Energy, Work and Systems

Complications of Energy

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CPR News

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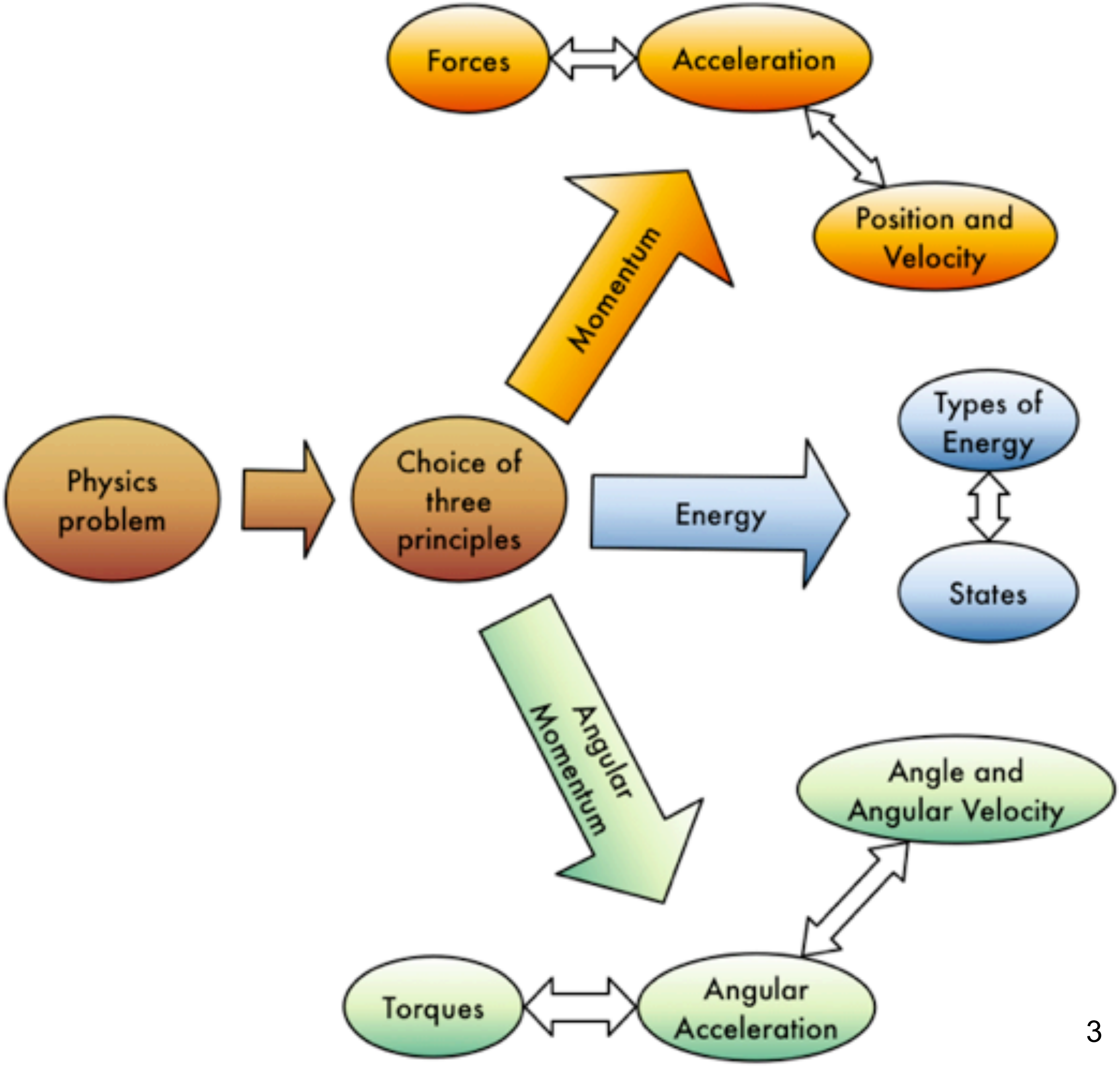
- It's my fault (well, Google's fault mostly but I didn't give clear enough instructions).
- Please assume that a good diagram was there when marking!
- A number of people have posted incorrect links - e-mail me if you get something to mark that you can't access.


Forces
Acceleration
Position and Velocity
Momentum
Physics problem
Choice of three principles
Energy
Types of Energy
States
Angular Momentum
Angle and Angular Velocity
Torques
Angular Acceleration

Back to Energy

- For weeks now we've been talking about force/momentum.
- Now lets go back to energy.
- We're now going to talk about "Work" done on a system, and thermal energy (Heat).

“Work”

- Be careful - we use this colloquially (“I had to work really hard on this week’s assignment”) but this word is used differently and very precisely in physics.
- Many things that you might think are hard work (carrying a heavy weight along a flat road, for example, or learning physics) do not involve physics work.

Two problems

You ride a sled down a snowy hill of height 100 m and slope 10°. How fast will you be going at the bottom?

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Energy

You ride a sled down a snowy hill of height 100 m and slope 10°. How fast will you be going at the bottom?

Most easily solved via energy. Initial energy (potential) turns into final energy (Kinetic).

