Class 18: Protostellar evolution

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Outline

- Models and methods
  - Timescale hierarchy
  - Evolution equations
  - Boundary conditions
  - The role of deuterium
- Qualitative results: major evolutionary phases
- Evolution of protostars on the HR diagram
  - The birthline
  - The Hayashi and Heyney tracks

Timescale hierarchy

Similarities and differences to main sequence stars

- Protostellar evolution governed by three basic timescales:
  - Mechanical equilibration: $t_{\mathrm{mech}} \sim (R \,/\, c_s) \sim (R^3 \,/\, GM)^{1/2} \sim$ few hours
  - Energy equilibration (Kelvin-Helmholtz): $t_{\mathrm{KH}} = GM^2 \,/\, RL \sim 1$ Myr
  - Accretion: $t_{\mathrm{acc}} = M \,/\, (\mathrm{d}M \,/\, \mathrm{d}t) \sim 0.1$ Myr
  - Numerical values for $M = \mathrm{M}_\odot$, $R = 3\ \mathrm{R}_\odot$, $L = 10\ \mathrm{L}_\odot$, $\mathrm{d}M \,/\, \mathrm{d}t = 10^{-5}\ \mathrm{M}_\odot\ \mathrm{yr}^{-1}$
- Implication: protostars are always in mechanical equilibrium, but are not generally in energy equilibrium while they are forming
- By contrast, main sequence stars in energy equilibrium because $t_{\mathrm{KH}} \ll$ lifetime
- This hierarchy applies to low-mass stars; somewhat different for high mass

Equations of stellar structure

For protostars

- Equations describing mass conservation, hydrostatic balance, and energy transport are exactly the same as for main sequence stars:

$$\frac{\partial r}{\partial M} = \frac{1}{4\pi r^2 \rho}$$ Mass conservation

$$\frac{\partial P}{\partial M} = -\frac{GM}{4\pi r^4}$$ Hydrostatic balance

$$\frac{\partial T}{\partial M} = -\frac{3\kappa_R L}{256\pi^2 \sigma_{\mathrm{SB}} T^3 r^4}$$ Radiative flux (or equivalent equation for convective flux in convective regions)

- Major difference is in energy equation: in a MS star, this would be $\frac{\partial L}{\partial M} = \epsilon_{\mathrm{nuc}}$

Nuclear energy generation rate

- However, this is only true if the stars is in energy equilibrium; we cannot assume that this is the case for a protostar

The non-equilibrium energy equation

How protostars differ from main sequence stars

- In a protostar, there is an additional potential source of energy: a mass shell can lose energy, which is added to the energy change across that shell
- To include this use fundamental thermodynamic relation $dU = P\,dV + T\,dS$: change in energy equals work done + temperature times change in entropy
- No change in volume ($dV = 0$), so energy equation changes to $\frac{\partial L}{\partial M} = \epsilon_{\rm nuc} - T\frac{\partial s}{\partial t}$
- Computational procedure in practice: (1) given starting state $r(M)$, $T(M)$, solve flux equation to get $L(M)$; (2) use $L(M)$ in energy equation to find ds/dt in every shell; (3) update entropy of every shell; (4) solve equation of hydrostatic equilibrium to find new $r(M)$, $T(M)$, fixing entropy in each shell; (5) repeat

Boundary conditions for protostars

The second difference

- System of equations for $r(M)$, $T(M)$, $P(M)$, $L(M)$ needs four boundary conditions
- Two at inner boundary are same as for main sequence stars: $r(0) = 0$, $L(0) = 0$
- If protostar is not accreting, outer boundary conditions are the same as for main sequence stars as well: $P(M_{\mathrm{tot}}) = 0$ (or something small if we are being more sophisticated), $L(M_{\mathrm{tot}}) = 4\pi r^2(M_{\mathrm{tot}})\sigma_{\mathrm{SB}}T^4(M_{\mathrm{tot}})$
- BCs for accreting star are different because (1) accretion flow potentially provides finite pressure at outer radius, (2) accretion flow restricts escape of energy from star

Spherical accretion flows

Also known as “hot accretion”

- If accretion flow is spherical, ram pressure of infall is $\rho_i v^2$
- Density determined by mass accretion rate: $\dot{M} = 4\pi r^2 \rho_i v$
- If material arrives at free-fall speed, bounding pressure is $P(M) = \frac{\dot{M}}{4\pi}\sqrt{\frac{2GM}{R^5}}$
- Luminosity at surface takes form $L(M) = 4\pi R^2 \sigma_{\rm SB} T^4 + f_{\rm in}\frac{GM\dot{M}}{R}$
- First term is usual blackbody radiation, second is mechanical energy provided by accretion flow — but only a fraction $f_{\rm in}$ of this is advected inward at accretion shock, with rest escaping immediately as radiation
- Evaluation of shock properties suggests $f_{\rm in} \approx 1/4$

