A Logical Problem Solving Strategy

Introduction

At one level, problem solving is just that, solving problems. Presented with a problem you try to solve it. If you have seen the problem before and you already know its solution, you can solve the problem by recall. Much of the time, however, you have never experienced this situation before (if you had, you would not call it a problem). Solving real problems involves making a logical chain of decisions which lead from an unclear situation to a solution. Solving physics problems is not very different from solving any kind of problem. In your professional life, you will encounter new and complex problems (after all, if your employers knew how to solve these problems, why would they pay you?). The skillful problem solver is able to invent good solutions for these new problem situations. But how does the skillful problem solver create a solution to a new problem? And how do you learn to be a more skillful problem solver?

A Logical Problem-Solving Strategy

The problem-solving strategy presented here is based on research done in a variety of disciplines such as physics, medical diagnosis, engineering, project design and computer programming. There are many similarities in the way experts in these disciples solve problems. The most important result is that experts follow a *general strategy* for solving all complex problems. That is, experts solve *real* problems in several steps. Getting started is the most difficult step. In the first and most important step, you must accurately *visualize the situation*, *identify the actual problem*, and *identify information relevant to the problem*. At first you must deal primarily with the qualitative aspects of the situation. You must interpret the problem in light of your own knowledge and experience. This enables you to decide what information is important, what information can be ignored, and what additional information may be needed, even though it was not explicitly provided. In this step drawing a useful picture of the problem situation is crucial to getting started correctly. A picture is worth a thousand words (if it is the right picture).

In the second step, you must *represent the problem* in terms of formal concepts and principles, whether these are concepts of engineering design, concepts of medicine, or concepts of physics. These formal concepts and principles use the accumulated knowledge of your field and thus enable you to simplify a complex problem to its essential parts. Frequently, your field has developed a formalized way to diagram the situation which helps show how the concepts are usually applied to a problem. Third, you must use your representation of the problem to *plan a solution*. Planning results in an outline of the logical steps required to obtain a solution. In many cases the logical steps are conveniently expressed as mathematics. Fourth, you must determine a solution by actually *executing* the logical steps outlined in your plan. Finally, you must *evaluate* how well the solution resolves the original problem.

The general strategy can be summarized in terms of five steps.:

(1) Comprehend the problem.

(2) Represent the problem in formal terms.

(3) Plan a solution.

(4) Execute the plan.

(5) Interpret and evaluate the solution.

The strategy begins with the qualitative aspects of a problem and progresses toward the quantitative aspects of a problem. Each step uses information gathered in the previous step to translate the problem into more quantitative terms and to clarify the decisions which you must make. These steps should make sense to you. You have probably used a similar strategy, without thinking about it, when you have solved problems before.

The Importance of Writing

Solving a problem requires that you constantly make decisions. This is very difficult to do if you must also remember many pieces of information and the relationships between those pieces of information. Soon you overload your brain which has only a small number of short term memory locations. You could forget important parts of the problem or the steps in a mathematical procedure. The chain of decisions you construct may even have logical flaws. Drawing pictures and diagrams and writing your procedures using words, symbols, and mathematics makes the paper a part of your extended memory. Your brain is then free to deal with the decision-making process. The single biggest mistake of novice problem solvers is not writing down enough in a form which is organized to be a useful aid to their memory. If you have had the experience of understanding how to solve a problem when someone shows you how but "getting lost" when you try to do a similar problem yourself, the effective use of writing could be your primary trouble.

A Physics-Specific Strategy

Each profession has its own specialized knowledge and patterns of thought. The knowledge and thought processes that you use in each of the steps will depend on the discipline in which you operate. Taking into account the specific nature of physics, we choose to label and interpret the five steps of the general problem solving strategy as follows:

1. **Focus the Problem**: In this step you develop a qualitative description of the problem. First, visualize the events described in the problem using a sketch. Write down a simple statement of what you want to find out. Write down the physics ideas which might be useful in the problem and describe the approach you will use. When you finish this step, you should never have to refer to the problem statement again.

2. **Describe the Physics**: In this step you use your qualitative understanding of the problem to prepare for a quantitative solution. First, simplify the problem situation by describing it with a diagram in terms of simple physical objects and essential physical quantities. Restate what you want to find by naming specific mathematical quantities. Using the physics ideas assembled in step 1, write down equations which specify how these physical quantities are related according to the principles of physics or mathematics. The results of this step contains all of the relevant information so you should not need to refer to step 1 again.

