Class 17: Protostar formation

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Outline

- Thermodynamics of a collapsing core
  - The gas / dust cooling transition
  - The isothermal / adiabatic transition
  - The first core
  - Second collapse and the second core
- The protostellar envelope
  - Accretion luminosity
  - The dust destruction front
  - Temperature structure beyond the dust destruction front

Thermodynamics of collapse

General considerations

- In bulk of GMC ($n \lesssim 10^3$ cm^{-3}), gas temperature is set by balance between CR heating and line cooling
- CR heating rate per atom nearly independent of density, line cooling rate only weakly dependent on density due to high optical depth
- However, there are other processes whose rates increase with density, so as density increases, these start to take over heating and cooling
- As a result, a collapsing core passes through a few different thermodynamic regimes, where different processes dominate — our goal is to explore these

The line - dust cooling transition

First step

- Dust particles are solid, so absorb and emit much more efficiently than gas; as a result, grains always reach equilibrium with radiation field

- Coupling between dust and gas is via collisions; exchange rate per atom:

$$\underbrace{\Psi_{\rm gd}}_{\text{Energy exchange per atom}} = \underbrace{\alpha_{\rm gd}}_{\text{Coupling coefficient: depends on dust abundance and size distribution}} \underbrace{n}_{\text{Linear dependence on density}} \underbrace{T_g^{1/2}}_{\text{Depends on gas temperature due to change in mean particle velocity}} \underbrace{(T_d - T_g)}_{\text{Mean energy transfer / collision proportional to temperature difference}}$$

- Linear dependence on density means dust-gas coupling dominates over CRs and line cooling once *n* is high enough

- For MW conditions, dust dominates above ~10^4 - 10^5 cm^{-3}

- Above that density, gas temperature locked very close to grain temperature

Physics question: how do you expect the grain-gas coupling density to differ for galaxies of lower metallicity than the MW?

Collapse in dust-dominated phase

Implications of dust-gas coupling

- Except close to local sources, dust maintains ~constant temperature due to uniform background ISRF; small variations due to varying levels of extinction
- This keeps gas close to isothermal as long as it is well-coupled to dust
- Isothermal gas able to collapse freely, because thermal pressure term in virial theorem is ~constant, while gravitational term increases as collapse proceeds
- In order to slow collapse, thermal pressure term must be able to rise
- For spherical geometry with $P \propto \rho^\gamma$ ($T \propto \rho^{\gamma-1}$), stable hydrostatic configurations exists only for $\gamma > 4/3$; so when does this condition begin to hold?

The isothermal - adiabatic transition

The optically thin case

Thermal energy / mass

Heating rate / mass

Cooling rate / mass

- Energy conservation: $\frac{de}{dt} = \Gamma - \Lambda$

Volume / mass

Pressure

- At high *n*, main heating is *P* d*V* work by gravitational compression: $\Gamma = -P \frac{d}{dt}\left(\frac{1}{\rho}\right)$
- Time derivative term must be of order $1/\rho t_{\mathrm{ff}}$, so $\Gamma \approx c_s^2 \sqrt{4\pi G \rho}$
- Cooling is via thermal radiation by dust grains; first consider case where region is optically thin, so $\Lambda \approx 4\kappa_{\mathrm{P}}\sigma_{\mathrm{SB}}T^4$

Planck mean opacity of dust, κ_{P} ~ 0.01 cm^2 g^{-1} in MW

- In this case, heating begins to exceed cooling ($\Gamma > \Lambda$) for

$$\rho \gtrsim \frac{4\kappa_{\mathrm{P}}^2\sigma_{\mathrm{SB}}^2\mu^2 m_{\mathrm{H}}^2 T^6}{\pi G k_{\mathrm{B}}^2} \approx 5\times 10^{-15}\ \mathrm{g\ cm^{-3}}$$