Magnetically-channeled accretion flows

Also known as “cold accretion”

- Accretion flow onto star may cover only a small portion of surface due to channeling by stellar magnetic field
- In this case, material being added to the star provides no confining pressure, and radiates away all its energy (and thus is added to star “cold”, with some specified, usually much lower, entropy) — then BCs are the same as for main sequence stars
- This generally produces much smaller predicted radii for protostars, because material being added to star has far lower entropy
- Reality probably somewhere between hot and cold extremes, but still debated

The importance of deuterium

The first thing that burns

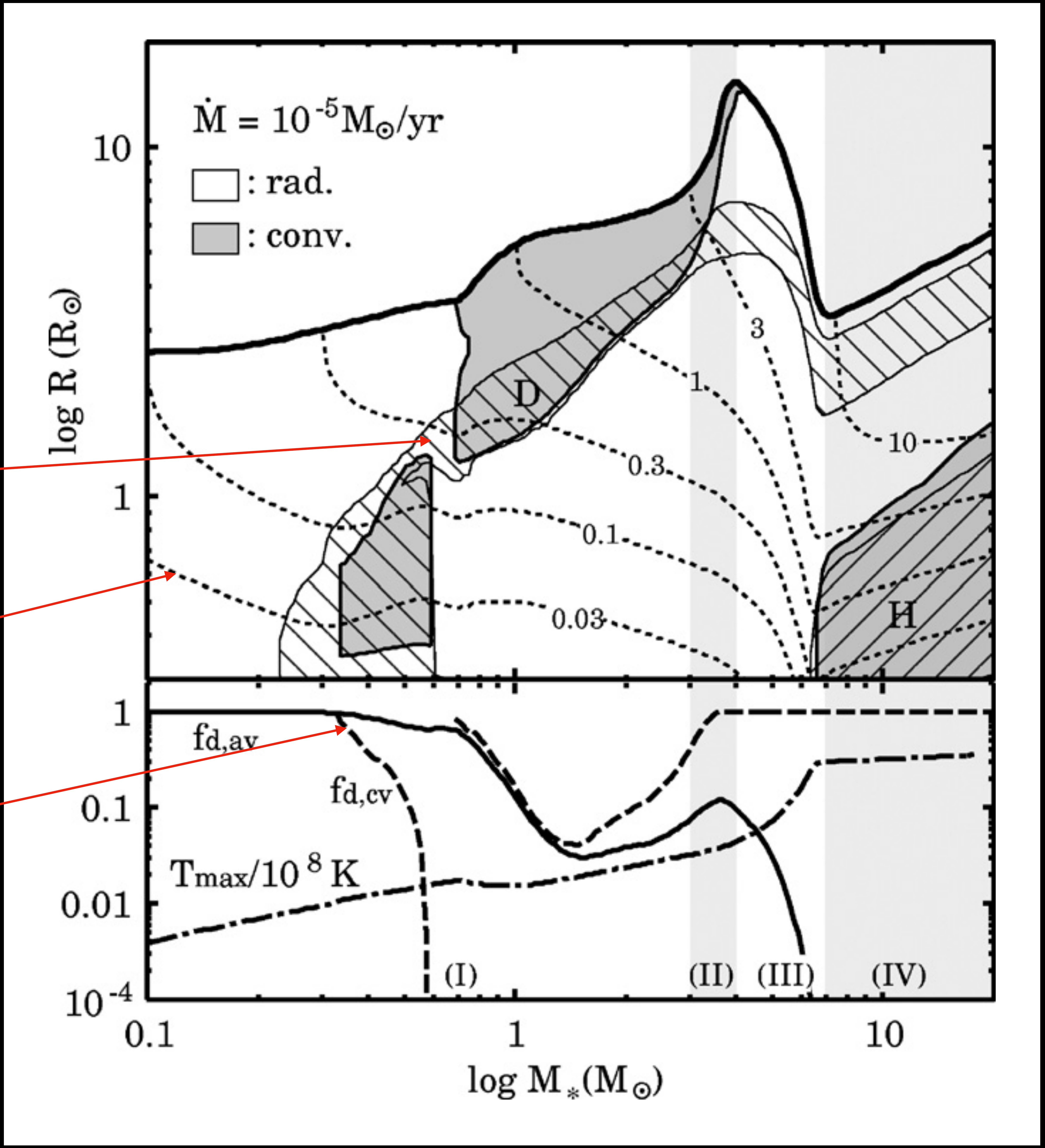
- By definition, protostellar phase is phase before star ignites H and settles onto main sequence
- However, BBN produced $\approx 2 \times 10^{-5}$ D/H, and D burns more easily than H because there is no need to wait for the weak nuclear force to convert protons to neutrons; ignites at $\sim 10^6$ K (compared to $\sim 10^7$ K for H)
- Basic reaction is $^2H + {}^1H \rightarrow {}^3He$; releases 5.5 MeV / D burned
- Timescales: energy release comparable to H burning (7 MeV / nucleon), but fuel supply smaller by factor of 2×10^{-5}; since H lasts $\sim 10^{10}$ yr, expect D to last $\sim$few $\times 10^5$ yr — comparable to accretion time