3. **Plan the Solution**: In this step you translate the physics description into a set of equations which represent the problem mathematically by using the equations assembled in step 2. Each equation should have a specific goal to find a single unknown quantity in the problem. An equation thus used may involve a new unknown quantity which must be determined using another equation. In other words, solving the original problem usually involves creating and solving sub-problems. As you do the mathematical operations to isolate your unknown quantities, you create an outline of how to arrive at a solution. You will find that most of your effort will go into deciding how to construct this logical chain of equations with less effort spent on mathematical operations.

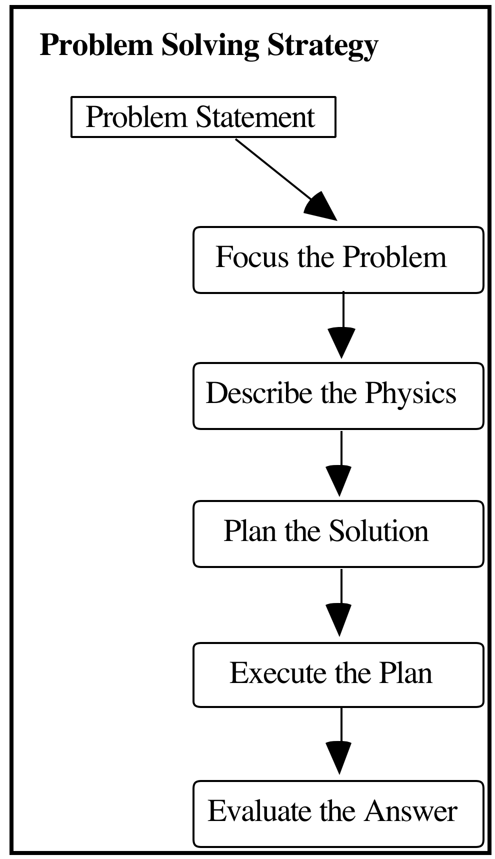
4. **Execute the Plan**: In this step you actually execute the solution you have planned. Plug in all of the known quantities into the algebraic

solution, which is the result of step 3, to determine a numerical value for the desired unknown quantity(ies).

5. **Evaluate the Answer**: Finally, check your work to see that it is properly stated, not unreasonable, and actually answers the question asked.

Consider each step as a translation of the previous step into a slightly different language. You begin with the full complexity of real objects interacting in the real world and through a series of decisions arrive at a simple and precise mathematical expression.

The solution to the following problem illustrates each step. On the right side of the page is the actual solution, as you might construct it. On the left side of the page are brief descriptions of each step of the solution. We have used a familiar situation so that you can concentrate on understanding how the strategy is applied.



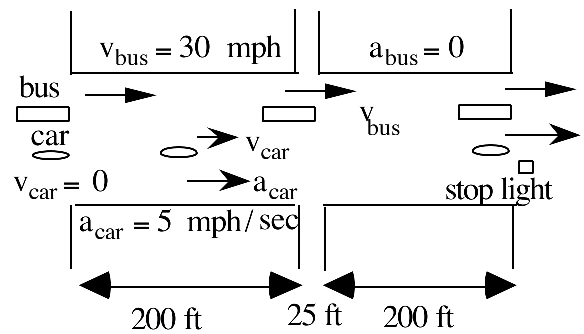
Example: Just as you turn onto the main avenue from a side street with a stop sign, a city bus going 30 mph passes you in the adjacent lane. You want to get ahead of the bus before the next stoplight which is two blocks away. Each block is 200 ft long and the side streets are 25 ft wide, while the main avenue is 60 ft wide. If you increase your speed at a rate of 5 mph each second, will you make it?

(1) <u>Focus the Problem</u>: In this step of the problem solving strategy construct your initial qualitative understanding of the problem situation. Write down what you know, what you want to know, the physics you will use, and the assumptions you will make. This understanding can be usefully expressed as follows:

Picture & Given Information:

What's happening? Visualize the problem situation and make a sketch of the important objects and events.

Decide which given information may be useful and write it down on the sketch.



Question(s):

What is(are) the question(s)? Express it as some quantity to be found.

Find the distance the car travels to catch up to the bus. See if it is less than 425 feet.

