Class 14: The IMF: Theory

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Outline

- General considerations
- The IMF tail
  - Competitive accretion
  - Turbulent fragmentation
- The IMF peak
  - Setting the peak from the galactic scale
  - Setting the peak from small scales

Theory of the IMF

General considerations

- The IMF consists of two main parts: a broad plateau centred at ~0.2-0.3 $M_{\odot}$, and a power law tail at high mass
- These features may vary slightly with environment, but, if so, by a surprisingly small amount — e.g., the slope may be ~0.2 flatter in the densest star clusters in the local universe, or the mass to light ratio in ellipticals may be a factor of ~2 higher than in spirals
- A successful theoretical model must explain both parts of the IMF, and also why they vary so little in response to many order of magnitude changes in density, redshift, galactic environment, etc.

Limits of isothermal models

Part I

- Basic equations for a magnetised, self-gravitating, isothermal fluid:

$$
\begin{aligned}
\frac{\partial \rho}{\partial t} &= -\nabla \cdot (\rho \mathbf{v}) \\
\frac{\partial}{\partial t}(\rho \mathbf{v}) &= -\nabla \cdot (\rho \mathbf{v}\mathbf{v}) - c_s^2 \nabla \rho + \frac{1}{4\pi}(\nabla \times \mathbf{B}) \times \mathbf{B} - \rho \nabla \phi \\
\frac{\partial \mathbf{B}}{\partial t} &= -\nabla \times (\mathbf{B} \times \mathbf{v}) \\
\nabla^2 \phi &= 4\pi G \rho
\end{aligned}
$$

- Define characteristic length L, velocity V, mean density ρ_0, mean magnetic field B_0, make change of variables:

$$
\mathbf{x} = \mathbf{x}' L \quad t = t'(L/V) \quad \rho = r\rho_0 \quad \mathbf{B} = \mathbf{b}B \quad \mathbf{v} = \mathbf{u}V \quad \phi = \psi G \rho_0 L^2
$$

Class exercise: write down the equations of motion with this change of variable. What dimensionless ratios appear in your equations?

Limits of isothermal models
Part II

- Non-dimensional equations are:

$$\frac{\partial r}{\partial t'} = -\nabla' \cdot (r\mathbf{u})$$

$$\frac{\partial}{\partial t'}(r\mathbf{u}) = -\nabla' \cdot (r\mathbf{u}\mathbf{u}) - \frac{1}{\mathcal{M}^2}\nabla' r + \frac{1}{\mathcal{M}_A^2}(\nabla' \times \mathbf{b}) \times \mathbf{b} - \frac{1}{\alpha_{\mathrm{vir}}} r\nabla'\phi$$

Derivative with respect to *x'*

$$\frac{\partial \mathbf{b}}{\partial t'} = -\nabla' \times (\mathbf{b} \times \mathbf{u})$$

$$\nabla'^2\psi = 4\pi r$$

- The dimensionless quantities appearing in these equations are:

$$\mathcal{M} = V/c_s \qquad \mathcal{M}_A = V/V_A = V\sqrt{4\pi\rho_0}/B_0 \qquad \alpha_{\mathrm{vir}} = V^2/G\rho_0 L^2$$

- The evolution is *entirely* determined by these quantities + the initial conditions

