# Class 6: Magnetic fields and magnetised turbulence

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# Outline

- Motivation for considering magnetic fields
  - General physics considerations
  - Zeeman effect
  - Dust polarisation and the Chandrasekhar-Fermi method
- MHD turbulence
  - Flux freezing and the magnetic Reynolds number
  - The Alfvén Mach number
  - Non-ideal effects

# Why magnetic fields?

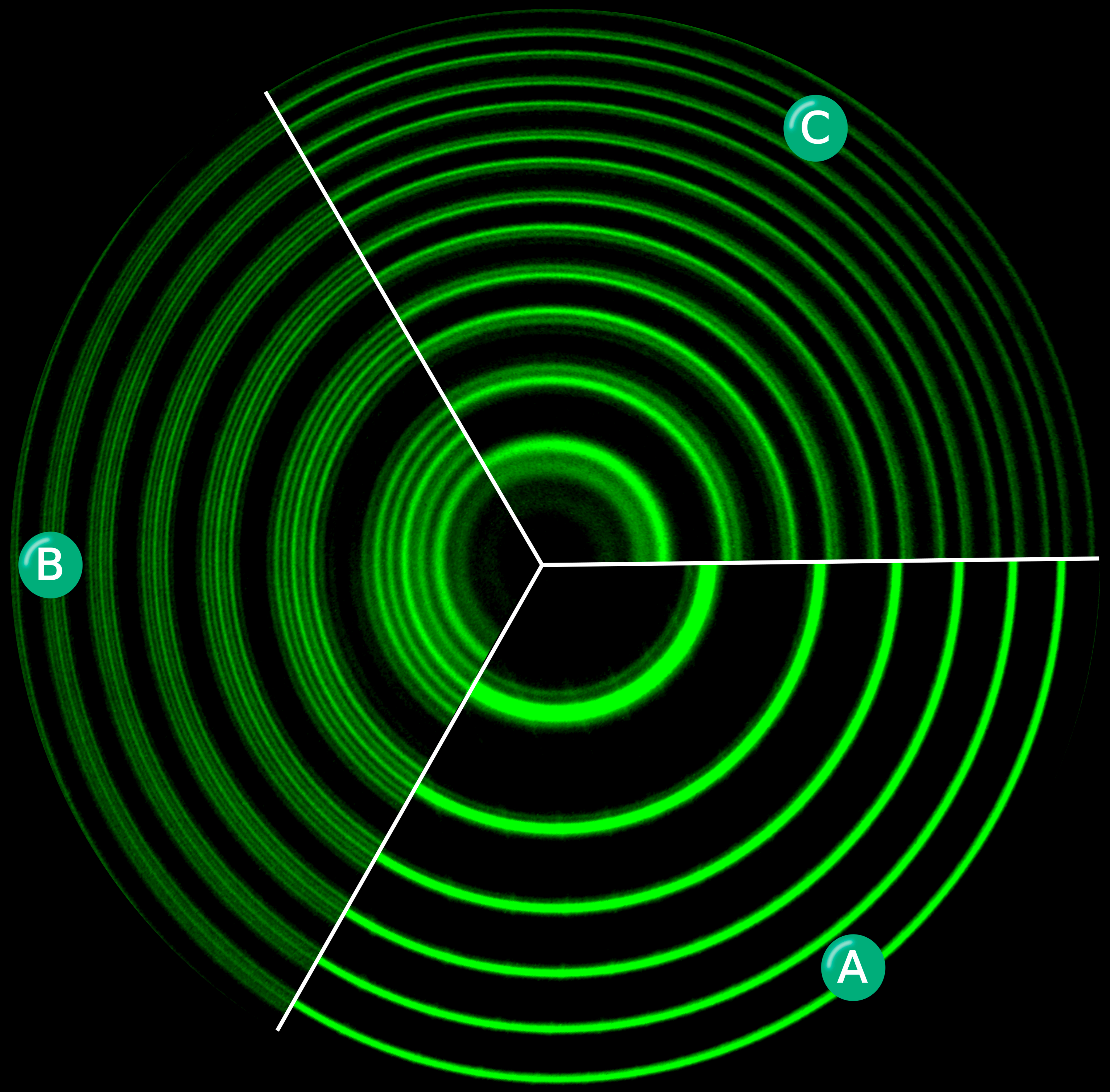
## A *very* cursory primer on plasma physics