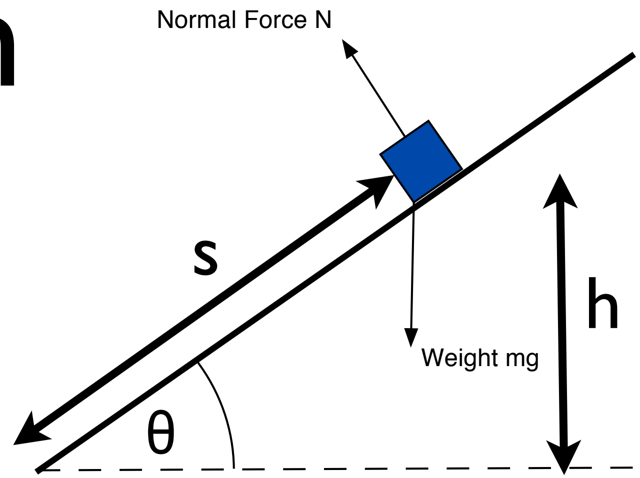
$$\frac{1}{2}mv^2 = mgh$$

so

$$v = \sqrt{2gh}$$

Done!

Force/Momentum



Could also be solved by force/ momentum

You ride a sled down a snowy hill of height 100 m and slope 10°. How fast will you be going at the bottom?

Choose axes along slope and perpendicular to it.
Force component downward along slope is mg sin(θ), so acceleration is g sin(θ).
Use $v^2 = u^2 + 2as$.

$$\sin\theta = \frac{h}{s}$$

u = 0 (starting from rest)

So $$v^2 = 2g\sin\theta\frac{h}{\sin\theta}$$

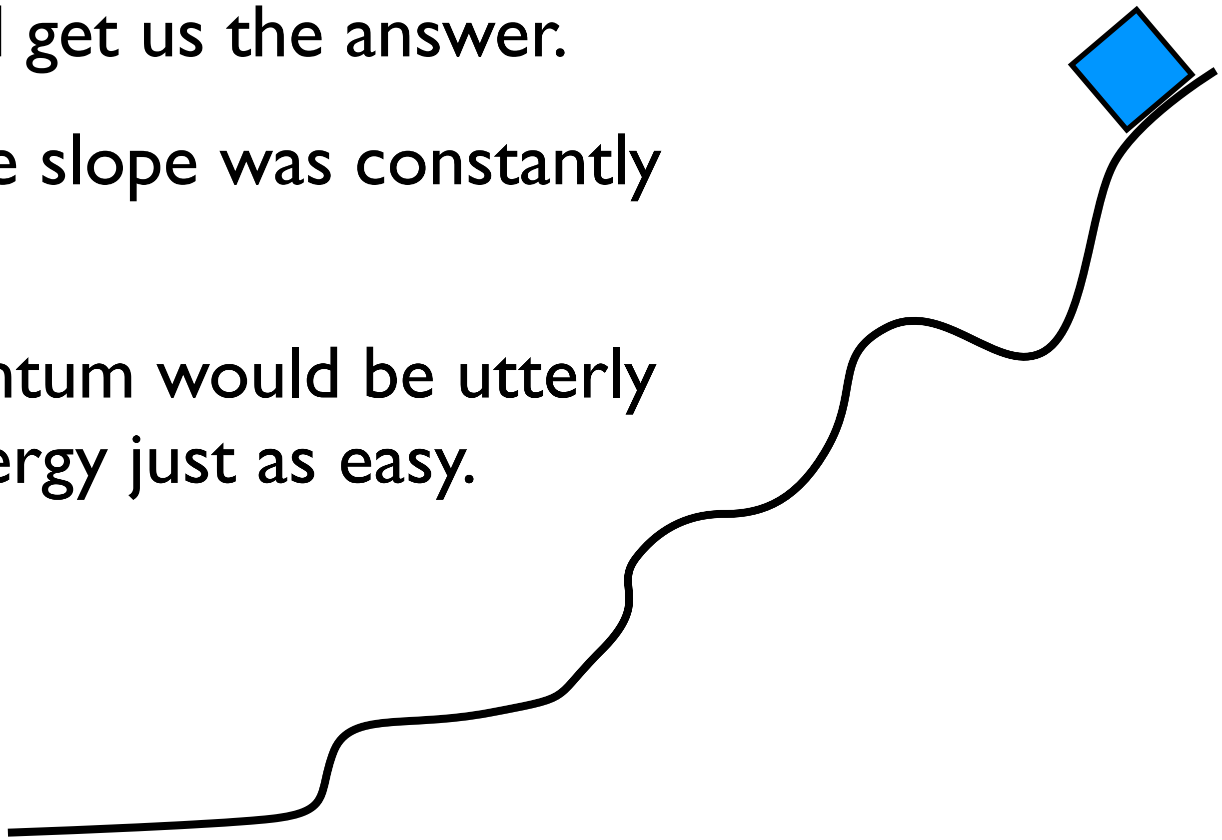
And rearranging...

$$v = \sqrt{2gh}$$

Same as before, but rather more work

And if we didn't know the slope

- we couldn't have used this way - only Energy would get us the answer.
- Similarly if the slope was constantly changing.
- Force/Momentum would be utterly horrible - Energy just as easy.



What about the Second type of question?

- You could solve this using the force/momentum principle.
- Just like the previous solutions, but with an additional friction force added.
- Wouldn't it be nice if we could use energy?

You ride a sled down a snowy hill of height 100 m and slope 10°. The snow applies a friction force of 100N to the sled. How fast will you be going at the bottom?

Why can't we use energy?