Basic outline of evolution

Nuclear burning regions

Radii of Lagrangian mass shells

Fraction of initial D remaining in all of star (solid) and in convective zone (dashed)

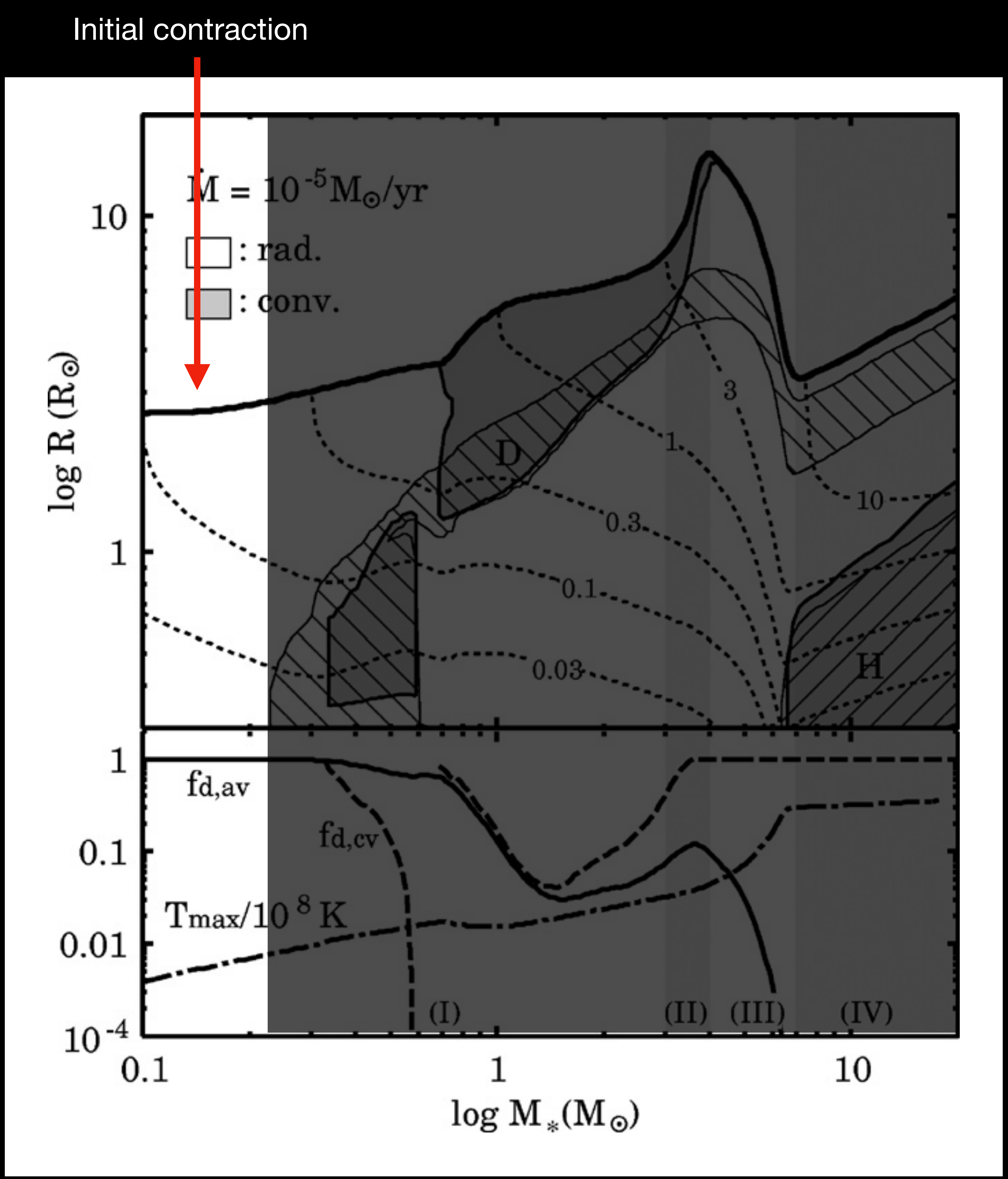


Kippenhahn diagram — Hosokawa & Omukai (2009)