The isothermal - adiabatic transition

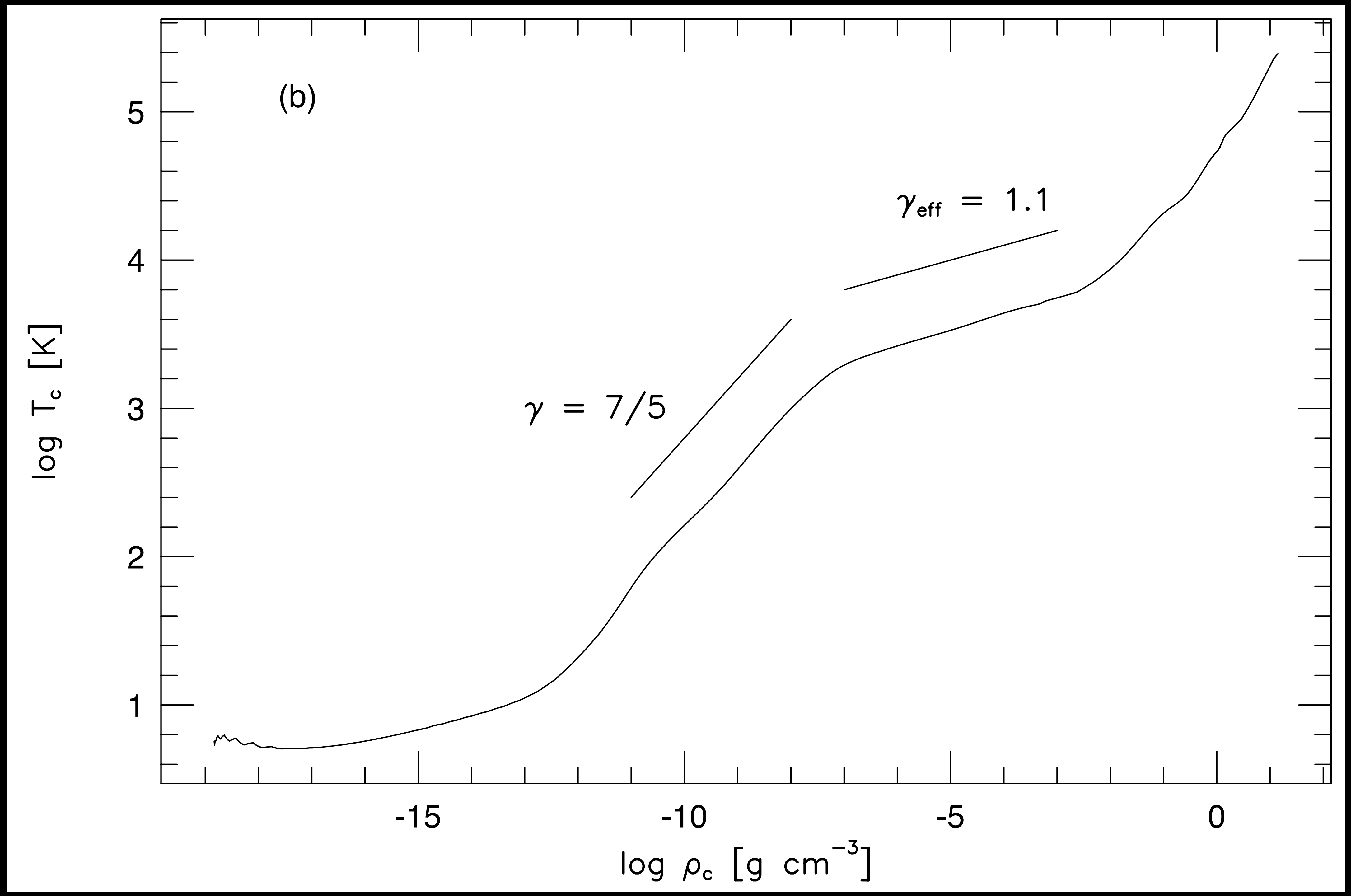
The optically thick case

- Consider collapsing region of characteristic size $R \sim \lambda_J = (\pi c_s^2 / G\rho)^{1/2}$
- Region is optically thick if $\tau = 2\kappa_P \rho R \gtrsim 1$ → $\rho \gtrsim \frac{\mu m_H G}{4\pi\kappa_P^2 k_B T} \approx 1.5 \times 10^{-13} \text{ g cm}^{-3}$
- In MW, thin-thick transition density > density where optically thin heating can't keep up with cooling, so can use thin case — however, depends on κ_P and T
- Thus may need to consider thick case too; in this case cooling rate reduced by factor of ~$1/\tau$ compared to thin case → $\Lambda \sim \frac{\kappa_P \sigma_{SB} T^4}{\kappa_P^2 \rho^2 R^2}$
- For this case, heating > cooling for $\rho \gtrsim \left(\frac{G\sigma_{SB}^2 \mu^4 m_H^4 T^4}{4\pi^3 k_B^4 \kappa_P^2}\right)^{1/3} \approx 5 \times 10^{-14} \text{ g cm}^{-3}$