- Because of the friction - some energy is being turned into a form (heat) that we can't easily handle.
- But there is a way...

Use “Work”

- Helpful in any situation where you know two of the following and want to work out the remainder:
  - Energy
  - Force
  - Distance

Define a System

- Just as before, a system can be absolutely anything you like. Your nose, your whole body, your whole body plus Taronga Zoo, the Earth, The Solar System, etc...
- Is energy in any given system always constant?

No way!

- Energy can flow into our out of a system.
- One way is heat flow (we'll cover that later)
- Another way that energy can enter or leave a system is "Work"

Work Done

- The rule is that if you apply a net force to your system, it does an amount of work

$$W = \vec{F} . \vec{D}$$

where $\vec{F}$ is the (vector) force applied and $\vec{D}$ is the displacement during the time over which the force was applied.

Note - this is a dot product.

Dot product: Algebraic

- If $\vec{A} = (x_A, y_A, z_A)$
- and $\vec{B} = (x_B, y_B, z_B)$
- Then $\vec{A}.\vec{B} = x_A x_B + y_A y_B + z_A z_B$

- i.e. a scalar, made up by multiplying together the x, y and z components and adding up the results.

Dot Product: Geometric

$$\vec{A}.\vec{B} = \left|\vec{A}\right|\left|\vec{B}\right| Cos(\theta)$$

- i.e. multiply the magnitudes of the two vectors together and multiply by the cosine of the angle between them.

So the work done…

- is the dot product of the force applied and the distance moved.
- Another way to think of it is the force times the component of the distance moved along the direction of the force.

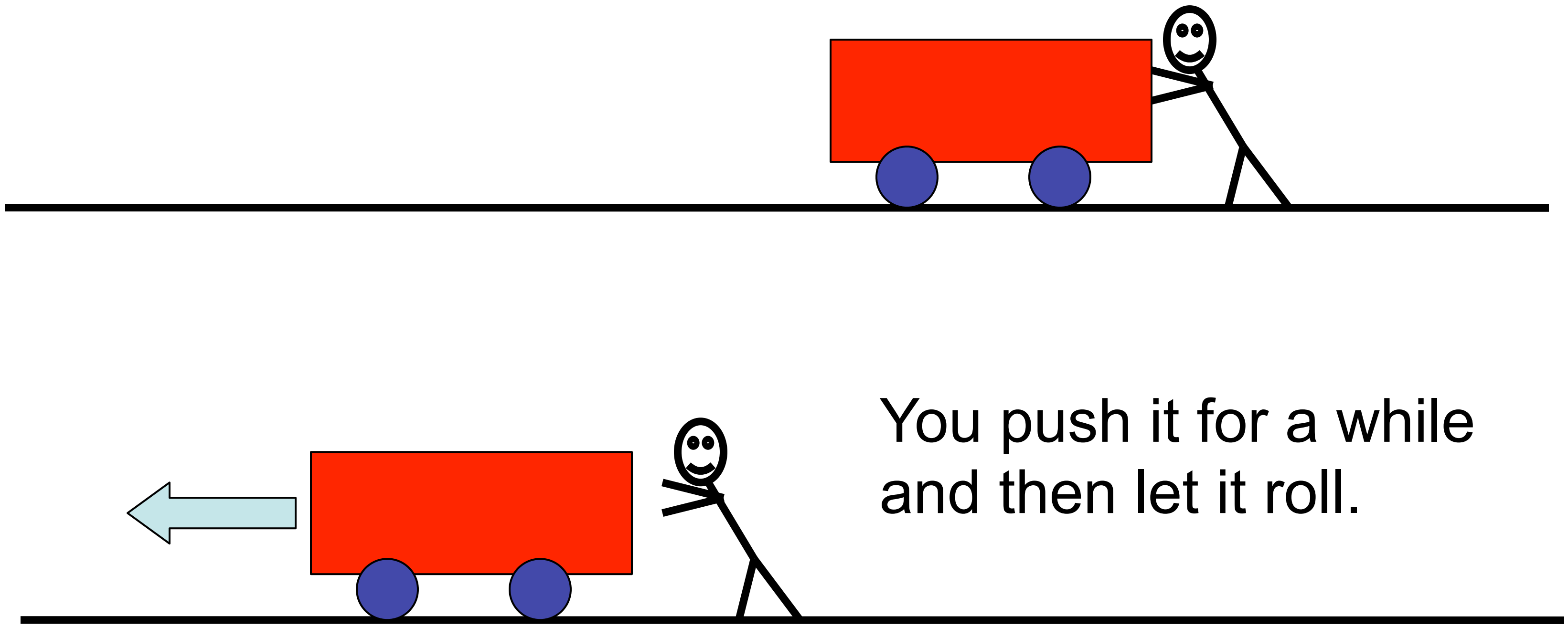
So...

- Pick a system
- If no work is done ON THAT SYSTEM, the energy in that system remains constant. It can change from one form to another (e.g. spring energy to kinetic energy) but the total amount doesn't change.
- If work is done on that system, the total energy in the system changes by that amount.