Approach:

What approach shall I take? Outline the concepts which can relate the given information to the question.

Use the definition of average velocity for the bus since it travels at constant velocity.

Use the relationship between acceleration and position for the car since it travels at constant acceleration.

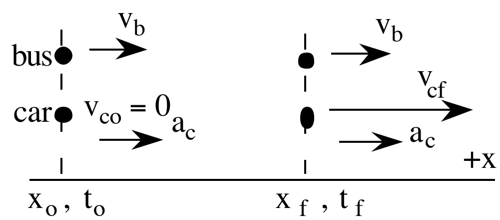
Initial time is when the bus and the car are first together. Final time is when the bus and the car are next together.

(2) **Describe the Physics**: In this step use your physics ideas to translate your initial understanding of the problem into a diagram of the actual problem. This diagram contains only idealized physical objects and representations of important physical quantities. Identify which of these physical quantities you need to find to answer the question. Write down the relationships between the quantities which will help you determine the unknowns. This information can be summarized with the following items:

Diagram & Define Quantities:

For kinematics problems, use a motion diagram. This diagram requires:
* Coordinate axes.
* Simplified representations (usually points) of objects.
* Indication of position, velocity and acceleration of objects at important times.

Identify known and unknown quantities.



$x_o = 0 \quad x_f = ?$

$t_o = 0 \qquad t_f = ?$

$v_{co} = 0 \quad v_{cf} = ?$

$v_b = 30 \text{ mph} \quad a_c = 5 \text{ mph/sec}$

Target Quantity(ies):

Decide which of your unknowns you will need to find in order answer the problem question.

$x_f = ?$

Quantitative Relationships:

Decide which physics principles or other mathematical relationships are applicable for the situation diagrammed above.

$$v_b = \frac{x_f - x_o}{t_f - t_o} = \frac{x_f}{t_f} \qquad v_b \text{ constant}$$

$$x_f = \frac{1}{2}a_c(t_f - t_o)^2 + v_{co}(t_f - t_o) + x_o$$

$$x_f = \frac{1}{2}a_c(t_f)^2 \qquad a_c \text{ constant}$$

(3) <u>Plan the Solution</u>: In this step translate your physics description of the problem into the particular equations, which will help you solve the problem. Always begin with an equation from your quantitative relationships containing the target quantity. If that equation contains additional unknowns, write down another equation from your quantitative relationships containing one of those unknowns. Continue until you have introduced a new equation for every unknown in your plan.

Construct Specific Equations:

Use your quantitative relationships to write specific equations relating unknown quantities to ones which are known.

<u>Find x_f</u>

<u>unknowns</u>

x_f

$$x_f = \frac{1}{2}a_c(t_f)^2$$

t_f

Find t_f

$$v_b = \frac{x_f}{t_f}$$

$$t_f = \frac{x_f}{v_b}$$

$$x_f = \frac{1}{2}a_c\left(\frac{x_f}{v_b}\right)^2$$

$$\frac{2v_b^2}{a_c} = x_f$$

Check Units:

Make sure the units on both sides of your equation are the same.

$$\frac{\left[\frac{mi}{hr}\right]^2}{\frac{mi}{hr^2}} = mi \qquad \text{OK}$$

(4) <u>Execute the Plan</u>: In this step carry out the mathematics specified in your solution plan in order to determine a numerical value for your target quantity(ies).

Calculate Target Quantity(ies):

Put numerical values of known quantities into the equation for the target quantity. Convert units if necessary and calculate a value for the target quantity.

$$x_f = 2\frac{\left(30\frac{mi}{hr}\right)^2}{\left(\frac{5\frac{mi}{hr}}{s}\right)}$$

$$x_f = 360\left(\frac{mi}{hr}\right)s$$

$$x_f = 360\left(\frac{mi}{hr}\right)s\left(\frac{hr}{3600\,s}\right) = \frac{1}{10}mi$$

Since 0.1 miles is 528 feet, which is more than 425 feet, you do not make it.

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(5) <u>Evaluate the Answer</u>: As a result of executing your plan, you have a numerical answer to the physics problem. In this final step, check that your answer is properly stated, not unreasonable, and complete.

Is Answer Properly Stated?:
Check that your answer has the appropriate units and sign.

Yes, miles are a correct unit for distance.

Is Answer Unreasonable?:
Check that the magnitude of your answer is not unexpectedly large or small.