Consequences of cooling not keeping up

The adiabatic phase

- Once cooling can't keep up, energy released by gravity goes into heat
- Gas at $\rho \gtrsim 10^{-13}$ g cm^{-3} is therefore close to adiabatic — constant entropy
- Temperature rises as $T \propto \rho^{\gamma-1}$, with $\gamma \approx$ 5/3 at low T (before H is rotationally-excited), $\gamma \approx 7/5$ at higher T
- Since $\gamma > 4/3$, pressure can halt collapse



Masunaga & Inutsuka 2000

The first core

Discovered by Richard Larson in 1969

- Switch to $\gamma > 4/3$ leads to formation of a pressure-supported object known as the "first core" or "first Larson core"
- Since gas is adiabatic, can model as a polytrope with $n = 1/(\gamma - 1) \approx 3/2$ or $5/2$
- Quoting general result for polytropes:

ξ_1 , $(-\xi^2\, d\theta/d\xi)_1$ = numerical constants of O(1) that just depend on n

$$R = a\xi_1$$

$$M = 4\pi a^3 \rho_c \left(-\xi^2 \frac{d\theta}{d\xi}\right)_1$$

$$a^2 = \frac{(n+1)}{4\pi G} K \rho_c^{(1-n)/n}$$

Polytropic constant, $K = P / \rho^\gamma$; set by gas specific entropy

Central density

Properties of the first core

From polytropic analysis

- Specific entropy of gas in first core fixed at point of isothermal-adiabatic transition: $P = \rho_{\mathrm{ad}} k_{\mathrm{B}} T_{\mathrm{ad}} / \mu m_{\mathrm{H}} \rightarrow K = \rho_{\mathrm{ad}}^{1-\gamma} k_{\mathrm{B}} T_{\mathrm{ad}} / \mu m_{\mathrm{H}}$
- Can plug this in to get mass and radius of first core as a function of ρ_{c}:

$$R = 2.2\ \mathrm{AU} \left(\frac{T_{\mathrm{ad}}}{10\ \mathrm{K}}\right)^{1/2} \left(\frac{\rho_{\mathrm{ad}}}{10^{-13}\ \mathrm{g\ cm^{-3}}}\right)^{-1/3} \left(\frac{\rho_{\mathrm{c}}}{10^{-10}\ \mathrm{g\ cm^{-3}}}\right)^{1/6}$$

$$M = 0.059\ \mathrm{M}_{\odot} \left(\frac{T_{\mathrm{ad}}}{10\ \mathrm{K}}\right)^{1/2} \left(\frac{\rho_{\mathrm{ad}}}{10^{-13}\ \mathrm{g\ cm^{-3}}}\right)^{-1/3} \left(\frac{\rho_{\mathrm{c}}}{10^{-10}\ \mathrm{g\ cm^{-3}}}\right)^{7/6}$$

N.B. numerical values for n = 3/2; nearly identical for 5/2

- Reason for choosing this ρ_{c} will become apparent shortly; bottom line for now: first core has mass ~few x 0.01 $\mathrm{M}_{\odot}$, radius ~ few AU

Observers are looking for first cores, and there are a few candidates. Based on what you know about them, how would you go about looking for a first core?