Initial contraction

Evolutionary phase I

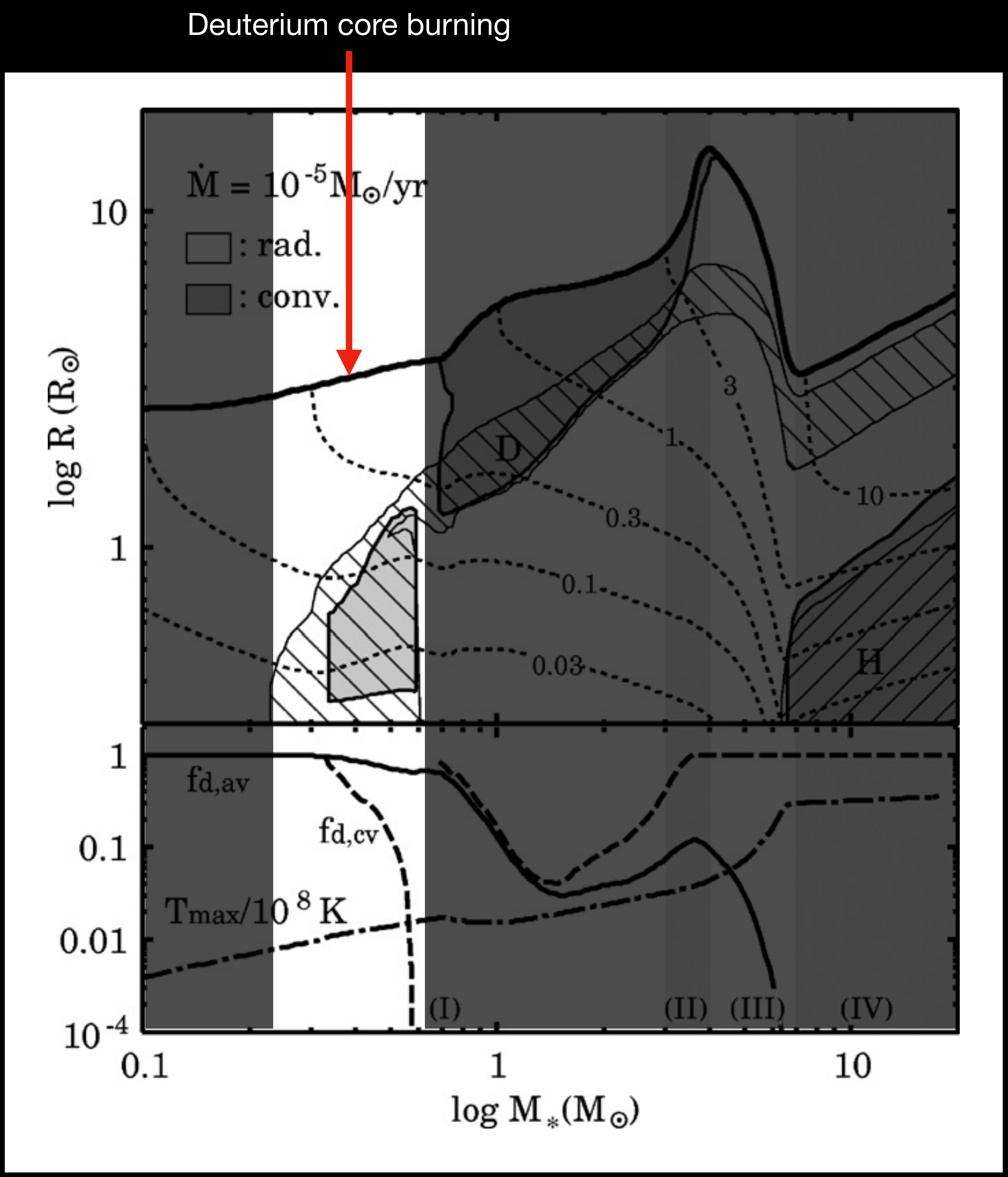
- At first core is too cool to burn anything, so star just radiates and contracts
- For hot accretion, radius fixed by *M* and accretion rate, due to self-regulation:
  - If R is too big, newly-accreted gas loses entropy easily and star shrinks
  - If R is too small, newly-accreted gas can't lose entropy, star grows
- Cold accretion → radius determined by choice of entropy of material added



Deuterium core burning

Evolutionary phase II

- Once core contracts and stellar mass rises enough to reach T ~ 10^6 K, D ignites
- D burning adds entropy to core, starting convection; star close to *n*=3/2 polytrope
- D burning very sensitive to temperature, so core temperature stays nearly fixed
- For polytrope, T_c determines surface escape speed: $v_{\rm esc}^2 = 2T_n k_B T_c/\mu m_{\rm H}$ → universal value of energy / mass accreted $\psi = v_{\rm esc}^2/2 \approx 2.5\times 10^{14}\ {\rm erg\ g^{-1}}$