The answer is only about 100 ft longer than the 2 block distance which is not unreasonable.

Is Answer Complete?:
Check that you have answered the original question.

The care does not make it answers the question

STEPS FOR SOLVING A PROBLEM

"What's going on?"
FOCUS on the PROBLEM

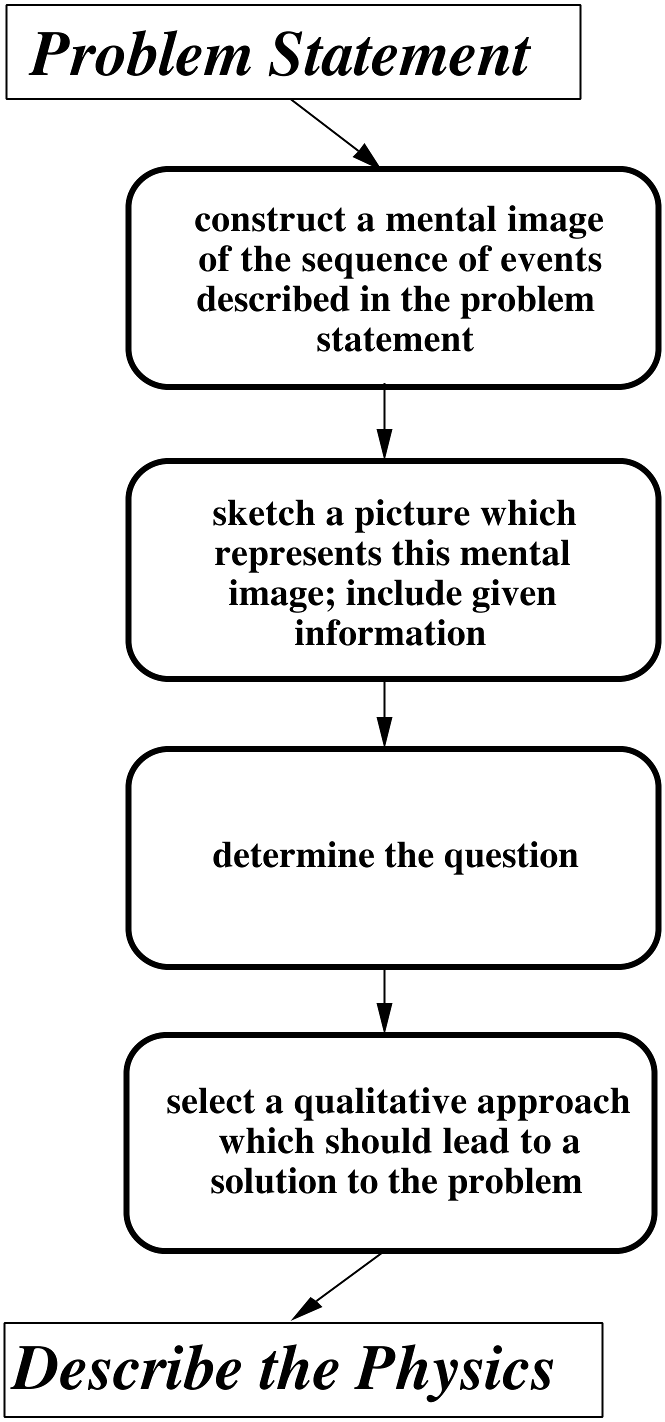
"What has this to do with physics?"
DESCRIBE the PHYSICS

"Can we find a solution?"
PLAN the SOLUTION

"What's the answer?"
EXECUTE the PLAN

"Can this be true?"
EVALUATE the SOLUTION

Focus the Problem



- What's going on?
- What objects are involved?
- What are they doing?

- Are all the important objects shown?
- Are the spatial relations between the objects shown?
- Are the important times represented?
- Are the important motions represented?
- Are the important interactions represented?

- Does the question ask about a specific measurable characteristic(s) about a particular object(s)? If not, reformulate it so it does.

- What is the system of interest?
- Which physics principles could be used to solve the problem?
- What information is really needed?
- Are there only certain time intervals during which one approach is useful?
- Should we make any approximations?