Limits of isothermal models

Part III

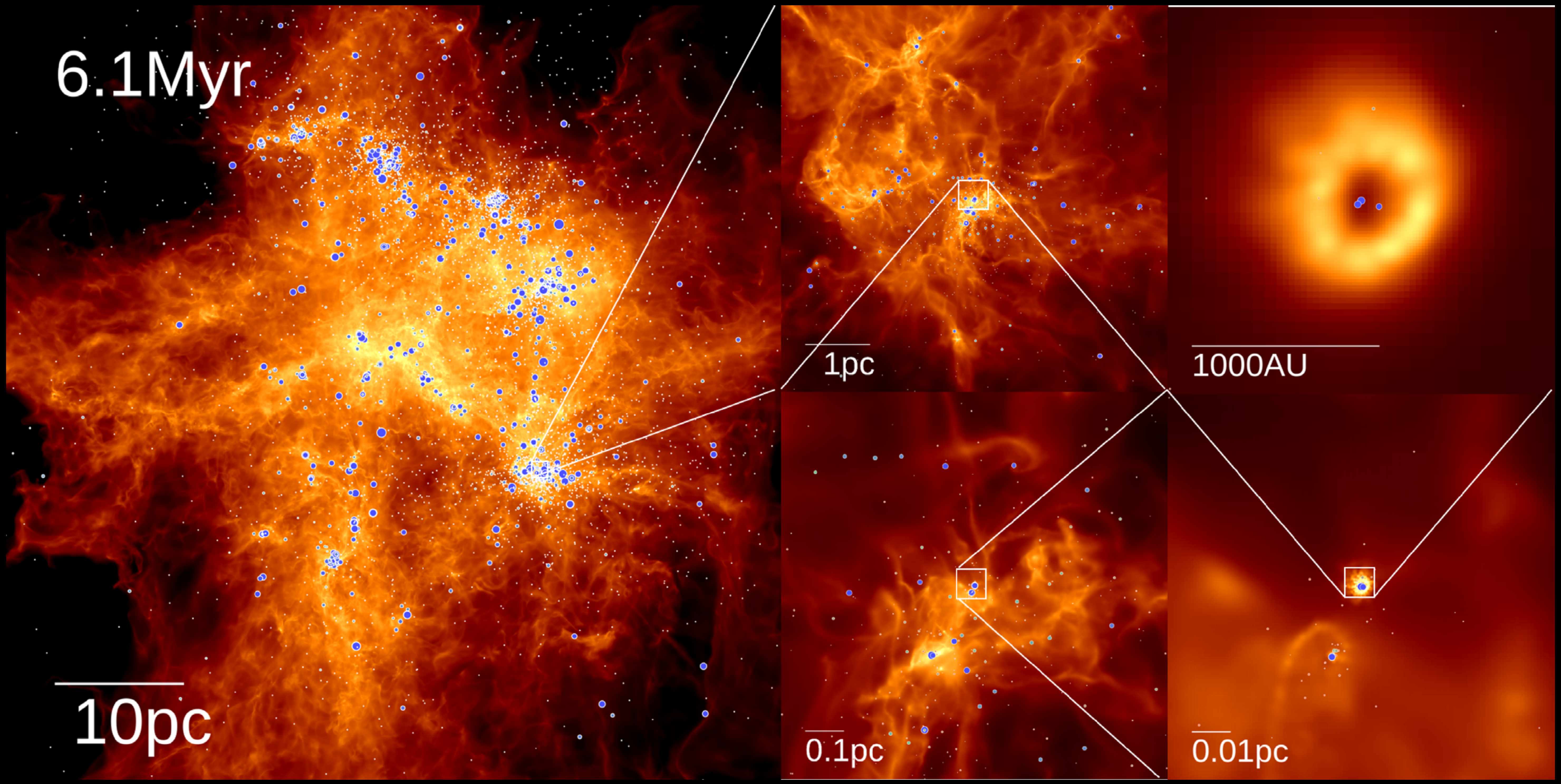
- Suppose a star forms in an isothermal cloud, with the material going into the star coming from a volume bounded by some initial surface *S* at time *t* = 0. The mass of the star is $M_* = \int_S \rho \, d^3x = \rho_0 L^3 \int_S r \, d^3x'$
- Now consider an identical cloud that has just been rescaled by a factor *y*, with
$$L' = yL \quad \rho_0' = \rho_0/y^2 \quad B_0' = B_0/y \quad V' = V$$
- One can verify by direct substitution that the rescaled cloud has exactly the same values of $\mathcal{M}, \mathcal{M}_A, \alpha_{\mathrm{vir}}$, so the evolution is identical — same star forms
- However, the resulting star has mass $M_*' = \frac{\rho_0' L'^3}{\rho_0 L^3} M_* = y M_*$, so the mass of the star is not the same — it can be changed arbitrarily by the choice of *y*