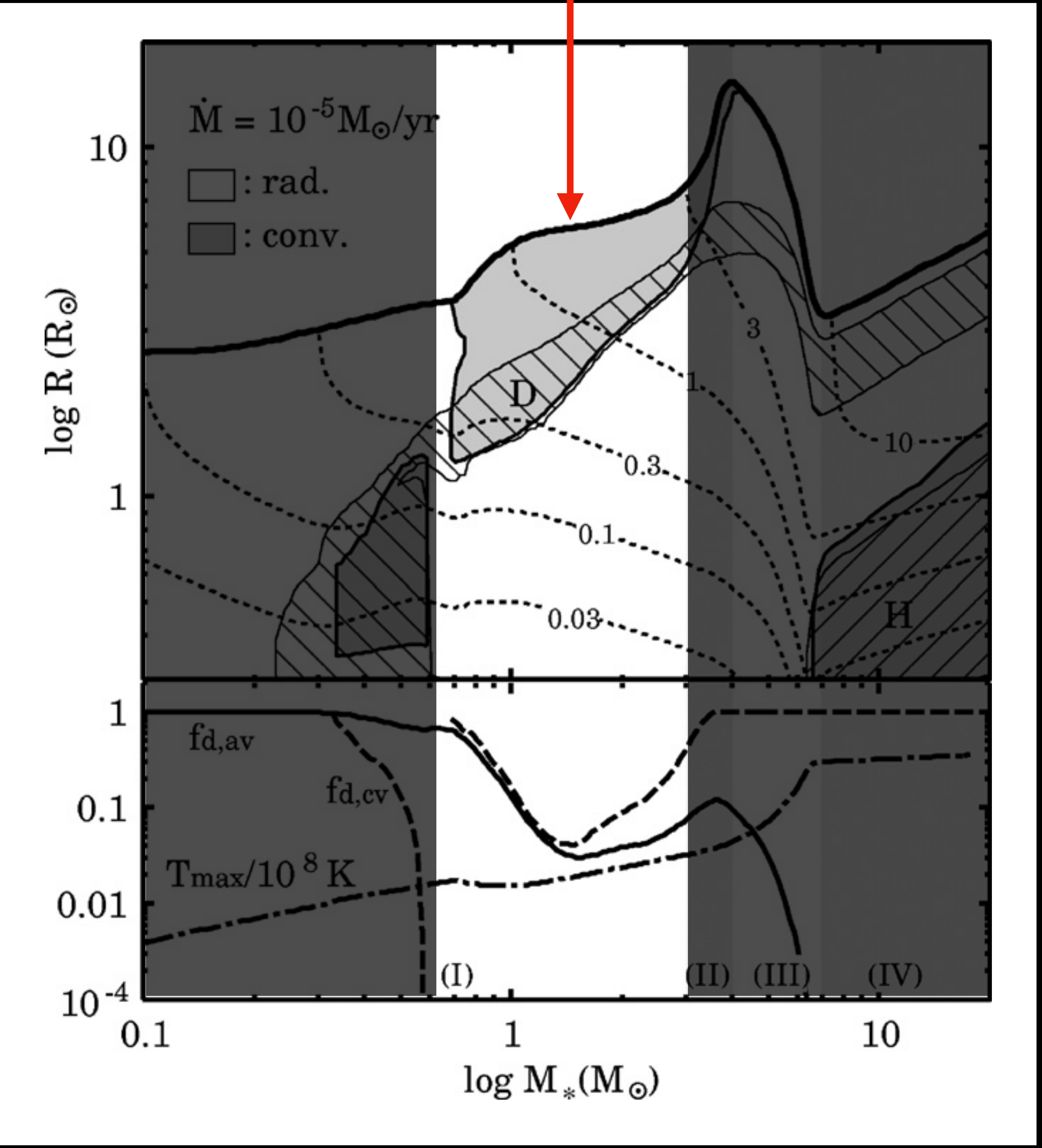


Deuterium shell burning

Evolutionary phase III

- After ~10^5 yr, D in core too depleted to prevent further contraction
- Core resumes heating, leading to falling opacity ($\kappa_R \propto T^{-3.5}$); eventually core switches from convective to radiative
- This cuts off supply of D to core, core stops burning: “radiative barrier”
- However, accreting D still burns in a shell above the core, driving convection