Describe the Physics

Focus the Problem

construct diagram(s) to show important space and time relationships of each object

- What coordinate axes are useful? Which direction should we call positive?
- Relative to the coordinate axes, where is (are) the object(s) for each important time?
- Relative to the coordinate axes, what is (are) the velocity and acceleration for each object at each important time?
- Are other diagrams necessary to represent the interactions of each object or the time evolution of its state?

make sure all symbols representing quantities shown on diagram(s) are defined

- What quantities are needed to define the problem mathematically using the approach chosen?
- Which symbols represent known quantities? Which symbols represent unknown quantities?
- Are all quantities having different values labeled with unique symbols?
- Does the diagram(s) have all of the essential information from the sketch?

declare a target quantity

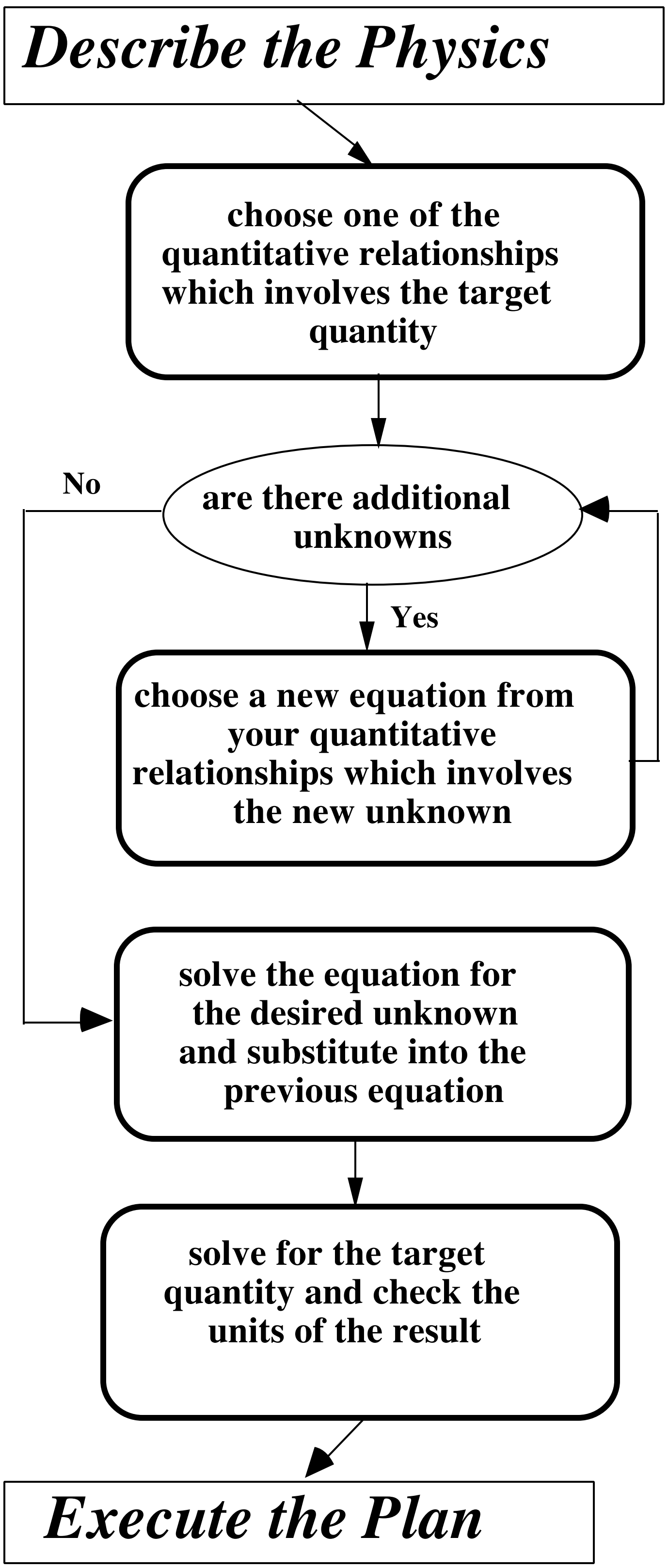
- Which of the unknowns defined on the diagram(s) answers the question?

state quantitative relationships from general principles and specific constraints

- What equations represent the general principles(s) specified in our approach and relate the physics quantities defined in the diagram?
- During what time intervals are those relationships either true or useful?
- Are there any equations that represent special conditions that are true for some quantities in this problem?

Plan the Solution

Plan the Solution



- Which quantitative relationship includes the target quantity?
- For what object does that equation apply?
- For what time interval does that equation apply?