Let's look at some examples

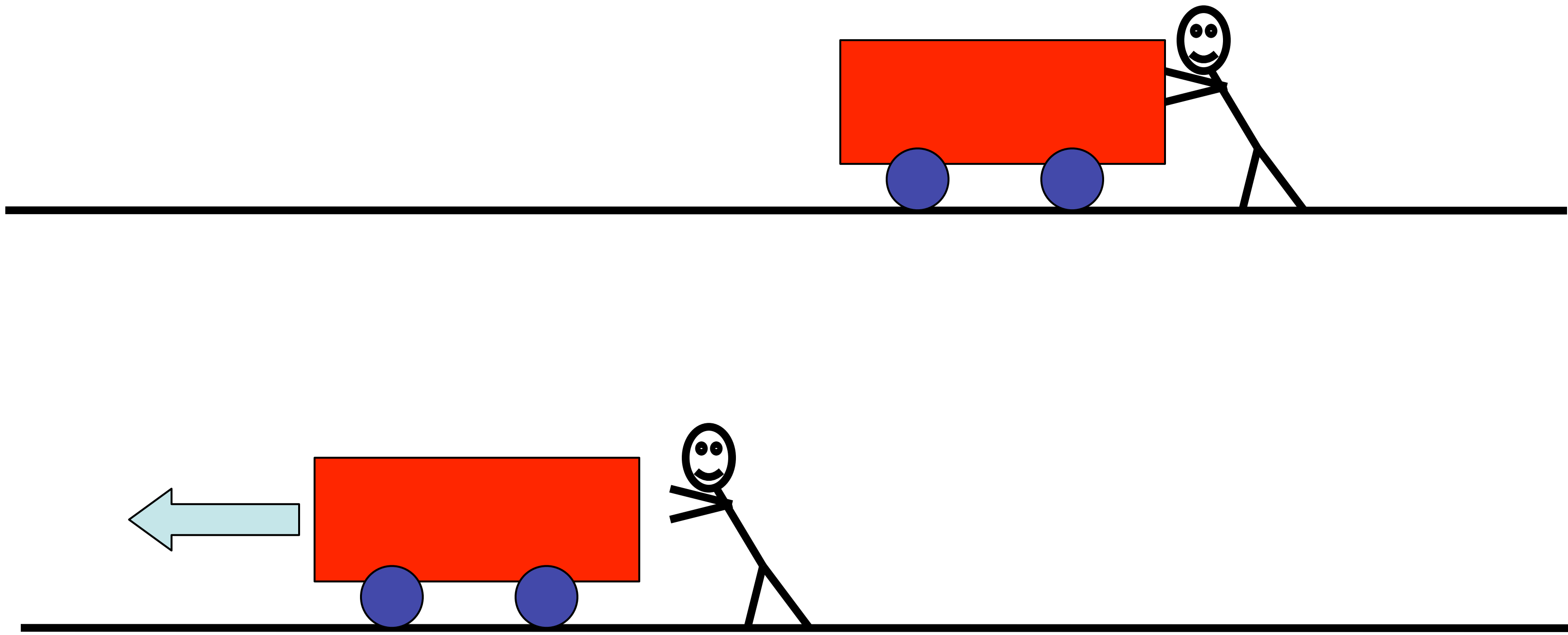
- You have been sent to work in a mine.
- You need to push a 1000 kg cart of iron ore along a flat track.
- You push it for 100m, applying a steady force of 200 N.
- How fast does it end up going?

Push a cart along a track

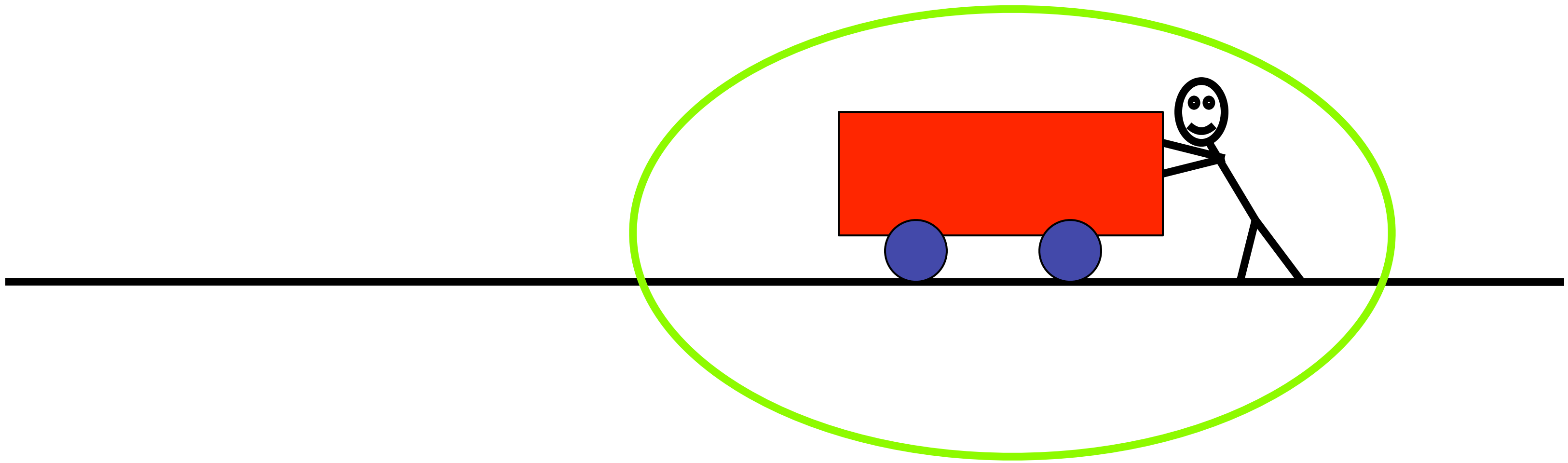


You push it for a while and then let it roll.