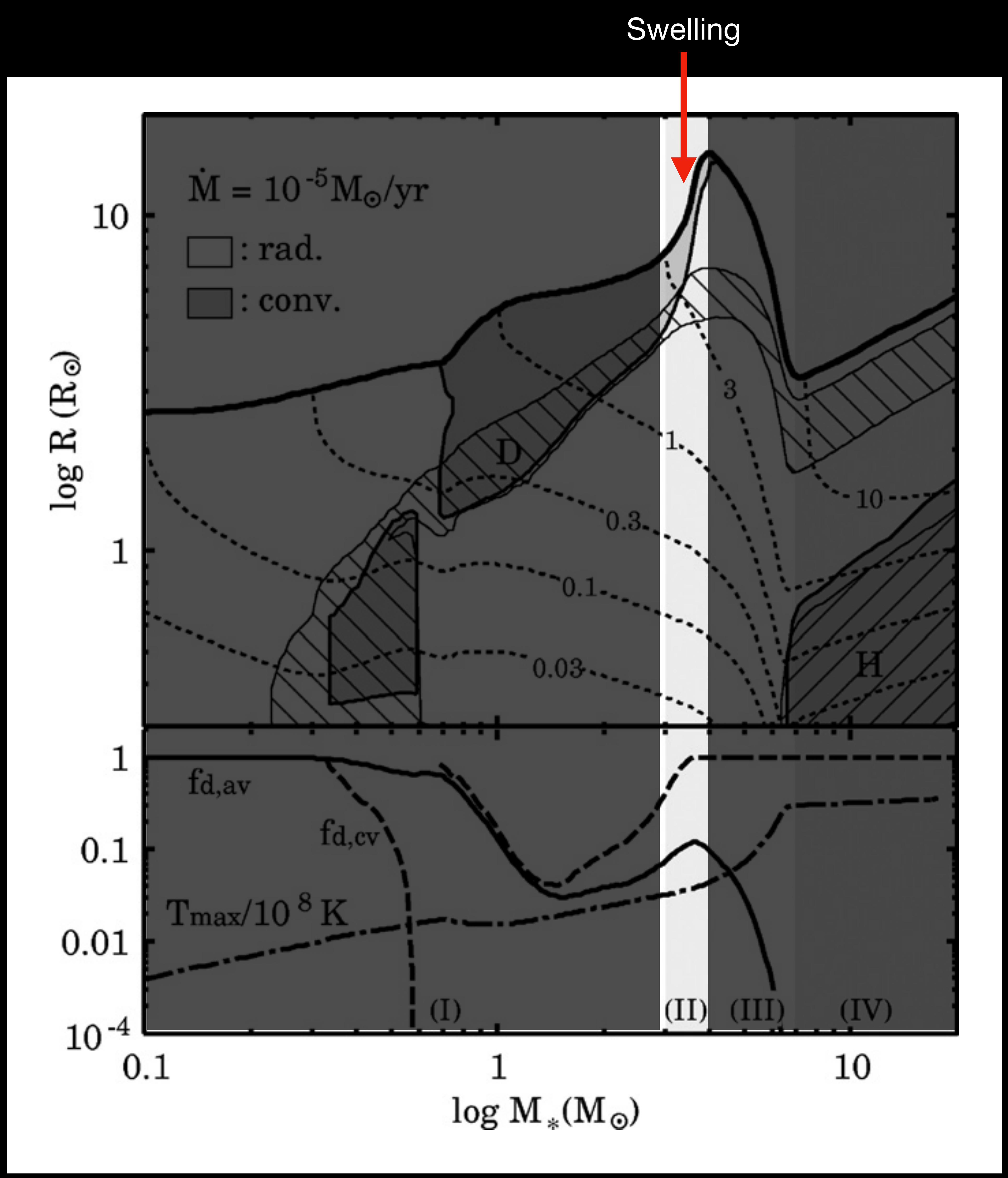
Deuterium shell burning



Swelling

Evolutionary phase IV

- Continued rise in T_c lowers core opacity, allowing rapid loss of entropy
- Entropy moves out in a wave, but is "trapped" in cooler outer layers of star, leads to rapid growth of stellar radius
- For high accretion rates, radius can reach ~100 $R_\odot$ (red giant size)
- Only occurs for stars $\gtrsim$ 3 $M_\odot$; less massive stars skip this phase