- Molecular gas is mostly neutral; however, CRs produce a small free charge fraction — typically ~$10^{-6}$ at GMC densities
- However, only a tiny free charge density is require to make a plasma a very good conductor; a molecular cloud is about as conductive as copper wire
- When a conductive fluid feels forces (e.g., pressure, gravity), the light electrons and heavy ions generally do not want to move in the same way; the resulting charge separations produce electric potentials that cause currents to flow — and any time there is current, there will be magnetic fields
- Thus magnetic fields are *inevitable* whenever one has moving, conductive fluids

# Detecting B fields

## The Zeeman effect

- Most direct method is Zeeman effect: splitting of degenerate energy levels by magnetic field
- Simplest example is LyA line of H:
  - With no B field, one line corresponding to 2p ($g$ = 4) → 1s ($g$ = 2) transition
  - In presence of B field, energy depends on orientation of electron spin and orbit relative to field → 6 distinct lines




Zeeman effect in mercury vapour (from wikipedia): (A) spectral lines with no magnetic field; (B) and (C) with magnetic field oriented transverse and long line of sight

# Zeemann effect in molecular clouds

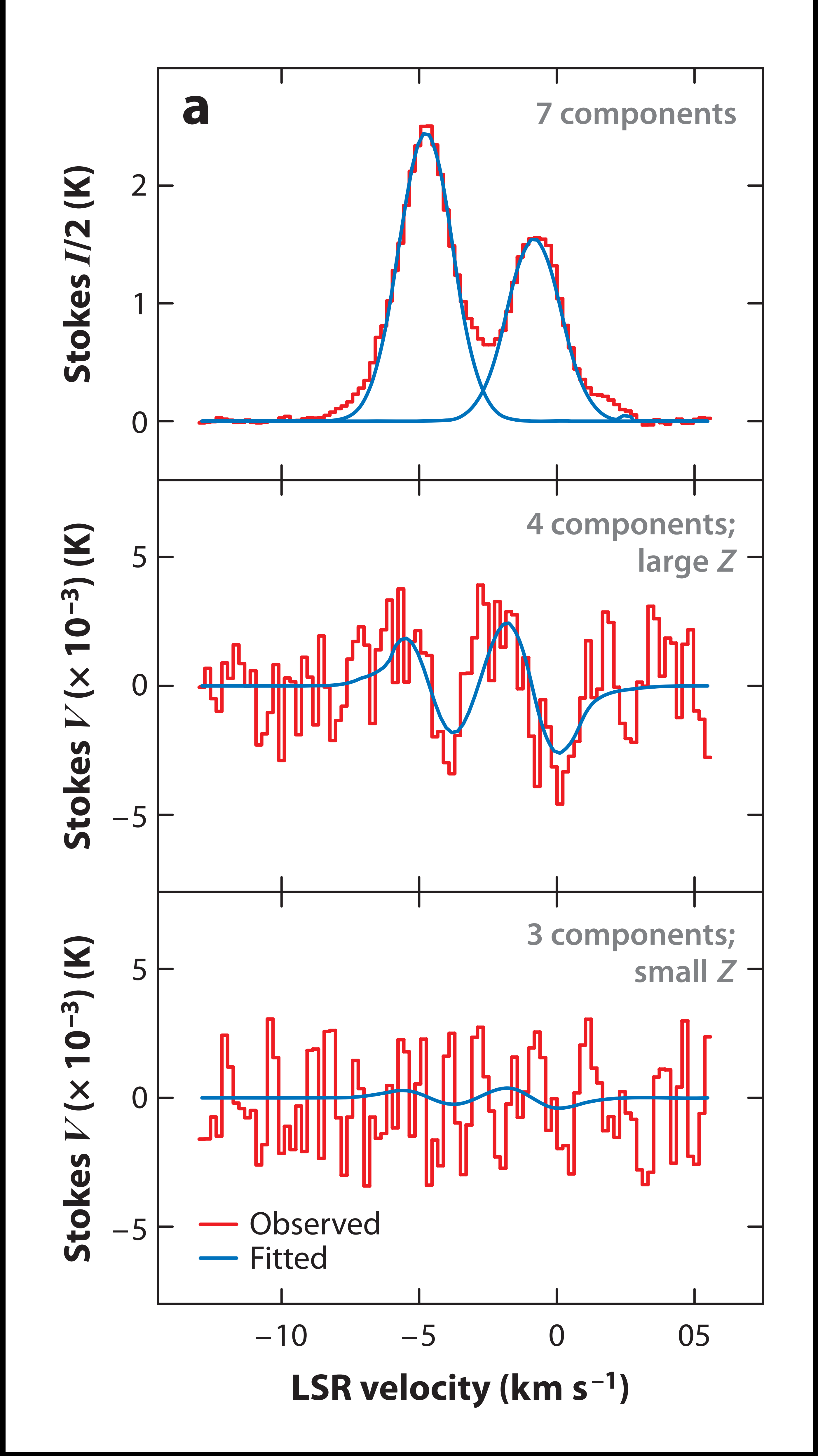
## Zeeman sensitivity and splitting

- Many molecules are “Zeeman sensitive”, meaning that they have levels that split, usually due to an unpaired outer shell electron; examples: OH, CN, CH