First, pick your system

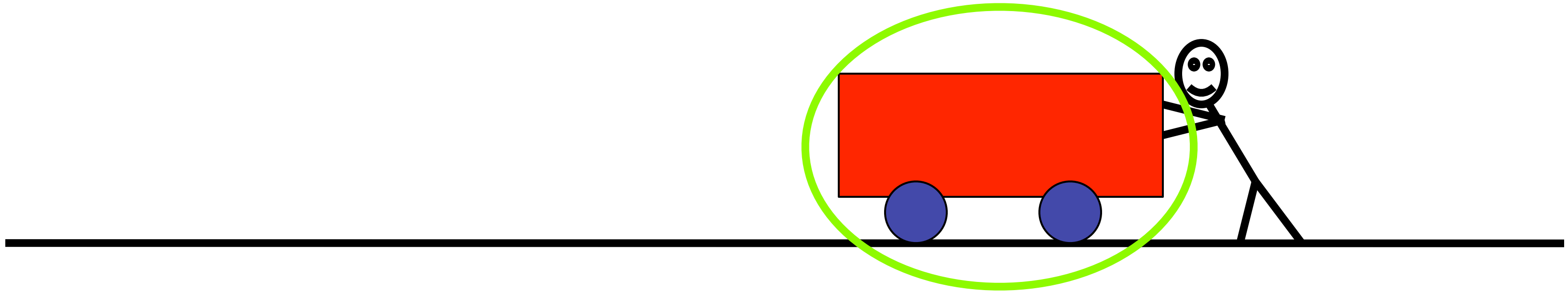


You, ground and cart?



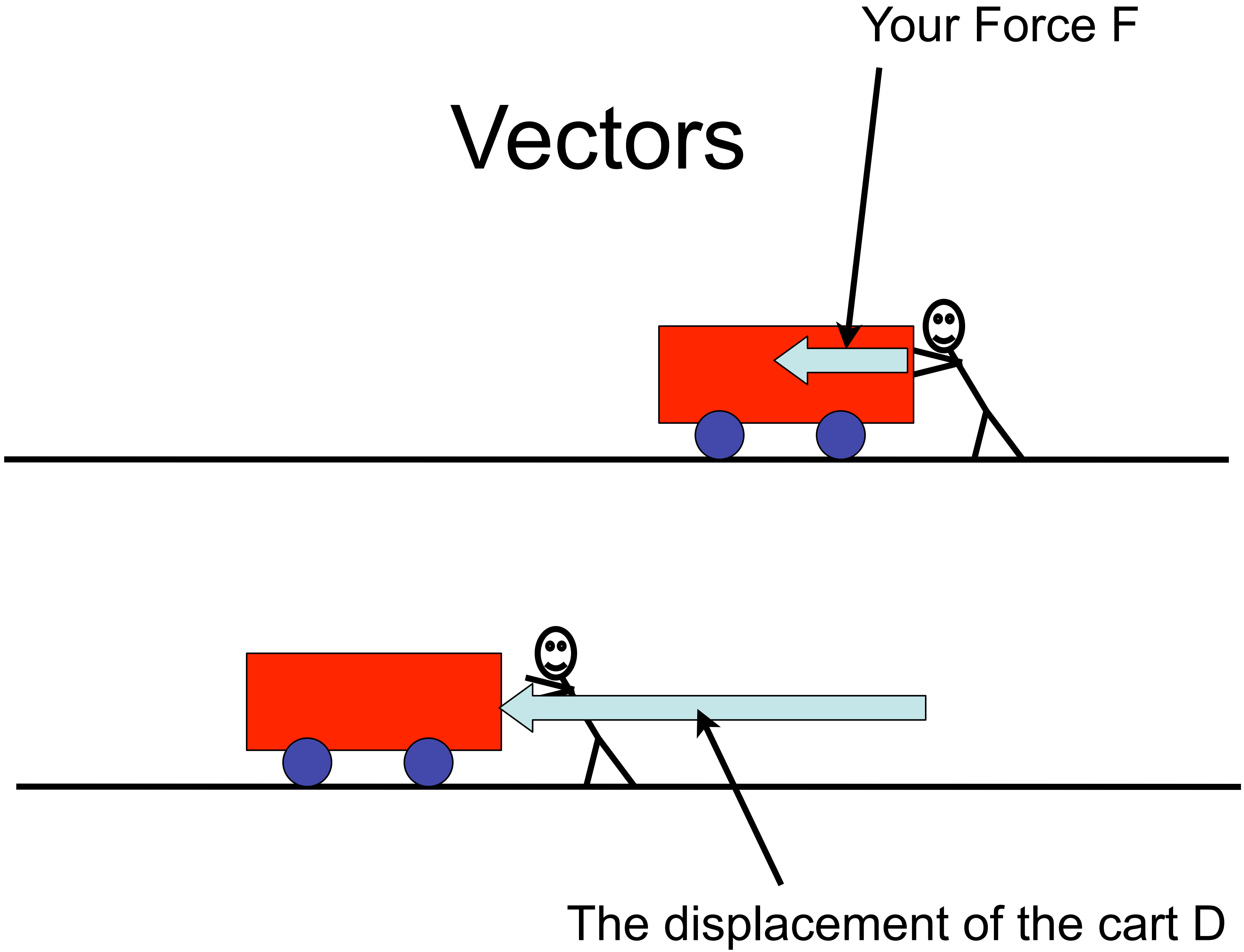
In this case, there are no external forces acting on the system.
So no work is being done on it.
So the energy inside your system is constant.
All that's happening is that chemical energy in your muscles is turning into kinetic energy in the cart.