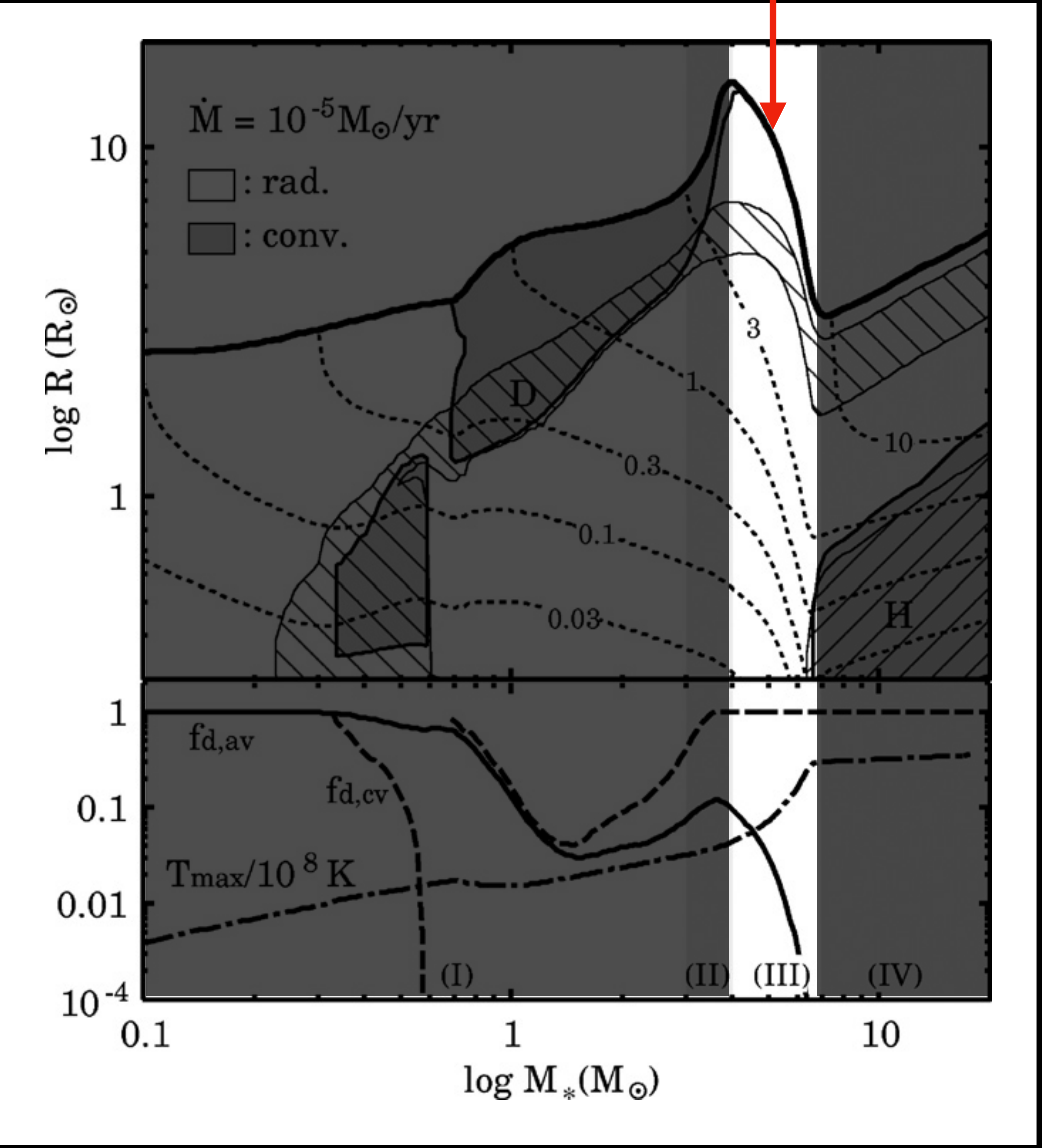


Final contraction

Evolutionary phase V

- Once entropy wave reaches surface, star rapidly shrinks
- D burning in a shell continues, but energy limited by rate at which new D falls onto star; not enough to hold up interior
- Contraction only halts once $T_c \sim 10^7$ K and H ignites; at this point star is on the main sequence

