Clickers

- If you haven't already got one, come and pick one up.
- Type in your university ID (if you haven't already done so)
- You will keep them all semester. Try not to lose yours (we'll have to charge for a replacement if you do). Bring them to Physics and Engineering lectures.

Uncertainties: Part 1

Why they matter and how to calculate them

Two Quotes

• The uncertainty is almost more important than the number itself (ANU researcher, 2010)

 Uncertainty is that stupid number you calculate to keep your lab demonstrator happy (ANU student, 2010)



 Imagine that you work for a car company. You are designing the doors of a new car model. You want the door to fit snugly into the door frame, so that it doesn't rattle, water can't get in, and to minimise wind drag.

Uncertainty

The door is supposed to be 1200 mm high. But the manufacturers cannot guarantee that every door and every door frame they produce is exactly 1200 mm high. There will be an uncertainty in this height. It might depend on how worn down the stamping machines are, how pure the steel is, how hot the shop floor was on the day of manufacture, and many other things.

Clicker Question

Let's say you get the following quotes from different manufacturers, to make a door frame and a door.

Frame size 1200±10mm, door size 1199±10mm, cheap
 Frame size 1200±1mm, door size 1199±1mm, average
 Frame size 1200±0.5mm, door size 1199±0.5mm, expensive
 Frame size 1200±0.1mm, door size 1199±0.1mm, very expensive

Which quote would you go for?

Answer

If you went for the first quote, you could have a frame size anywhere in the range 1190-1210mm and a door with a size in the range 1189-1209. So you could easily have a door larger than the frame (which wouldn't fit) or at the opposite extreme, a door 21mm smaller than the frame - such a big gap that water would almost certainly get through. Not a good quote to accept.

If you went with the second quote, you could still have a frame that was smaller than the door (if a particular frame had a below average size and the particular door had an above average size). Most of the time it would work but you'd have to throw out a certain fraction of doors and frames.

If you went with the third quote, the smallest frame you'd get would have a size of 1200-0.5=1199.5mm and the largest door you'd get would have the same size - so you're OK, they would always fit. So this would be acceptable. But you might end up with a 2mm gap, which might make the door rattle a bit.

With the last quote, the biggest gap would be 1.2mm. If the manufacturers uncertainties are believable, you could get away with a door size of 1199.9mm, in which case your gap would never be larger than 0.2mm, which would be very snug. Perhaps an option for luxury cars?

Conclusion

• Changing the uncertainties can make a given set of components usable or unusable.

 Ford Motor Company was simultaneously manufacturing a car model with transmissions made in Japan and the United States. Soon after the car model was on the market, Ford customers were requesting the model with Japanese transmission over the USAmade transmission, and they were willing to wait for the Japanese model. As both transmissions were made to the same specifications, Ford engineers could not understand the customer preference for the model with Japanese transmission. Finally, Ford engineers decided to take apart the two different transmissions. The Americanmade car parts were all within specified tolerance levels. On the other hand, the Japanese car parts had much closer tolerances than the USA-made parts - e.g., if a part were supposed to be one foot long, plus or minus 1/8 of an inch - then the Japanese parts were within 1/16 of an inch. This made the Japanese cars run more smoothly and customers experienced fewer problems.

> Aguayo, Rafael (1991). Dr. Deming: The American Who Taught the Japanese About Quality. Fireside. pp. 40–41.

Gravity Variations and Minerals

 One way of finding buried metal deposits is to measure the gravity at the surface of the Earth. As metal deposits are denser than other rocks, gravity is a little greater when you are directly over them.





present day. Continental Australia has a basic station spacing coverage of 11 kilometres, with South Australia, Tasmania and part of New South Wales covered at a spacing of 7 kilometres. Victoria has station coverage of approximately 1.5 kilometres. Recent Federal, State and Territory Government initiatives have funded systematic infill at a grid station spacing of 2, 2.5 or 4 kilometres to provide improved coverage in areas of scientific or economic interest. Other areas of detailed coverage have been surveyed by private companies for exploration purposes. Only open file data as held in the ANGD at May 2008 were used in the creation of the grid.

It is recommended that this map be referred to as: Bacchin, M., Milligan, P. R., Wynne, P., and Tracey, R., 2008, Gravity Anomaly Map of the Australian Region (Third Edition), scale 1:15 000 000, Geoscience Australia, Canberra. Cartography: S. Mezzomo.

The geodetic version of the grid used for the preparation of the image and the original point located data are available for free download via the internet using Geoscience Australia's Geophysical Archive Data Delivery System (GADDS) at: <u>http://www.geoscience.gov.au/gadds</u>

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Copies of this map may be downloaded from the Geoscience Australia internet site at: <u>http://www.ga.gov.au</u>, or by contacting: Sales Centre, Geoscience Australia GPO Box 378 Canberra, ACT 2601 Phone (02) 6249 9966 Facsimile (02) 6249 9960 Email: <u>sales@ga.gov.au</u>

Clicker question

- Let's say you suspect that there might be iron ore at a given location. If it was present, you'd expect gravity to be greater by more than 4µms⁻².
- Would you go to the great expense of doing some sample drilling if you measured an increase in gravity of:
 - I. 5±100 μms⁻²?
 - 2. $5\pm 2 \ \mu ms^{-2}$?
 - 3. $5\pm0.1\,\mu\text{ms}^{-2}$?

Answer

1. $5\pm0.1\mu ms^{-2}$?

If the uncertainty was $\pm 2\mu$ ms⁻², then the true value could be 3, which would not indicate useful ore deposits.