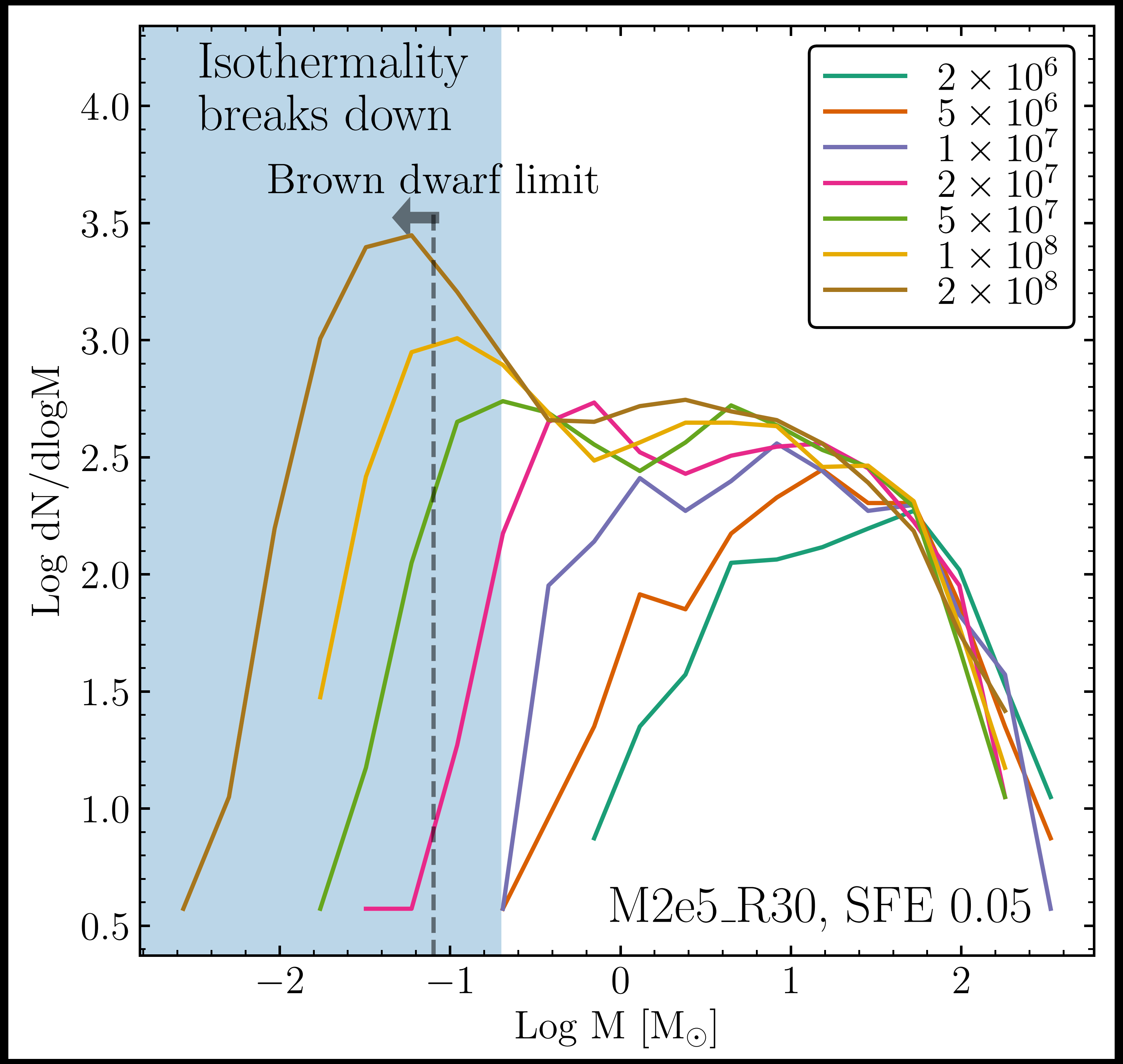
Limits of isothermal models

Implications

- The conclusion to draw is that isothermal gas does not have a mass scale — the characteristic masses of any stars it forms are not set by the physics of the cloud, but by the initial conditions
- Thus isothermal gas is capable of producing stars with a scale free (powerlaw) mass distribution, but if it produces a distribution that has a scale (like the IMF), that scale has to be set by the initial conditions
- This realisation motivates separate consideration of the IMF tail (which can potentially arise in purely isothermal gas) from the IMF peak (which must depend either on the initial conditions in GMCs, or on deviations from isothermal behaviour)

Numerical confirmation





Guszejnov+ 2020

The power law tail

Basic approaches

- Two natural candidate scale-free processes:
  - Accretion of gas by stars that form near the IMF peak — “competitive accretion”
  - Collapse of a distribution of structures with different masses that are created by turbulent flow — “turbulent fragmentation”
- Reality is almost certainly somewhere between these two extremes, but they provide useful ways of thinking about the problem

The competitive accretion model

Key papers: Bonnell+ (1997), Bonnell+ (2001), Bate+ (2005), Bate (2009)

- Consider a population of “seed” stars formed by fragmentation, all with similar initial masses, that gain mass by accreting additional gas
- Suppose accretion rate depends on current mass as $dm/dt \sim m^{\eta}$; for example, for a point mass accreting gas from a uniform, infinite medium, Bondi & Hoyle showed that this holds with $\eta = 2$

Dimensionless time parameter Starting accretion rate Starting mass

- Mass some time later is $m(t) = m_0 \begin{cases} [1-(\eta-1)\tau]^{1/(1-\eta)}, & \eta \neq 1 \\ \exp(\tau), & \eta = 1 \end{cases}$, where $\tau = t\dot{m}_0/m_0$
- Suppose all stars start at mass m_0, but have different starting accretion rates (e.g. because they are in gas of different density) or accrete for different amounts of time (e.g. because some form earlier) — what is resulting IMF?