Final contraction



Protostars in the HR diagram

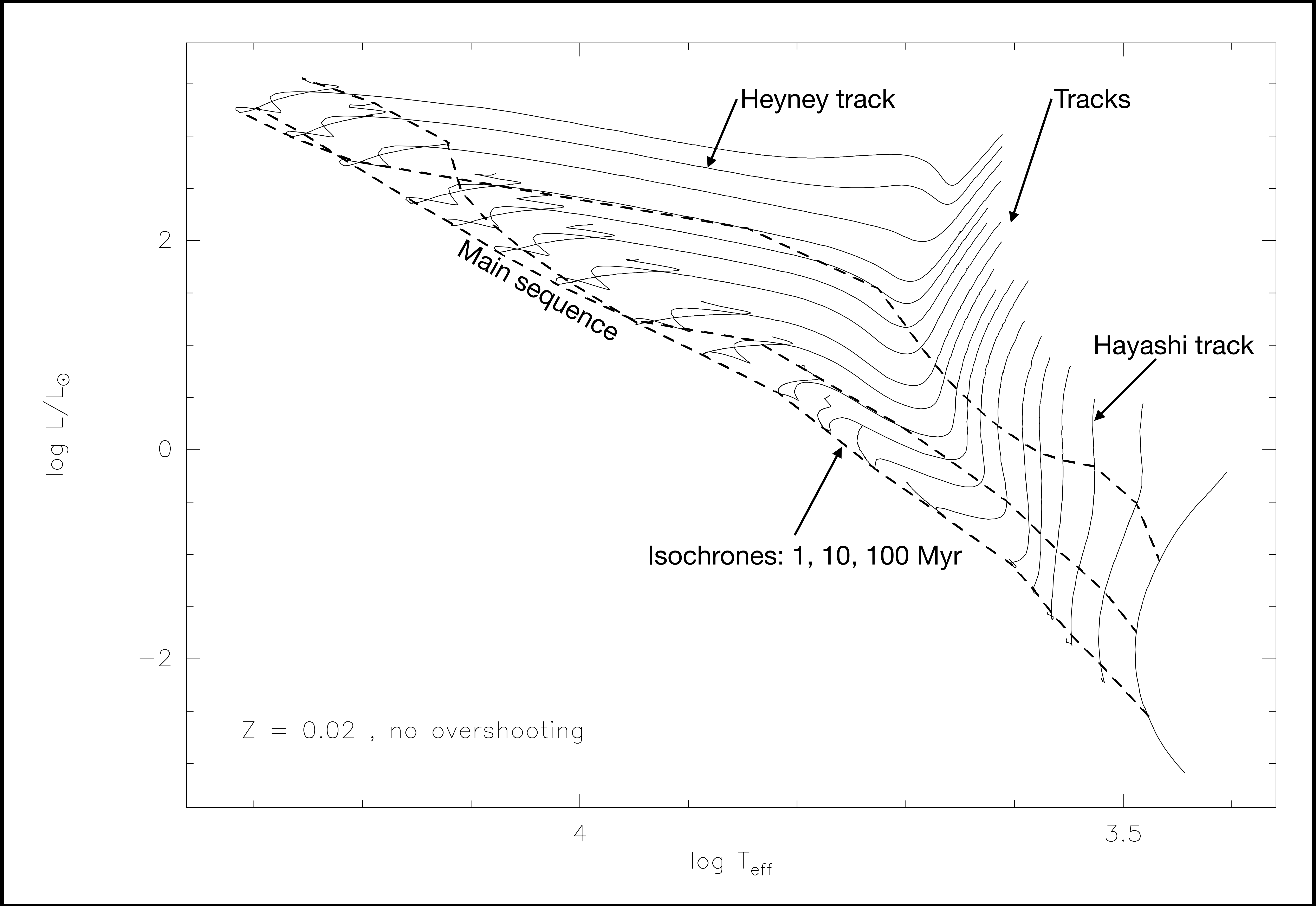
The birth line

- Protostars have larger radii → lower temperature and/or higher luminosity than main sequence stars of same mass, so they lie above the main sequence in the HR diagram

- Zone protostars occupy limited on low *L* side by main sequence, on high *L* side because stars not optically visible while accreting rapidly → can't be placed on HR diagram until accretion almost over, so not visible when *L* is at its highest

- Locus where stars stop accreting and become visible is called the "birth line"; thus zone on HR diagram occupied by protostars is region between birth line and main sequence

The HR diagram



Siess et al. 2000

The Hayashi limit

Dominant effect for low-mass protostars

- At $T \lesssim 5000$ K, H is almost entirely neutral; almost no free electrons, so very low opacity
- Dominant remaining source of opacity is H^-, produced by free electrons released by metal atoms with very low ionisation potentials
- Since this depends on thermal ionisation of metals, opacity incredibly sensitive to temperature, $\kappa_R \sim T^b$ with $b \sim 4 - 9$
- This effectively sets a minimum surface temperature ~3000 - 3500 K for stars: if temperature is below this, gas not heated enough by stellar radiation to be able to hold itself up against gravity

Consequences of the Hayashi limit

The Hayashi and Henyey tracks

- Low-mass protostars run up against the Hayashi limit, so the evolve toward the main sequence at nearly constant $T = T_{\mathrm{H}}$
- Temperatures slightly different for different masses, due to different strengths of gravity setting different minimum opacities for hydrostatic balance
- Consequence: series of parallel tracks in HR diagram that are nearly vertical — constant T, decreasing L — as stars evolve
- Once stars are above T_{H}, can start to evolve increase in effective temperature, luminosity changes much more gradually — nearly horizontal evolution in HR diagram, called Heyney track