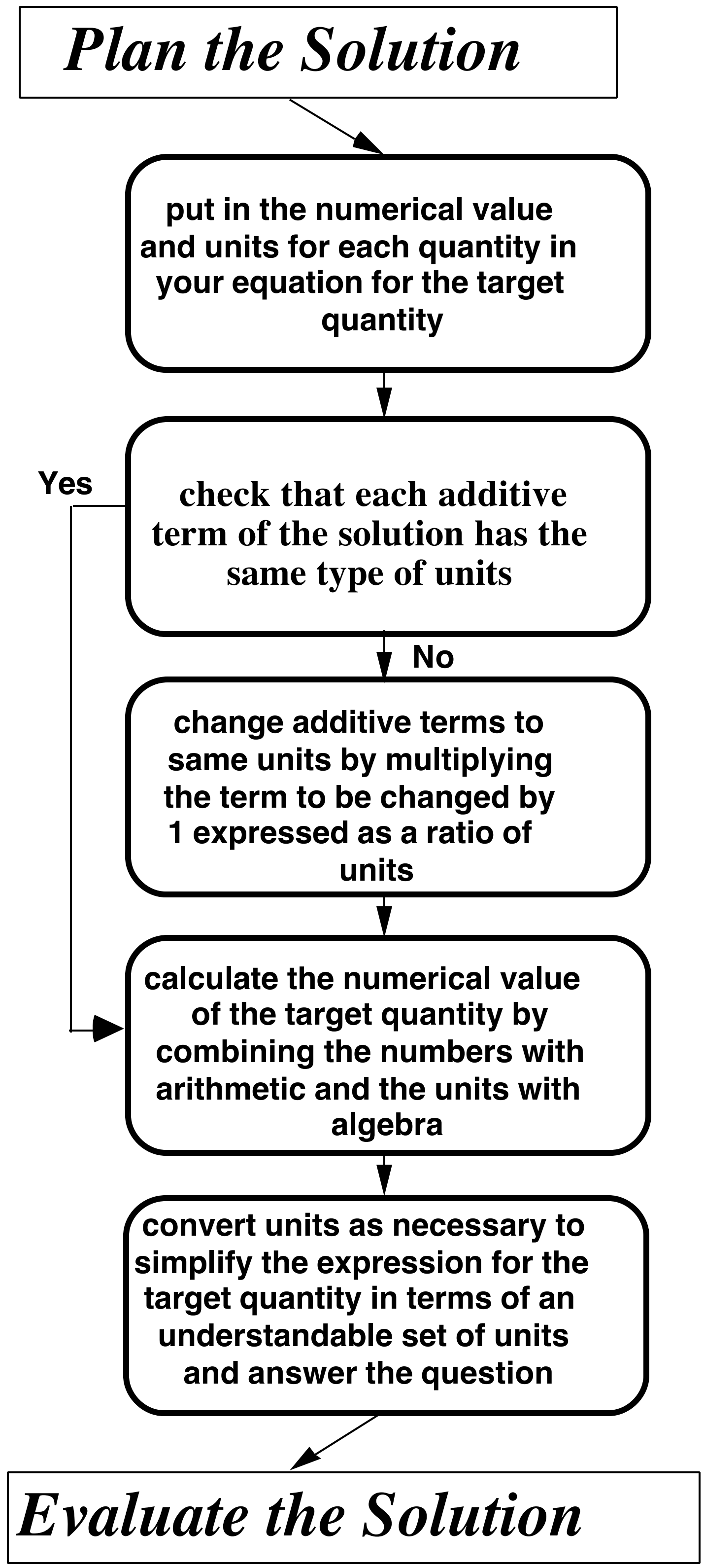
- Are there any unknowns in the equation other than the target quantity?
- Are there any unknowns that cancel out in the algebra?

- Which quantitative relationship includes the unknown quantity?
- For what object does that equation apply?
- For what time interval does that equation apply?
- Is this equation different from those already used in this problem?

- What unknown is the target of this specific equation?
- Which previous equations have that unknown?
- Are there any quantities that cancel out in the algebra?