- Most common case: one line to splits into 3, corresponding to angular momentum vector aligned with field, anti-aligned with field, transverse to field; frequency shift characterised by Zeeman sensitivity parameter $Z$: $\Delta\nu = \pm BZ$
- Example: for OH, $Z$ = 0.98 Hz / $\mu$G, so for typical ~10 $\mu$G field, split is ~10 Hz
- This is generally much smaller than the Doppler broadening: for OH $\nu_0$ = 1.67 GHz, so velocity dispersion $\sigma_v$ = 0.1 km/s induces frequency dispersion $\sigma_\nu = (\sigma_v/c)\,\nu_0 \approx 1$ kHz, so line is not visibly split — so how do we detect the splitting?

# Measuring the Zeeman split

## Polarisation to the rescue

- Zeeman shift set by orientation of angular momentum vector, but this also determines angular momentum of emitted photon
- Consequence: sub-levels make lines with different circular polarisations!
- Can separately measure each polarisation, then take difference to detect shift; amount of shift directly measures B field strength
- Important note: since only the component of the field along the line of sight causes polarisation, we measure only this component




Example Zeeman detection for CN (Crutcher 2012); the line has 7 subcomponents, 4 with large $Z$ and 3 with small $Z$.
(A) total intensity, shifting all 7 components to lie at the same velocity.
(B) difference in intensity between left and right circular polarisation, for 4 large $Z$ components.
(C) same as (B) for for the 3 small $Z$ components.

