1. **ISM injection of carbonaceous grains. [20 points]**
Late in their evolution, AGB stars eject dust grains into the ISM. Suppose a spherical grain of density ρ_{g} and radius r_{g} is injected with initial velocity v_{i} relative to the surrounding gas. The gas has number density n and temperature T, and you may approximate that it consists of pure hydrogen.

   (a) Initially the grain velocity will be radially outward from the star, but after some time t_{iso} the direction of travel will be randomised by collisions between the grain and atoms in the ISM. Roughly estimate t_{iso}. You may assume that (1) the grain mass $m_{\mathrm{g}} = (4\pi/3)\rho_g r_g^3 \gg m_{\mathrm{H}}$, where m_{H} is the hydrogen mass, (2) all collisions between grains and gas are elastic, and (3) v_{init} is much smaller than the mean speed of atoms in the surrounding ISM (not fully realistic, but makes life easier). [10 points].

   (b) Collisions between the grain and the gas will also gradually cause the grain to slow down, until its speed is close to the thermal equilibrium value – which is slower than the mean speed of gas atoms by a factor of $\sqrt{m_{\mathrm{g}}/m_{\mathrm{H}}}$. Estimate the time required for the grain to reach thermal equilibrium. How does it compare to the isotropisation time? [5 points]

   (c) Estimate the isotropisation and equilibration times for a grain of size $r_{\mathrm{g}} = 10$ nm and density $\rho_{\mathrm{g}} = 3$ g cm^{-3} injected at 5 km s^{-1} into gas with a density $n = 0.1$ cm^{-3} and temperature $T = 8000$ K, typical of the warm ISM. Given this result, do you expect grain directions to isotropise and/or equilibrate as a result of fluid processes? [5 points]

2. **Magnetic fields of clouds and stars. [10 points]**
Consider a 1 $M_\odot$ protostellar core that collapses to make a star like the Sun. A typical observed density for such an object is 10^5 H_2 molecules cm^{-3}.

   (a) Suppose that such an object has an initial uniform magnetic field, and that it is governed by ideal MHD during its collapse. Make an order of magnitude estimate of how much larger the magnetic field of the resulting star is compared to the field of the initial core. [5 points]

   (b) Consult the literature and look up typical observed magnetic field strengths for star-forming cores at a density of $\sim 10^5$ cm^{-3}, and for the surface magnetic fields of T Tauri stars, which are young stars that formed recently. Two suggested papers to consult are Johns-Krull (2007, ApJ, 664, 975) and Crutcher (2012, ARA&A, 50, 29), but feel free to use others. Are the observed magnetic field strengths of cores and T Tauri stars consistent with the idea that the collapse is governed by ideal MHD? If not, by what factor must the magnetic field be increased or decreased? [5 points]

3. **The C^+ ion. [20 points]**
Most carbon in the interstellar medium is in the form of C^+ ions. The ground electronic state of C^+ is $1s^2 2s^2 2p^1$, and the corresponding L-S coupling state consists of a ground state ${}^2P^o_{1/2}$ and excited state ${}^2P^o_{3/2}$. The transition between these two has a wavelength of 158 μm and an Einstein $A = 2.3 \times 10^{-6}$ s^{-1}. Atoms in the excited ${}^2P^o_{3/2}$ state can also absorb photons with a wavelength of 1335.7 Å and transition to a higher excited state $1s^2 2s^2 2d^1\ {}^2D_{5/2}$; the Einstein A for the corresponding downward transition is $A = 2.9 \times 10^8$ s^{-1}.

(a) For the two transitions discussed, is the transition allowed, semi-forbidden, or forbidden? How do you know? Explain in terms of selection rules. [5 points]

(b) Consider a collection of C^+ ions in the ${}^2P^o_{3/2}$ state, moving with a Doppler broadening parameter $b = 20$ km s^{-1}; assume that natural broadening is negligible. These atoms are exposed to radiation with a wavelength of 1335.7 Å that causes them to transition to the $1s^2 2s^2 2d^1\ {}^2D_{5/2}$ state. What is the absorption cross per C^+ atom at line centre? [5 points]

(c) Suppose that we observe a background source, with this collection of ions in the foreground, and find that the atoms absorb 10% of the light at line centre. What is the total column (i.e., how many per unit area) of C^+ ions in the ${}^2P^o_{3/2}$ state along the line of sight to the source? [5 points]

(d) The same ions that are *absorbing* light at 1335.7 Å will also be *emitting* light by spontaneously transitioning from the ${}^2P^o_{3/2}$ to the ${}^2P^o_{1/2}$ state, producing 158 μm photons. What is the surface flux of this emission, i.e., how much energy is emitted per unit time per unit area by these ions? [5 points]