1. **Closest approach in charged particle collisions**
   In this practice problem we will justify the assertion made in class that for a collision between an electron with charge e and an ion of charge Ze with impact parameter b and initial velocity v, the distance of closest approach r_{min} obeys

$$b = r_{\mathrm{min}} \left(1 + \frac{2Ze^2}{m_e v^2 r_{\mathrm{min}}}\right)^{1/2}.$$

   (a) Consider the system when the electron and ion are far apart, and write down expressions for the total kinetic energy and angular momentum of the system. You may work in the frame of the ion, and assume that the ion remains stationary throughout the collision due to its much larger mass.

   (b) Now consider the moment when the electron and ion are at their minimum distance. Again, write down expressions for the total system energy and angular momentum.

   (c) Since energy and angular momentum are both conserved, by equating their initial values and values at closest approach, you should arrive at two equations for the two unknowns r_{min} and v_{max}, the electron velocity at the point of closest approach. Solve these equations to obtain the expression above.

2. **Polycyclic Aromatic Hydrocarbons (PAHs)**
   Polycyclic aromatic hydrocarbons (PAHs) are large molecules that are common in the interstellar medium. They consist of linked benzene rings, and can have a very wide range of sizes. The smallest one (naphthalene, consisting of two linked rings) has an atomic weight of 128, while the largest PAHs can have masses of many thousands of amu and are akin to small dust grains. Our goal is to determine whether PAHs should be thought of as a fluid (like hydrogen atoms) or not (like larger dust grains). We will consider a neutral PAH of atomic weight A and cross-sectional area σ in a region of neutral hydrogen of density n and temperature T.

   (a) Compute the mean thermal velocities v_{PAH} and v_{H} of the PAH and of the hydrogen atoms. By what factor do they differ?

   (b) Suppose the PAH initially moves through the hydrogen at a velocity v_{init} such that $v_{\mathrm{PAH}} \ll v_{\mathrm{init}} \ll v_{\mathrm{H}}$. Roughly estimate the timescale required for the PAH's direction of motion to be randomised. You may assume that all collisions are elastic.

   (c) Suppose that $v_{\mathrm{init}} = 5$ km s^{-1}, $\sigma = 2 \times 10^{-17} A$ cm^{-2}, and the PAH is moving through a region of atomic hydrogen with $n = 0.1$ cm^{-3} and $T = 8000$ K. Estimate t_{iso} numerically as a function of A.

   (d) Based on this finding, should we think of PAHs as a fluid? For what range of masses A?