# Dust polarisation

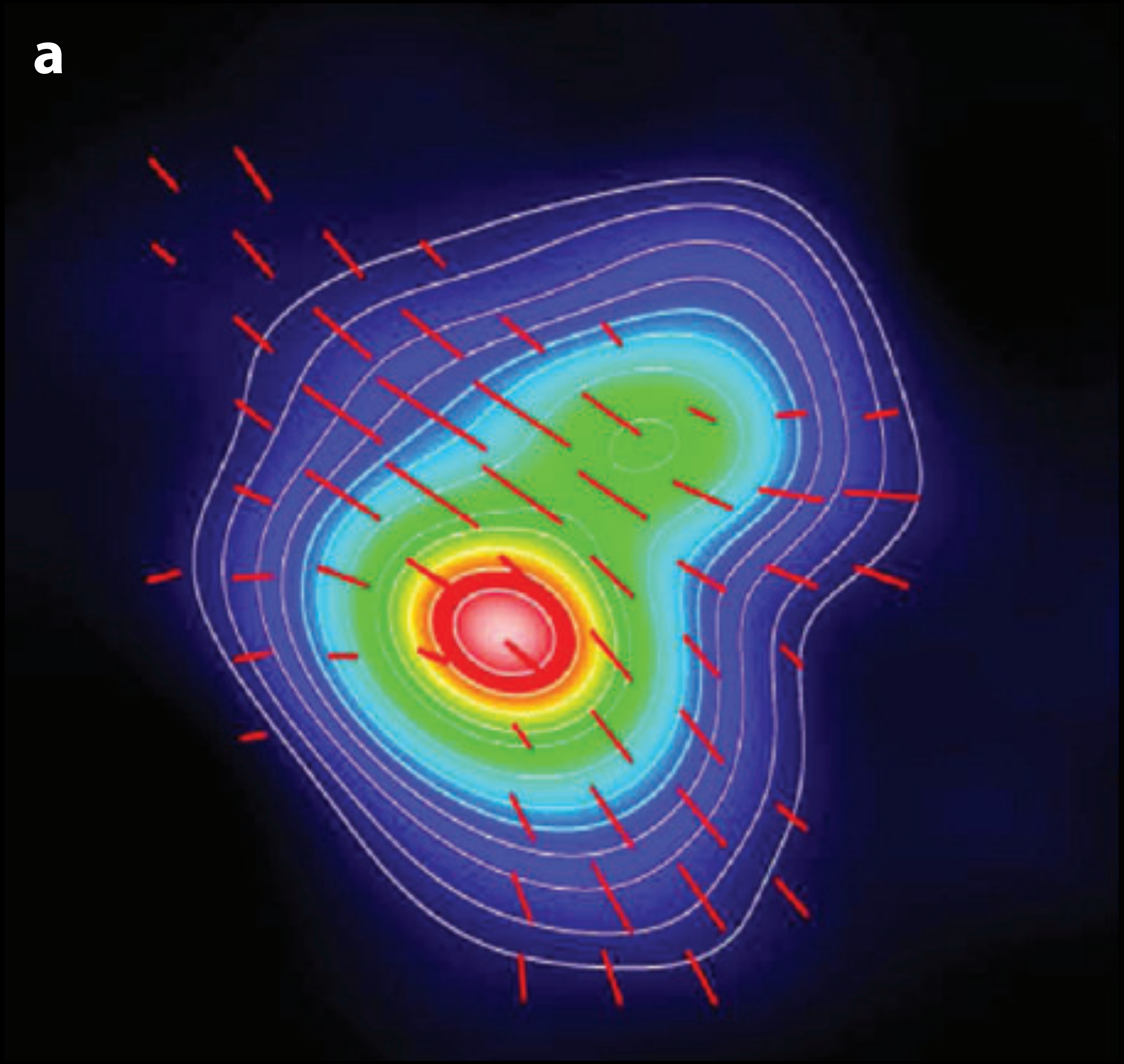
## A second method of detecting B fields

- Interstellar dust grains both emit and absorb light
- In general they are not spherical, so they emit or absorb more efficiently for light with linear polarisation along the long axis of the grain
- If grains are randomly oriented, there is no net effect, since averaging over many grains in different orientations averages out the efficiency
- However, in presence of B field, if grains are charged they can become preferentially aligned relative to field, so cancellation is imperfect: causes emitted or absorbed light to become linearly polarised
- Only sensitive to component of B field in the plane of the sky, not along LOS

# The CF method

## Estimating the field strength

- Polarisation vectors show direction of magnetic field, but not strength
- However, can estimate strength based on how much the vectors vary — called the Chandrasekhar-Fermi method
- Basic idea: if field is very strong compared to turbulence, magnetic tension holds field straight; if it is weak, field bends




Dust thermal emission (colour and contours) with polarisation vectors (red lines) (Crutcher 2012)

# Magnetised flows

## The magnetic Reynolds number

- Basic equation describing evolution of magnetic field is induction equation:

$$\frac{\partial \mathbf{B}}{\partial t} + \nabla \times (\underbrace{\mathbf{B}}_{\text{Magnetic field}} \times \underbrace{\mathbf{v}}_{\text{Velocity}}) = -\nabla \times (\underbrace{\eta}_{\text{Resistivity}} \nabla \times \mathbf{B})$$

- If resistivity is independent of position, this simplifies to

$$\frac{\partial \mathbf{B}}{\partial t} + \nabla \times (\mathbf{B} \times \mathbf{v}) = \eta \nabla^2 \mathbf{B}$$