Just the cart?



In this case, there is an external force being applied! Your push!
So you are doing work on the system (the cart) and the total energy of the cart must therefore increase (it moves faster).

Vectors



Same direction!

- F and D are in the same direction, so the work done is just W = F D.
- This must be equal to the increase in energy (Kinetic in this case) so

$$\frac{1}{2}mv^2 = FD$$

$$v = \sqrt{\frac{2FD}{m}} = 6.3ms^{-1}$$

Very simple

- Solved in just two easy lines.

Rocket

- A space-probe is floating stationary in space. A rocket onboard fires, exerting a force of 100 N for 10 seconds.
- At the end of this time, the space-probe has moved 5 metres.
- How much work was done to the space-probe?

1. 100 J
2. 105 J
3. 110 J
4. 500 J
5. 1,000 J
6. None of the above
7. Not enough information provided.

500 J

- Force and displacement are in the same direction.
- So just multiply the magnitudes together.
- You could use this to work out the speed at the end. Work done on the space-probe is converted into its kinetic energy, so

$$\vec{F}.\vec{D} = 500J = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2.500}{m}}$$

Back to our original problem

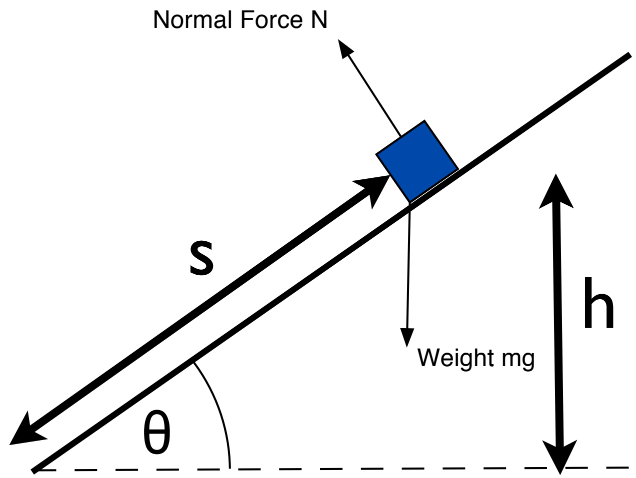
- How can we solve this using energy?
- Pick our system as skier.
- SNow does work on our system of force times distance.
- As force and distance are in opposite directions, the work is negative.

You ride a sled down a snowy hill of height 100 m and slope 10°. The snow applies a friction force of 100N to the sled. How fast will you be going at the bottom?

Work

Work is friction force F times distance s.