Second collapse

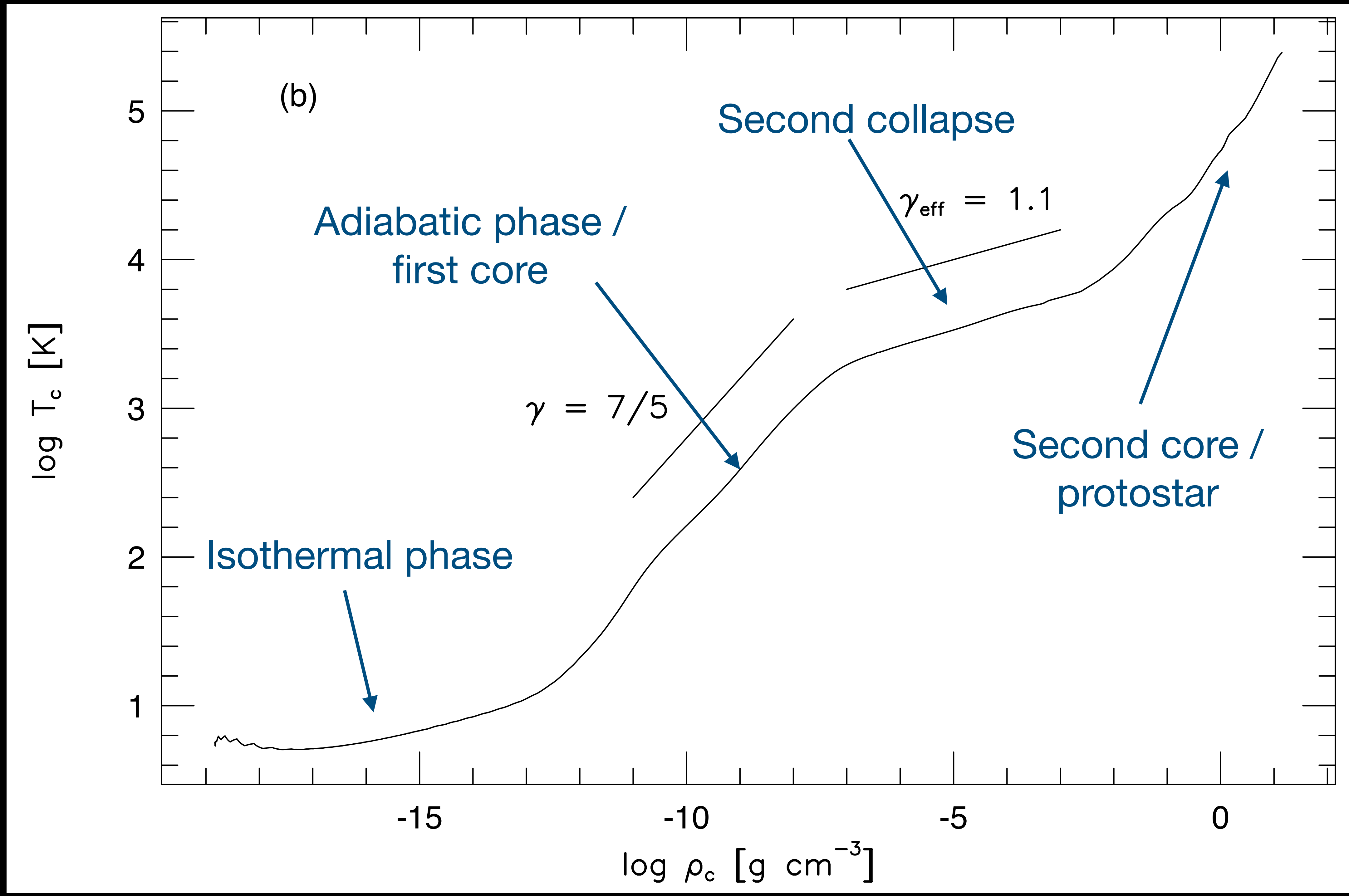
H_2 dissociation

- Central temperature of first core: $T_c = T_{\rm ad}\left(\frac{\rho_c}{\rho_{\rm ad}}\right)^{\gamma-1}$
- For $T_{\rm ad}$ = 10 K, $\rho_{\rm ad} = 10^{-13}$ g cm^{-3}, $\gamma = 5/3$, $T_{\rm c}$ = 1000 K for $\rho_{\rm c} = 10^{-10}$ g cm^{-3}: central temperature reaches ~1000 K once core reaches mass ~0.05 $M_\odot$
- Temperature of 1000 K is significant because binding energy of H2 is 4.5 eV = $5.4 \times 10^4\, k_{\rm B}$ K
- Rule of thumb: collisional dissociations start happening once temperature is high enough so that $k_{\rm B}T$ ~ few percent of binding energy — ~1000 K for H_2
- Thus collisional dissociation becomes significant at first core mass ~0.05 $M_\odot$

Physics question: once H_2 dissociation starts, how will this affect the temperature? Hint: this is like boiling water.