- Final term has exact same form as viscosity term in momentum equation, so by analogy with Reynolds number, define magnetic Reynolds number:

$$\mathrm{Rm} = \frac{LV}{\eta}$$

Length scale of flow
Velocity scale of flow

# The significance of Rm

## Part I

- To understand why Rm matters, consider the magnetic flux $\Phi$ threading a fluid element of area $A$ with normal vector **n**: $\Phi = \int_A \mathbf{B} \cdot \hat{\mathbf{n}}\, dA$
- The time derivative of this is

Change in flux due to B field changing

Change in flux due to fluid element moving

$$\frac{d\Phi}{dt} = \int_A \frac{\partial \mathbf{B}}{\partial t} \cdot \hat{\mathbf{n}}\, dA + \oint_{\partial A} \mathbf{B} \cdot \mathbf{v} \times d\mathbf{l}$$

$\mathbf{v} \times d\mathbf{l}$ = area per time swept out by element $d\mathbf{l}$ of the boundary of $A$ as it moves

$$= \int_A \frac{\partial \mathbf{B}}{\partial t} \cdot \hat{\mathbf{n}}\, dA + \oint_{\partial A} \mathbf{B} \times \mathbf{v} \cdot d\mathbf{l}$$

- N.B. In second line we exchanged dot and cross products using $\nabla \cdot \mathbf{B} = 0$

# The significance of Rm

## Part II

- Now apply Stokes theorem to second term:

$$\frac{d\Phi}{dt} = \int_A \frac{\partial \mathbf{B}}{\partial t} \cdot \hat{\mathbf{n}}\, dA + \int_A \nabla \times (\mathbf{B} \times \mathbf{v}) \cdot \hat{\mathbf{n}}\, dA$$

- Note that integrand now is just LHS of induction equation, so we can replace it with the RHS; for constant resistivity, this is:

$$\frac{d\Phi}{dt} = \eta \int_A \nabla^2 \mathbf{B}\, dA = \frac{LV}{\mathrm{Rm}} \int_A \nabla^2 \mathbf{B}\, dA$$

- Implication: in the limit Rm → ∞, then d$\Phi$ / dt → 0; flux is frozen into a fluid element, and cannot change over time: “beads on a wire”

# Resistivity of a molecular cloud

## A very brief sketch

- Resistivity $\eta$ related to conductivity $\sigma$ by $\eta = c^2 / 4\pi\sigma$; conductivity is relates current density $J$ and applied electric field $E$: $J = \sigma E$
- Current is carried by electrons, $J = e\, n_e\, v_e$, so $\sigma = e\, n_e\, v_e / E$
- Electron speed $v_e$ set by balance between electric acceleration and scattering by $H_2$ molecules; acceleration ~ $E$, so we have $v_e$ ~ $E$ and $\sigma$ independent of $E$
- Detailed calculation gives $\sigma = \dfrac{n_e e^2}{m_e n_{\mathrm{H}_2} \langle \sigma v \rangle_{e-\mathrm{H}_2}} \sim 10^{-17} x \ \mathrm{s}^{-1}$ $\quad \eta \sim (10^3/x)\ \mathrm{cm}^2\ \mathrm{s}^{-1}$

  Ionisation fraction, typically ~$10^{-6}$

  Electron - $H_2$ cross section times velocity, averaged over Boltzmann distribution
- For L ~ 10 pc, V ~ few km $s^{-1}$, Rm ~ $10^{16}$!

# The Alfvén Mach number

## The last dimensionless number today… I promise

- Momentum conservation including Lorentz forces is

$$\frac{\partial}{\partial t}(\rho \mathbf{v}) = -\nabla \cdot (\rho \mathbf{v} \otimes \mathbf{v}) - \nabla P + \rho \nu \nabla^2 \mathbf{v} + \frac{1}{4\pi}(\nabla \times \mathbf{B}) \times \mathbf{B}$$

- Order of magnitude estimate of terms:

$$\frac{\rho V^2}{L} \sim \frac{\rho V^2}{L} + \frac{\rho c_s^2}{L} + \frac{\rho \nu V}{L^2} + \frac{B^2}{L} \quad \rightarrow \quad 1 \sim 1 + \frac{1}{\mathcal{M}^2} + \frac{1}{\mathrm{Re}} + \frac{B^2}{\rho V^2}$$