$$\sin\theta = \frac{h}{s}$$

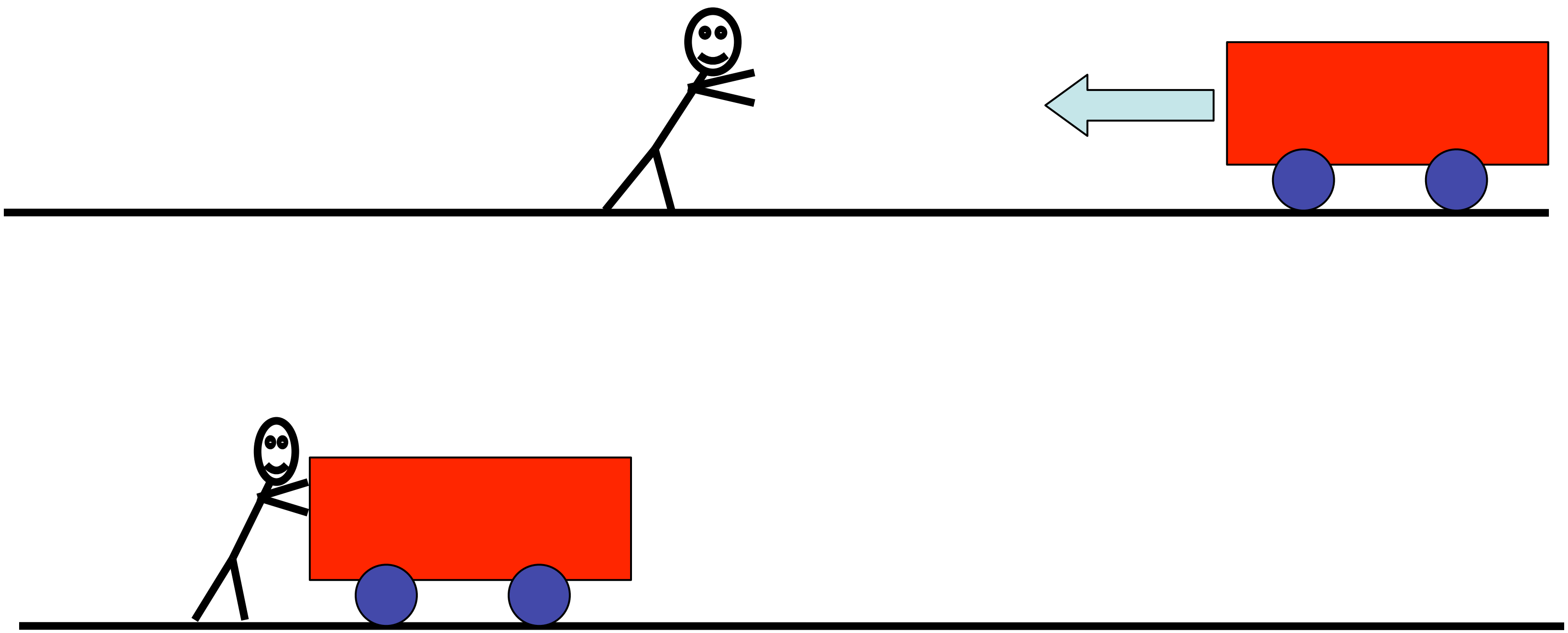


So write down energy balance:

$$mgh - F\frac{h}{\sin\theta} = \frac{1}{2}mv^2$$

$$v = \sqrt{2h\left(g - \frac{F}{m\sin\theta}\right)}$$

But now let's try it reversed.



A cart is rolling towards someone, who pushes against the motion and slows it to a halt.

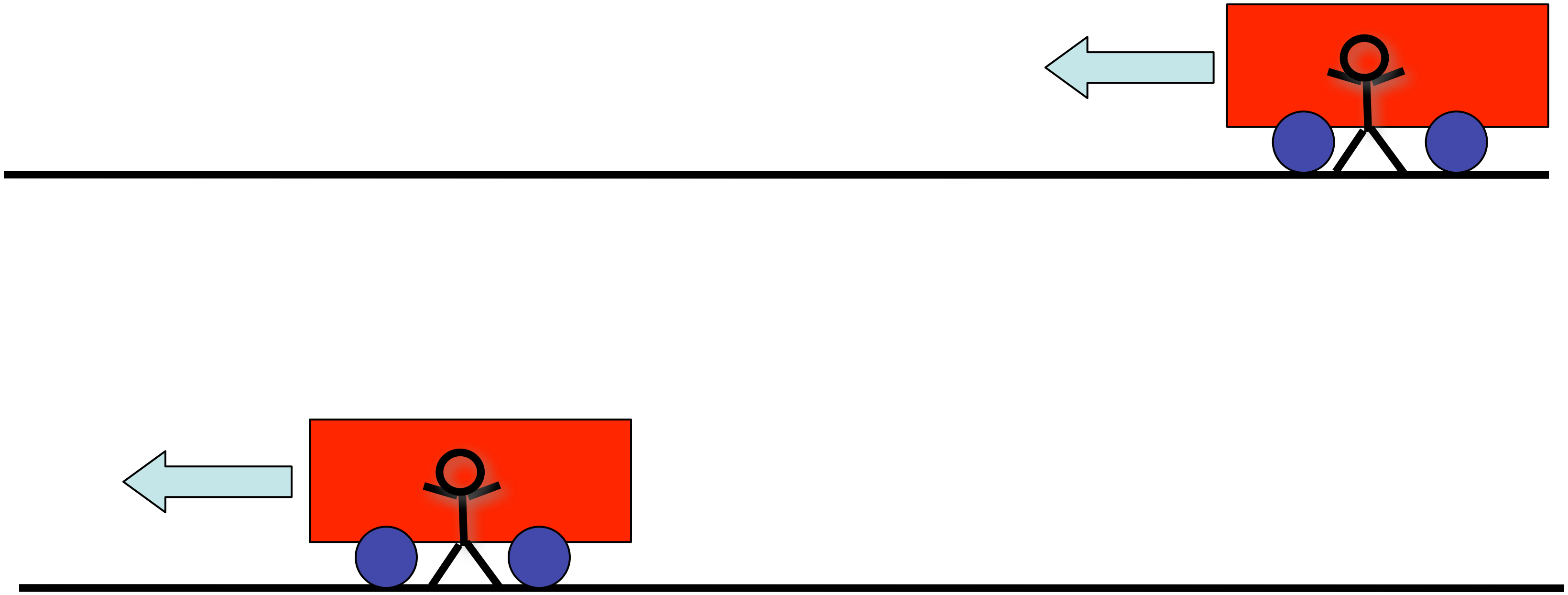
How much work was done now?

1. Same as would be needed to speed it up to the same speed
2. More than the same amount
3. Less than the same amount but still positive
4. Zero.
5. Minus the same amount needed to speed it up.
6. None of the above

Negative work

- Work - being a scalar, can be negative.
- In this case, the force and the displacement were in opposite directions.
- So the dot product was negative.
- How can you do negative work? You are removing energy from the cart.
- The cart is slowing down, so energy must be going somewhere.
- It is leaving the system and going into you - most likely heating you up.

Sideways push



A cart rolls across at a uniform speed - you push it sideways while running alongside.

How much work do you do?

1. The same positive amount as when speeding it up.
2. A smaller positive amount
3. Zero
4. A small negative amount.
5. The same negative amount as when slowing it down.
6. None of the above.

Zero

- Force and displacement are at right-angles, so no work is done ($Cos(\theta) = 0$)

Stationary cart?

- What if the cart were stuck?
- You push and push but nothing moves.
- According to the equation, you are doing no work.
- Does this make sense?

It feels like you are working

- But if you lean something against the side of the stuck cart, it applies a force to the cart, but nobody thinks that it is doing work.