Consequences of collisional dissociation

- Dissociation make nearly isothermal again ($\gamma \approx 1.1$), because all energy from compression goes into dissociation
- Since $\gamma < 4/3$, core cannot be stable → near-free fall collapse begins again
- H ionisation energy (13.6 eV) only slightly > H_2 binding energy, so collisional ionisation occurs soon after dissociation
- Collapse only stops when all H is fully ionised → second core = protostar



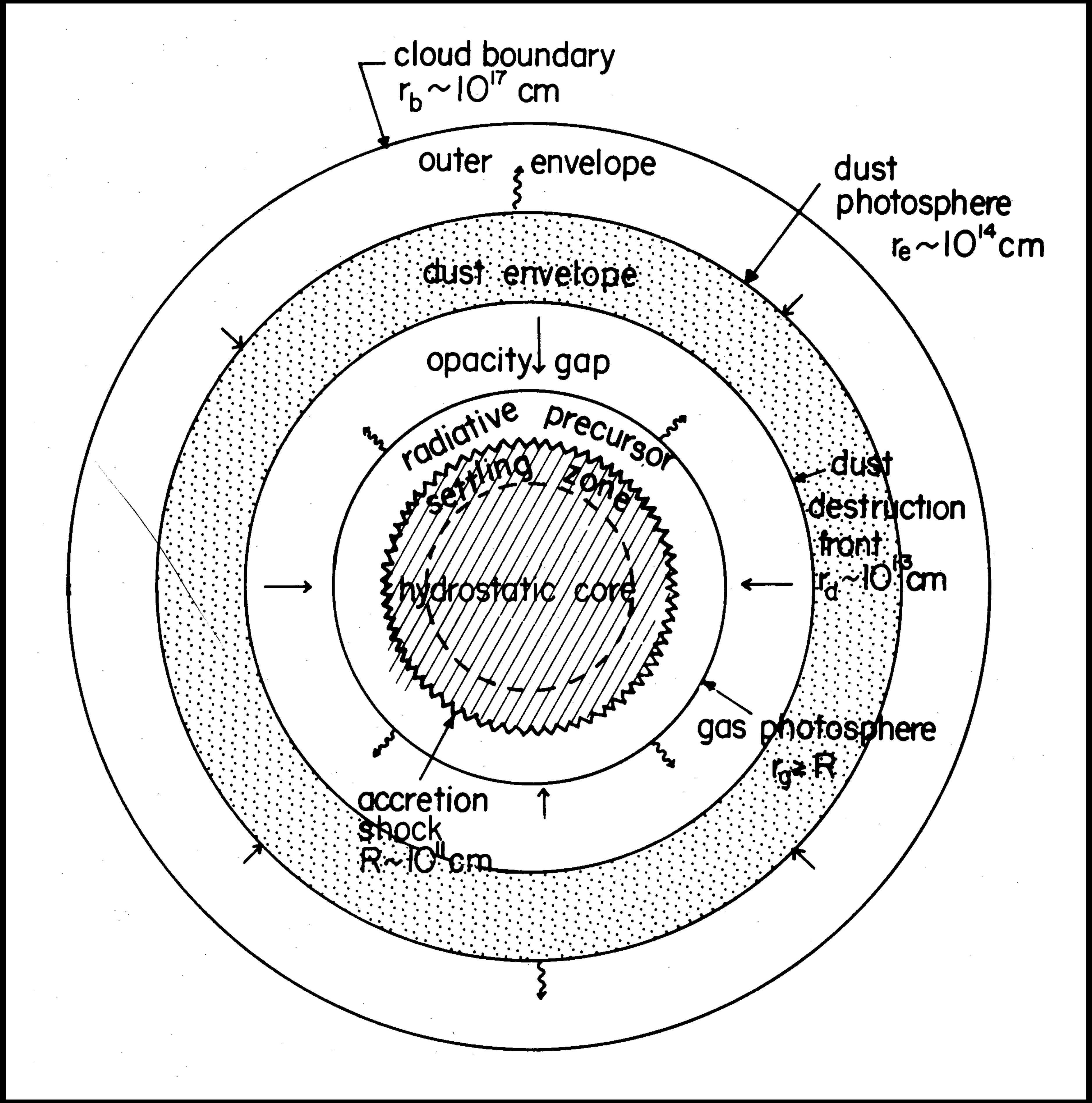
Masunaga & Inutsuka 2000

Question: this story says that protostars form once the first core exceeds few $\times$ 0.01 $M_\odot$. What happens to brown dwarfs, which may never reach this mass? Do they remain first cores forever? If not, why not?

Structure of the envelope

Overall picture

- Once hydrostatic protostar forms, matter rains down on it and stops in a shock
- Radiation escapes shock, propagates outward; close to shock, no dust grains survive because temperature is too high
- Radiation free-streams through dust-free zone until it hits dust, then is absorbed
- Radiation diffuses outward through dusty zone, eventually escaping



Stahler, Shu, & Taam 1980

The accretion shock

Basic properties

- Matter falling onto star loses kinetic energy at rate

$$L = \frac{GM_* \dot{M}_*}{R_*} = 30 L_\odot \left(\frac{\dot{M}_*}{10^{-5}\ M_\odot\ \mathrm{yr}^{-1}}\right) \left(\frac{M_*}{M_\odot}\right) \left(\frac{R_*}{10\ R_\odot}\right)^{-1}$$

- Most of this goes into radiation; only a small fraction trapped inside star