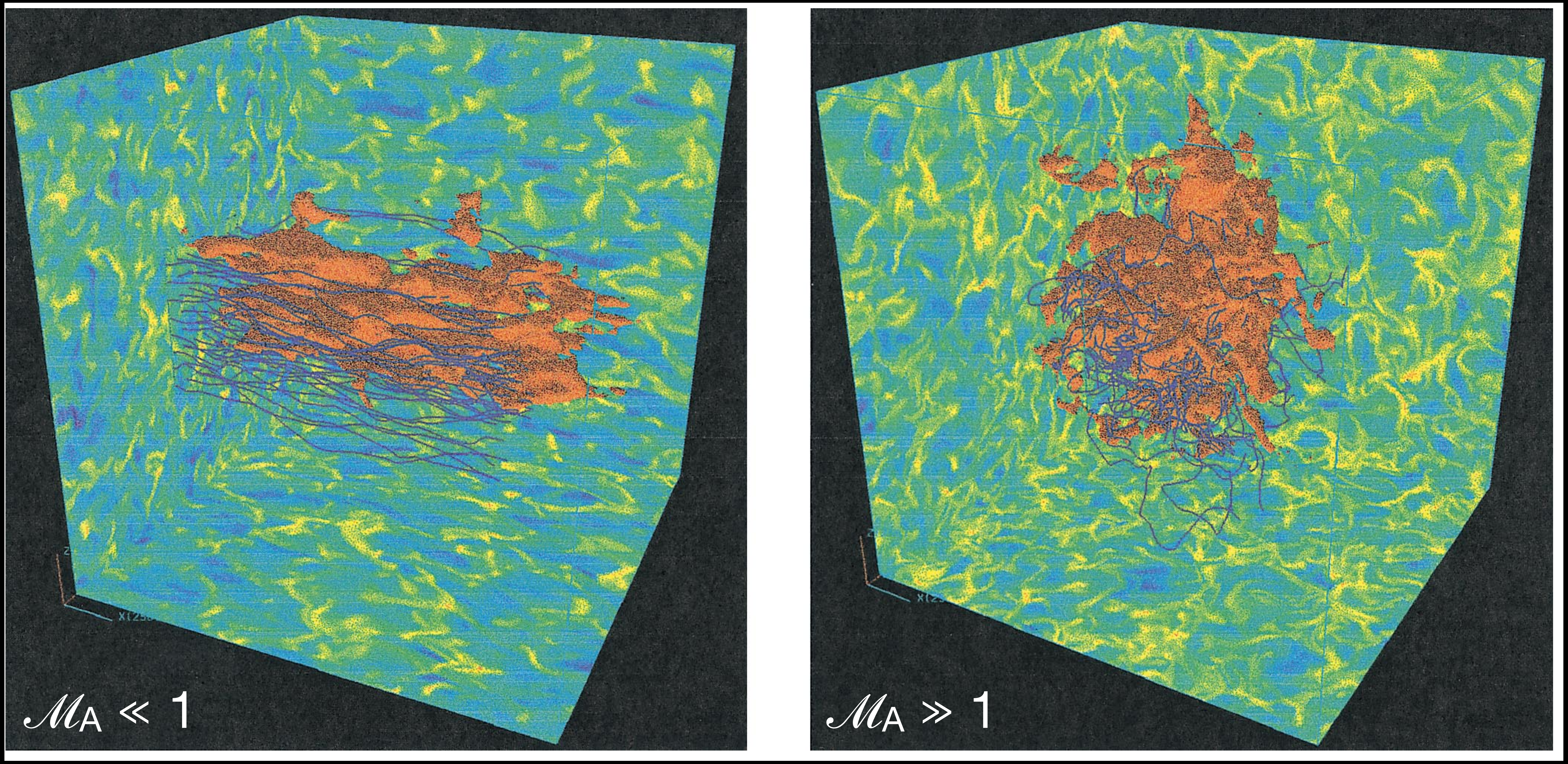
- Use the last term to define the Alfvén Mach number:

$$\mathcal{M}_A = \frac{V}{v_A} \qquad v_A = \frac{B}{\sqrt{4\pi\rho}}$$

- $\mathcal{M}_\mathrm{A}$ describes magnetic forces: $\mathcal{M}_\mathrm{A} \gg 1$ unimportant, $\mathcal{M}_\mathrm{A} \ll 1$ dominant