It might be suggestive - but far from proof.

And there are far more areas without minerals than with minerals. So you would probably find lots of spurious deposits by accident this way.

A STIS absorption spectrum

A beam of light coming to Earth from a distant quasar passes through numerous intervening gas clouds in galaxies and in intergalactic space. These clouds of primeval hydrogen subtract specific colors from the beam. The resulting 'absorption spectrum,' recorded by Hubble's Space Telescope Imaging Spectrograph (STIS), is used to determine the distances and chemical composition of the invisible clouds.

Cumulative absorption spectra

Final absorption spectrum recorded by STIS

Subtracted – by cloud 1

> Subtracted J by cloud 2

Subtracted by cloud 3

Wednesday, 2 March 2011





How to determine uncertainties

 There is no magic formula for this. You have to think through how you are making your measurement or building your components, think of what could vary, and by how much.

Brainstorm

- Let's say you are making bolts on a computer controlled milling machine.
- Why wouldn't they all come out the same size?
- Air pressure, Wear on the machine, Temperature, internal mechanics, human error, manufacture of machine, material imperfections...

Brainstorm 2

 Let's say you are trying to measure whether one person is short enough to be in a particular sporting category. Why might there be an uncertainty in their height?

Another way to measure uncertainties

- Make repeat measurements.
- See how much they vary.

Clicker Question

- A factory sends you ten bolts, claiming that they should have a length of 450±20 mm.
- You very accurately measure the length of these bolts. Here are the answers you get.
- 476.3, 493.9, 412.9, 423.7, 430.0,
 439.1, 456.1, 499.2, 434.0, 435.7 mm
- Do you believe their claim?

Answer

- No the claim is crap.
- Lots of the bolts lie outside the range 430-470 mm.
- Range is 412.9-499.2 mm
- The average is right but the uncertainty they quote is much too small.

Warning

- The "do lots of repeat measurements and see how great the scatter is" method is usually the most accurate way to determine uncertainty.
- This is because it is hard to work out uncertainty from first principles, and most people underestimate it - often by a lot.

Drawbacks

- But it is expensive.
- And the repeat measurements have to be "Independent" - i.e. whatever is causing the scatter must be different.
- For example, if some of the scatter is due to temperature changes and all your measurements were made on a hot day, they are not really independent.

How to quote uncertainties

- Best way explicitly state it. For example:
- Global temperature will rise by between 2.3 and 5.4 degrees.
- 96±3% of students think Paul talks too fast
- Unbalanced uncertainties (sometimes needed if the uncertainty from the best value is bigger one way than the other: e.g.

$$H_0 = 71^{+3.2}_{-4.7} \text{ km } s^{-1} \text{Mpc}^{-1}$$

Significant Figures

- You should always quote uncertainties explicitly if they matter.
- If you don't think they matter, you can imply them by the number of significant figures you quote. The rule is:
- "The second last significant figure should be certain the last one can be uncertain."

Example

- If you say that the mass of a ball is 34.53 kg, you know that the 34.5 is accurate, but that the final 3 is less certain.
- It could be 34.53±0.01 or 34.53±0.04, but if the uncertainty was more like 34.53±0.2, it should have been quoted as 34.5 as the 5 is now uncertain.
- None of this is hard-and-fast many cases are ambiguous. Many people are sloppy in how they do this, and many cases are ambiguous.

Common mistake

- You won't be penalised in this course is you quote one too few or to many significant figures.
- But a common mistake is to quote FAR too many - all the figures given by your calculator.
- e.g. the ball dropped 4.5628468264826 m
- We (the exam markers) don't like that...

Propagation of Uncertainties

- A common situation. You know the uncertainties in some variables, but need to know the uncertainty in another variable that depends on them in some way.
- For example you fire a ball at some (uncertain) angle - how much does the uncertainty in the angle affect how far it will travel?

More examples

- You measure the time it takes for an athlete to run 100 m. The length, start and end time all have uncertainties.
- What is the uncertainty in the speed?

Simple way to solve

 Work out the best and worst case scenarios consistent with the uncertainties
 plug in the numbers and see what happens.



Example

- The dangerous sports club at a certain elite British university built a giant trebuchet, and used it to fire undergraduates across a field into a net.
- If it fires someone at a speed v and angle θ to the horizontal, that person will (in the absence of wind resistance) travel a distance

Calculate the size of the net required

- if $\theta = 10 \pm 1$ degrees and v = 25 \pm 2 m/s.
- If $\theta = 10$ degrees and v=25 m/s, D=21.8 m.
- But what combination will give the smallest distance?

Answer



Substitute this is

- So plug in θ=10-1=9 degrees, and v=25-2 = 23 m/s into the equation.
- You get D=16.7 m, down from 21.8 m.
- The furthest you could fling someone would be for θ=10+1=11 degrees, and v=25+2=27 m/s, in which case D is 27.8 m.
- So D lies in the range 16.7 27.8 m (so you need a BIG net), and you can quote $D = 21.8^{+6.0}_{-5.1}$ m

Note

- Today I've given you the simple way to handle uncertainties.
- There are much fancier techniques which are sometimes appropriate - we'll come back to them just before the mid-semester break.
- Use these simple techniques in the labs and homework for now.

Key Points

- Uncertainties matter. Both for engineering and science.
- Pretty much everything has an uncertainty.
- The best way to estimate them is to do repeat measurements and look at the scatter.
- You can work out the effect of them by plugging in best- and worst-case scenarios.

Clicker Question