- Pre-shock velocity is $v = v_{\mathrm{ff}} = \sqrt{\frac{2GM_*}{R_*}} = 200\ \mathrm{km\ s^{-1}} \left(\frac{M_*}{M_\odot}\right)^{1/2} \left(\frac{R_*}{10\ R_\odot}\right)^{-1}$
- Can compute post-shock temperature from RH jump conditions: for Mach number >> 1,

$$T_{\mathrm{ps}} = 2\frac{(\gamma-1)}{(\gamma+1)^2}\frac{\mu m_{\mathrm{H}}}{k_{\mathrm{B}}} v^2 = 1.2 \times 10^6\ \mathrm{K} \left(\frac{M_*}{M_\odot}\right) \left(\frac{R_*}{10\ R_\odot}\right)^{-1}$$

The accretion shock

Effective temperature

- Post-shock temperature guarantees that gas falling onto star is very opaque — lots of metal lines to absorb radiation at EUV frequencies

- As a result, radiation is trapped near star until it thermalises, star radiates a nearly blackbody-spectrum

- Effective temperature set by energy balance:

$$L = 4\pi R_*^2 \sigma_{\mathrm{SB}} T_*^4 \qquad \Longrightarrow \qquad T_* = 4300\ \mathrm{K} \left(\frac{\dot{M}_*}{10^{-5}\ M_\odot\ \mathrm{yr}}\right)\left(\frac{M_*}{M_\odot}\right)\left(\frac{R_*}{10\, R_\odot}\right)^{-3/4}$$

- Therefore most protostars have reddish intrinsic colours

The dust destruction zone

Grain temperatures

- Consider a dust grain of size *a* at distance *r* from accreting protostar; its equilibrium temperature is set by balance between absorption and emission:

$$\Gamma = \frac{L_*}{4\pi r^2}\pi a^2 = \pi a^2 \sigma_{\rm SB} T_*^4 \left(\frac{R_*}{r}\right)^2$$

$$\Lambda = 4\pi a^2 \sigma_{\rm SB} Q T_d^4$$

Q < 1: efficiency of emission relative to blackbody — < 1 for grain size smaller than wavelength, = 1 for grain size > wavelength