# Why the Alfvén Mach number matters

## in one picture




Stone+ 1998

# Non-ideal MHD effects

## Why and where

- We call the regime where Rm ≫ 1 ideal MHD — perfect flux freezing; deviations from this are non-ideal MHD effects
- Resistivity is the simplest non-ideal effect, but it is generally unimportant in star-forming regions, as we will show in a moment
- Two most important for star formation are (probably) ion-neutral drift (also called ambipolar diffusion) and turbulent reconnection
- Other non-ideal effect sometimes considered is Hall effect, but we will not discuss it in this class; maybe important in discs, but not elsewhere

# Ion-neutral drift

## The basics

- Only ions feel the Lorentz force from the magnetic field; ions then transmit force to neutrals via collisions
- If ion density is low enough, neutrals may go a long time before colliding with an ion; consequently, appreciable differences between ion and neutral velocities can build up
- Since B field is tied to ions, this effect allows the neutrals (which dominate the mass) to “slip through” the magnetic field lines — violation of flux freezing
- This effect is potentially critical to explaining weak B fields of stars (see homework)

# Ion-neutral drift

## Calculation of the effective resistivity I

- Ions feel Lorentz force and drag force:

Drag coefficient; set by microphysics, $\gamma \approx 9 \times 10^{-13}$ cm$^3$ s$^{-1}$

Neutral and ion densities

Difference in ion and neutral velocities

$$\mathbf{f}_L = \frac{1}{4\pi} (\nabla \times \mathbf{B}) \times \mathbf{B} \qquad \mathbf{f}_d = \gamma \rho_n \rho_i (\mathbf{v}_i - \mathbf{v}_n)$$

- Since ion density is very low, reasonable to approximate that they have no inertia, are always in balance between these forces, so drift speed is

$$\mathbf{v_d} = \mathbf{v}_i - \mathbf{v}_n = \frac{1}{4\pi\gamma\rho_n\rho_i} (\nabla \times \mathbf{B}) \times \mathbf{B}$$

- Assume Rm = ∞ for ions alone, so induction equation for magnetic field is

$$\frac{\partial \mathbf{B}}{\partial t} + \nabla \times (\mathbf{B} \times \mathbf{v}_i) = 0$$

# Ion-neutral drift

## Calculation of the effective resistivity II

- Starting from induction equation for ions, replace ion velocity with neutral velocity using calculated drift speed:

$$\frac{\partial \mathbf{B}}{\partial t} + \nabla \times (\mathbf{B} \times \mathbf{v}_n) = \nabla \times \left\{ \frac{\mathbf{B}}{4\pi\gamma\rho_n\rho_i} \times [\mathbf{B} \times (\nabla \times \mathbf{B})] \right\}$$

- Recall induction equation with resistivity is $\frac{\partial \mathbf{B}}{\partial t} + \nabla \times (\mathbf{B} \times \mathbf{v}) = -\nabla \times (\eta \nabla \times \mathbf{B})$, so term on RHS looks like a (non-constant, vector) resistivity

- Just taking an order of magnitude estimate for analysis purposes, we have

$$\eta_{in} \approx \frac{B^2}{4\pi\gamma\rho_i\rho_n} \qquad \mathrm{Rm} \approx \frac{4\pi L V \gamma\rho_i\rho_n}{B^2}$$

