called "Precession"

- And has immense implications in all sorts of areas.

Magnetic Resonance Imaging

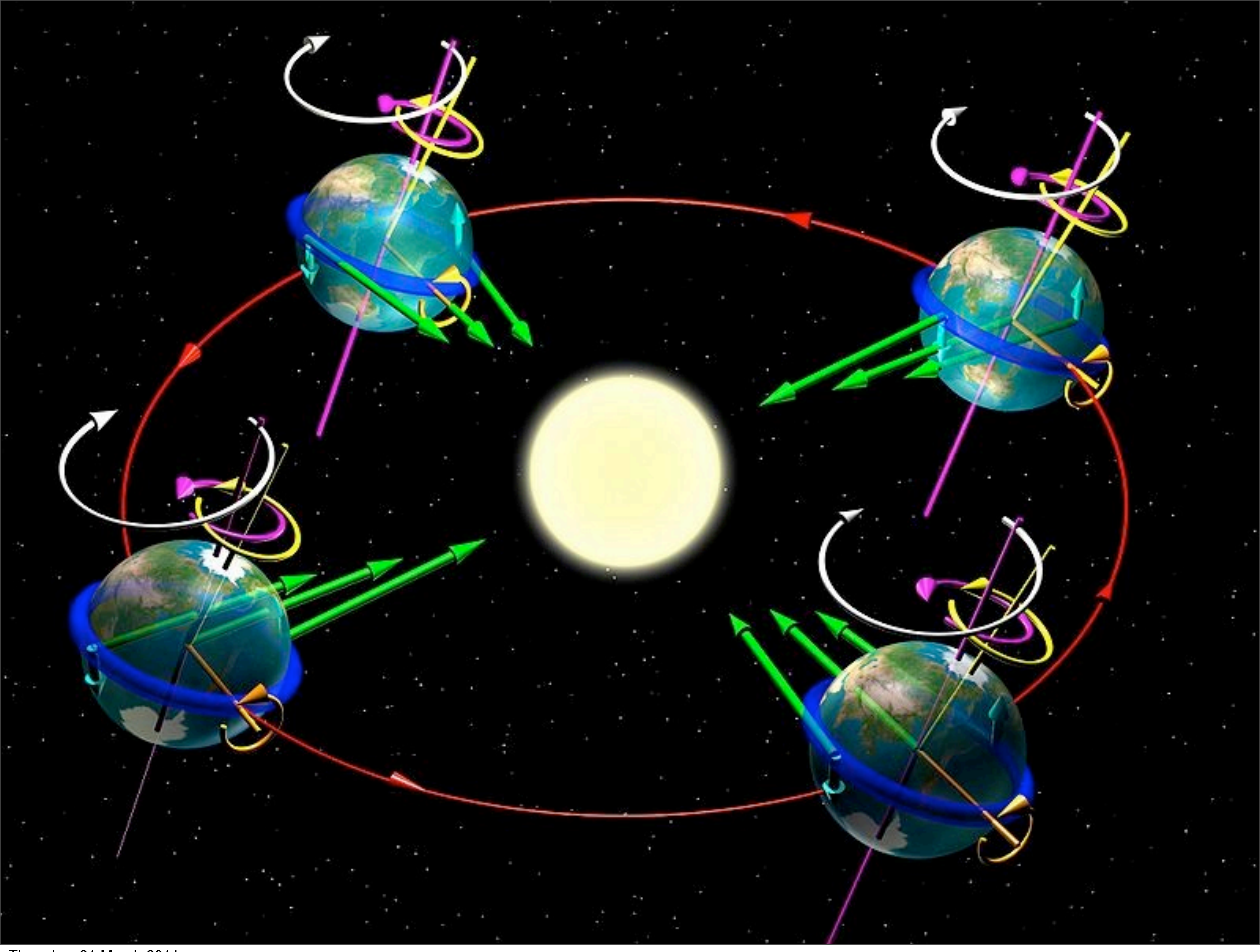


This works via precession

- Use a strong magnetic field to line up all the magnetism of all the hydrogen nuclei.
- Zap them with an electromagnetic pulse - hence applying a torque.
- The torque causes all the nuclei to precess, due to their angular momentum.
- This generates electromagnetic radiation we can measure and map.

Why horoscopes are wrong

- The Earth precesses.
- This is because it is not a sphere - the Earth bulges around the middle, due to its rotation (the equatorial diameter is 42 km greater than the polar diameter)
- The gravity of Sun and Moon acts on this bulge to cause a net torque.



This causes the axis to precess

- Which means that exactly what star is overhead at a given time changes.
- Thus pushing the constellations out of position.
- Most horoscopes refer to your star signs as defined in ancient Babylon - but the Earth has precessed since then and star signs are no out by months.

That's the end of Angular Momentum

- Key points - everything is an analogue of the normal force/momentum principle.
- Torques and angular momentum depend on the perpendicular distance.
- They are vectors, at right-angles, using the right-hand rule.