Is the chair doing work on me?

But would the (lower) me be doing work in this situation?

It feels like a lot of work applying a large force while not moving.
But according to the definition, no work is done.

This is actually a peculiarity of our muscles.
They consume energy (or to be precise turn it into heat) even when pushing a stationary object.
Chairs, bones, springs and almost everything else, however, can apply forces for ever without needing energy.

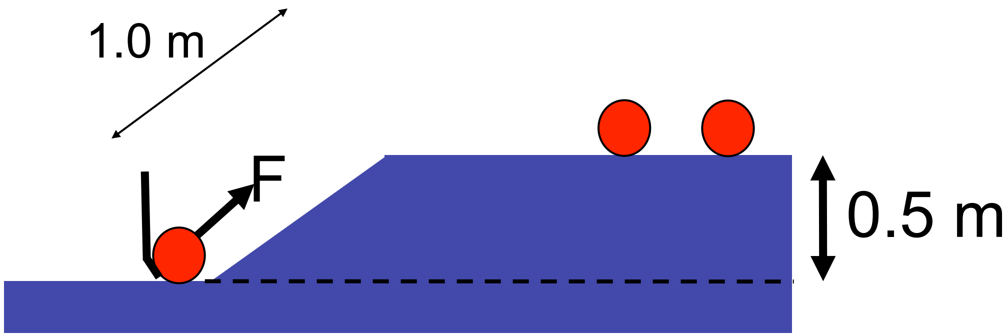
Pushing

- So if you push something, you only do work if it moves in the same direction while you are pushing.
- If it moves the opposite way, the work is negative.
- And pushing something that doesn't move does no work.
- Though it can still feel hard if your muscles are inefficient.

Look at energy difference!

- Another way to think of it.
- As a result of your push, does the cart gain energy? If so, you've done (positive) work on it.
- As a result of your push, does the cart lose energy? If so, you've done negative work on it.

Bowling



- At the bowling alley, the ball-feeder mechanism must exert a force to push the bowling balls up a 1.0-m long ramp. The ramp leads the balls to a chute 0.5m above the base of the ramp. Approximately how much force must be exerted on a 5.0 kg bowling ball?

1. 200 N
2. 50 N
3. 25 N
4. 5.0 N
5. None of the above
6. Impossible to determine

25 Newtons

- The ball has gained energy (potential energy in this case).
- This must have come from the work done on it.
- The work done must equal the gain in potential energy mgh = 25 J
- As the distance is 1 m, the force must be 25 N.

When do you need this?

- Whenever you know two of energy change, displacement and force, and want to work out the third.
- The energy change may be a speed change (kinetic energy), a height change (potential energy) or some other form of energy (rotational, spring, thermal etc).

If you know force and time…

- Then you probably should probably calculate the impulse (force times time) which gives you the change in momentum, instead of the work.

Double Counting

- One tricky thing that causes lots of errors - knowing when to use potential energy and when to use work.
- For example, say you throw a ball 10 m up into the air.
- How fast must you have thrown it?
- You can do this three ways…

Method 1: Momentum

- Gravity is applying a downward force mg to the ball.
- This is the rate of change of momentum.
- So in time t, the momentum will be reduced by mgt.
- When this equals the initial momentum (mv) the ball must have stopped.
- So mv = mgt, hence t=v/g

Method 1 continued

- This gives you the time needed to reach the top of its arc.
- But not the height. You'll have to integrate to get the height (or use $s=vt+0.5at^2$ which is the result of such an integration)
- $h = v(v/g) -0.5\ g(v/g)^2 = 0.5\ v^2/g$
- So $v = \sqrt{2gh}$

Method 2: Potential Energy

- Gain in potential energy = mgh
- This comes from the initial kinetic energy 1/2 mv^2
- so

$$\frac{1}{2}mv^2 = mgh$$

$$v^2 = 2gh$$

$$v = \sqrt{2gh}$$

Method 3: Work

- To push the weight upwards, you must apply a force mg (i.e. the weight).
- Work done is force times distance; i.e. -mgh. Note that the force is in the opposite direction to the motion so the work done is negative.
- When this negative work equals the initial kinetic energy, no energy is left in the ball so it stops, so $1/2\ mv^2 = mgh$ - as before.

Tricky Bit - difference between methods 2 and 3.

- Why didn't we factor the potential energy into method 3?
- Or factor the work done into Method 2?

System…

- The rule is - potential energy is internal to a system.
- Work is external to a system.

Consider a ball being thrown up

Pick the ball as your system

Is there an external force acting on this system?

1) Yes
2) No
3) Not enough information

Yes

Pick the ball as your system

The gravity from the Earth acts on this system.

So as the ball moves, the gravity does work on the system.

Now pick the Earth plus ball as your system.

Is there an external force?

Yes - gravity from the Sun, Moon, Galaxy etc acts on both.

But if we ignore these effects (you don't normally take tides into effect when calculating a ball's motion) there are no external forces.

So where does the kinetic energy go when the ball flies up?

Into potential energy.

Similarly in other situations

- For example a spring pushing a mass - if the spring is part of the system you treat it as gaining or losing potential energy.
- If you just pick the mass as your system, then force times distance from the spring does work on it.

Same answer

- You will always get the same answer whatever system you pick.
- You just need to be careful that you don’t double count. Always be sure what is and is not in your system. Things inside the system have potential energy. Things outside do work. Nothing does both.