# Ion-neutral drift

## Ionisation fraction and dissipation scale

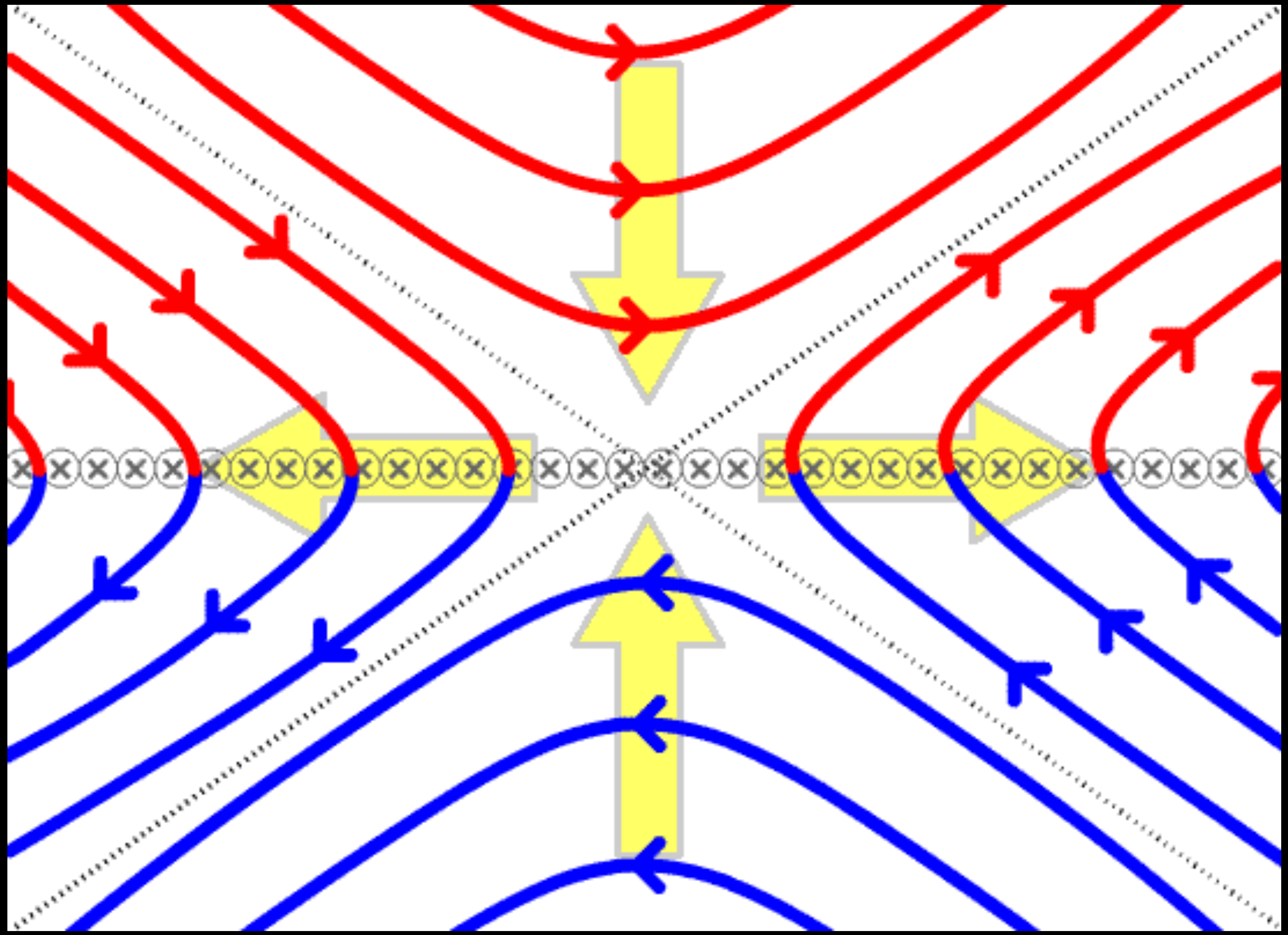
- Typical ionisation fraction at a density of $n \sim 100$ cm$^{-3}$ is $\sim 10^{-6}$, so $\rho_n \sim 100$ $m_H$, $\rho_i \sim 10^{-6}\rho_n$ → for $L \sim$ few × 10 pc, $V \sim$ few km s–1, $B \sim 10$ $\mu$G, we have Rm ~ 50
- Can also define an associated ion-neutral dissipation length scale = value of $L$ for which Rm ~ 1; this is $L_{in} \sim 0.1$ pc
- Physical meaning: $B$ field frozen in to matter on larger scales, but on small scales neutrals are able to slip through field lines

# Turbulent reconnection

- Reconnection occurs when magnetic field lines of opposite direction are forced into each other by the flow; the field rearranges its topology

- Expected reconnection rate for high Rm is very low, but observed reconnection rates in laboratory plasmas and Solar flares are much faster than expected — not understood

- May occur in molecular clouds too; currently not known



# Turbulent reconnection

## A second possibility

- Reco