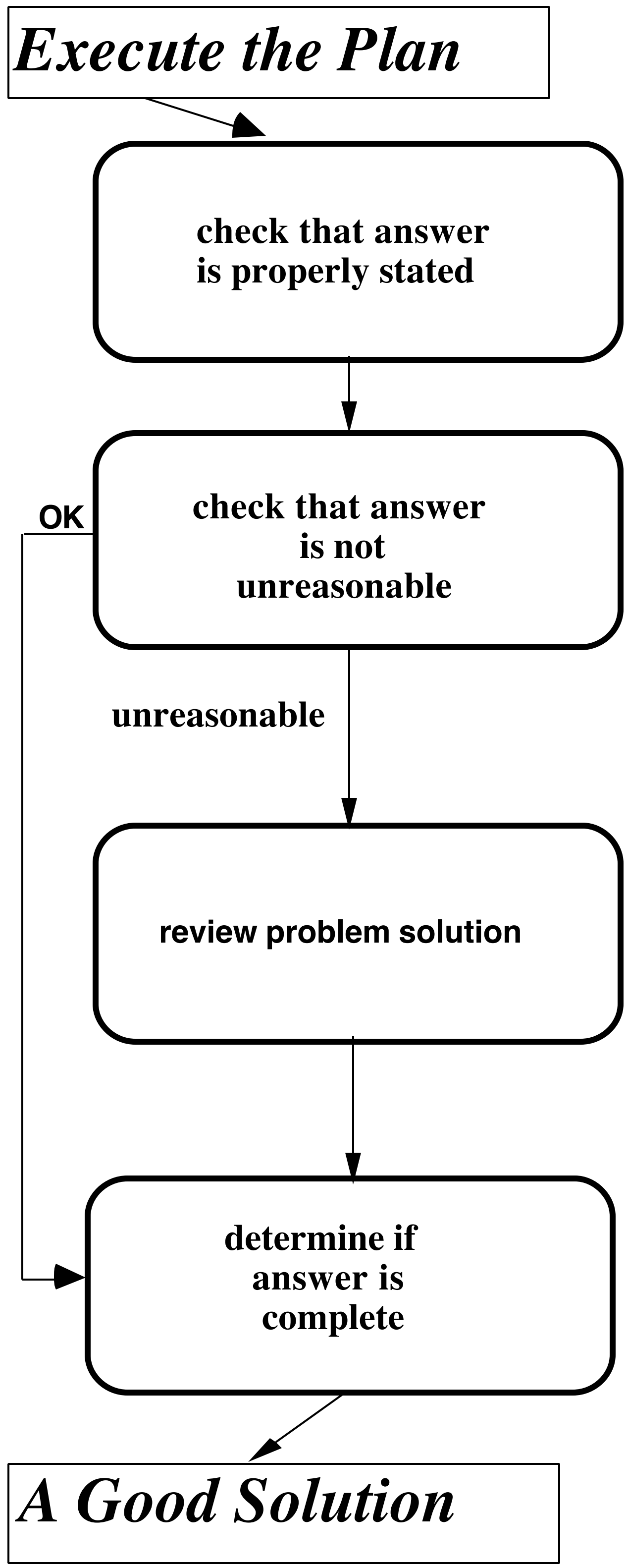
- After all the substitution for unknowns, is the only unknown left the target quantity?
- Are the units the same on both sides of the equation?

Execute the Plan



- Which values (numbers with units) from the physics description should be put into the equation for the target quantity?
- Do you need to convert units?
- What ratio of units equals 1?
- Use a calculator for the numbers and algebra for the units.
- Do any units cancel?
- Do we need to convert any units?
- What is the most reasonable set of consistent units for this problem?

Evaluate the Answer



- Do the units make sense?
- Do vector quantities have both magnitude and direction?
- If someone else read just your answer, would they know what it meant?

- Does the answer fit with your mental picture of the situation?
- Is the answer the magnitude you would expect in this situation?
- Do you have any knowledge of a similar situation that you can compare with to see if the answer is reasonable?
- Can you change the situation (and thus your equation for the target quantity) to describe a simpler problem to which you know the answer?

- Is your physics description complete?
- Are the definitions of your physics quantities unique?
- Do the signs of your physics quantities agree with your coordinate system?
- Can you justify all of the mathematical steps in your solution plan?
- Did you use units in a consistent manner in your execution?
- Is there a calculation mistake in the execution?

- Have you answered the question from the Focus the Problem step?
- Could someone else read and follow the solution plan?
- Are you sure you can justify each mathematical step in the plan?