Dust temperature

- Solve for dust temperature: $T_d = \left(\frac{R_*}{2r}\right)^{1/2} Q^{-1/4} T_* \geq \left(\frac{R_*}{2r}\right)^{1/2} T_*$

- Temperature falls with radius; close to star, T_d approaches T_*

- However, solid dust will vaporise at temperatures above ~1000 - 1500 K (depending on composition) → no dust grains survive too close to star

Dust destruction radius

Where grains can survive

- Given grain sublimation temperature T_s, can solve for minimum radius at which any grains can survive:

$$r_d = \frac{R_*}{2}\left(\frac{T_*}{T_s}\right)^2 = 0.4 \text{ AU} \left(\frac{T_s}{1000 \text{ K}}\right)^{-2} \left(\frac{\dot{M}_*}{10^{-5}\ M_\odot \text{ yr}^{-1}}\right)^{1/2} \left(\frac{M_*}{M_\odot}\right)^{1/2} \left(\frac{R_*}{10\ R_\odot}\right)^{-1/2}$$

- Since star emits almost no ionising photons (due to low T_*), and only dust can absorb most non-ionising photons, region inside r_d is effectively transparent — known as the "opacity gap"
- Radiation impacting dust destruction front therefore has same frequency distribution as that leaving star

The dusty envelope

Structure of the dust destruction front

- Density at dust destruction front is approximately given by:

$$\dot{M}_* = 4\pi r_d^2 \rho v_{\rm ff} \qquad \Longrightarrow \qquad \rho = \frac{\dot{M}_*}{\sqrt{8\pi^2 G M_* r_d^3}}$$

$$\rho = 4\times 10^{-13}\ \mathrm{g\ cm^{-3}} \left(\frac{\dot{M}_*}{10^{-5}\ M_\odot\ \mathrm{yr}^{-1}}\right)^{1/4} \left(\frac{M_*}{M_\odot}\right)^{-7/4} \left(\frac{R_*}{10\ R_\odot}\right)^{3/4} \left(\frac{T_s}{1000\ \mathrm{K}}\right)^{3}$$

- Stellar radiation spectrum peaks at wavelength

$$\lambda \approx \frac{hc}{4k_B T} = 440\ \mathrm{nm} \left(\frac{\dot{M}_*}{10^{-5}\ M_\odot\ \mathrm{yr}^{-1}}\right) \left(\frac{M_*}{M_\odot}\right)^{-1/4} \left(\frac{R_*}{10\ R_\odot}\right)^{3/4}$$

- MW dust opacity at 440 nm is κ ~ 8000 cm^2 g^{-1}, so mean free path for photons is 1 / $\kappa\rho$ ~ 10^9 cm ~ 10^{-4} AU → dust destruction front is **very** thin

The dusty envelope

Diffusive transport of radiation

- Dust at dust destruction front is at temperature T_s ~ 1000 K — lower than stellar surface temperature by factor of ~4
- Radiation re-emitted by dust is therefore at factor of ~4 longer wavelength, ~2 μm — near IR
- Opacity scales as $\kappa \sim \lambda^{-2}$, so mean free path of re-radiated photons is a factor of ~16 larger — but this is still tiny!
- Implication: radiation re-emitted by dust must diffuse outward, cannot free-stream and escape either

The dusty envelope

Temperature structure

- In diffusive zone, radiation flux obeys $F = -\frac{c}{3\rho\kappa_R}\nabla(a_R T^4)$

  Radiation energy density

  Rosseland mean opacity
- Luminosity passing through shell at radius r is just $4\pi r^2 F$ — but this must equal stellar luminosity in order to conserve energy
- Thus temperature obeys $L_* = -\frac{16\pi c a_R r^2}{3\rho\kappa_R} T^3 \frac{dT}{dr}$
- If $\kappa_R \sim T^\alpha$ and $\rho \sim r^{-\beta}$, temperature is a power law $T \sim r^{-k}$ with $k = \frac{\beta+1}{4-\alpha}$
- Typical $\alpha \sim 0.8$, $\beta \sim 1.5$, so $k \sim 0.8$ — temperature falls from ~1000 K at ~0.5 AU to ~10 K at ~150 AU; actual drop-off a bit shallower, due to failure of diffusion approximation at larger radii