Competitive accretion

Predicted IMF

- Varying initial accretion rate or time corresponds to varying τ
- For a distribution of τ values $\mathrm{d}n / \mathrm{d}\tau$, resulting mass distribution is

$$\frac{dn}{dm} = \frac{dn/d\tau}{dm/d\tau} = \left(\frac{dn}{dt}\right) m^{-\eta}$$

- Thus accretion converts an initial δ-distribution in mass (all stars at mass m_0) into a power law distribution
- BH accretion ($\eta = 2$) is a bit too shallow compared to observed IMF, but in a crowded environment likely to have somewhat larger η due to tidal truncation of feeding zones — plausible explanation for IMF slope

Turbulent fragmentation

Key papers: Padoan & Nordlund (2002), Hennebelle & Chabrier (2008), Hopkins (2012)

- Basic idea: consider density field smoothed at some size scale ℓ, and imagine drawing iso-density contours at density ρ
- Define mass m enclosed by each contour — typically most numerous contours will be as small as possible on the smoothing scale, so $m \sim \rho\ell^3$
- What is the distribution of m values? Can estimate from the density PDF: total mass dM of objects with density from ρ to $\rho + d\rho$ is $dM \sim \rho\, p(\rho)\, d\rho$
- Thus number of contours of mass m must follow $\frac{dn}{dm} = \frac{dM}{m} \sim \frac{1}{\ell^3} \int p(\rho)\, d\rho$

Turbulent fragmentation

The barrier function

- Not all of the identified contours will be gravitationally bound, so not all will make stars; IMF will come from distribution of bound contour masses
- Contour of mass m, size scale ℓ is bound if $Gm^2 / \ell \gtrsim m\sigma(\ell)^2$
- Can express this condition in terms of a minimum density: $\rho \gtrsim \sigma(\ell)^2 / G\ell^2$
- Using LWS to get $\sigma(\ell) = c_s (\ell / \ell_s)^{1/2}$: $\rho \gtrsim c_s^2 / G\ell\ell_s$
- This sets lower limit to integral over PDF; integrate over all smoothing scales ℓ, and, given model for smoothed PDF, calculate resulting IMF; result is a power law

The IMF peak
Basic approaches

- Mass scale of IMF peak must be set by two possible mechanisms:
  - “Initial conditions”, i.e., properties of molecular clouds at the Galactic scale
  - Deviations from isothermal behaviour caused by additional physical processes
- Numerous papers published using both approaches, but consensus seems to be moving toward deviations from isothermality, for reasons that will become clear as we go through the arguments

The peak from initial conditions

Key papers: Bate+ (2005), Hennebelle & Chabrier (2008), Hopkins (2012)

- Simplest hypothesis is that peak mass set by Jeans mass in molecular cloud at largest scale: $M_{\mathrm{peak}} \sim c_s^3 / (G^3\rho)^{1/2}$
- Problem: mean density ρ in molecular clouds varies by a lot from galaxy to galaxy, and even within galaxies
  - Example: mean GMC density near Sun is $n \sim 100$ cm^{-3}, but near MW centre $n \sim 10^4$ cm^{-3}
  - Predicted change in M_{peak} by factor of ~10 — ruled out by observations
- Can be partly compensated by change in c_s, but requires considerable fine-tuning to explain the small amount of observed variation in IMF peak mass

The peak from initial conditions

Revised version

- More refined hypothesis: peak mass set by sound speed and sonic length, $M_{\mathrm{peak}} \sim c_s^2 \ell_s / G$
- Can rewrite in terms of dimensionless parameters using LWS plus definition of virial parameter, $\sigma(L) = c_s (L / \ell_s)^{1/2}$ and $\alpha_{\mathrm{vir}} \sim \sigma^2 L / GM \rightarrow M_{\mathrm{peak}} \sim c_s^4 / \alpha_{\mathrm{vir}} G \Sigma^2$
- Advantage of this approach: predicts constant peak mass in GMCs of fixed surface density and virial parameter, so predicts no variation in Milky Way
- However, still has problems with external galaxies where GMCs can have (much) higher Σ, but no big changes in peak mass are observed

The IMF peak from non-isothermality

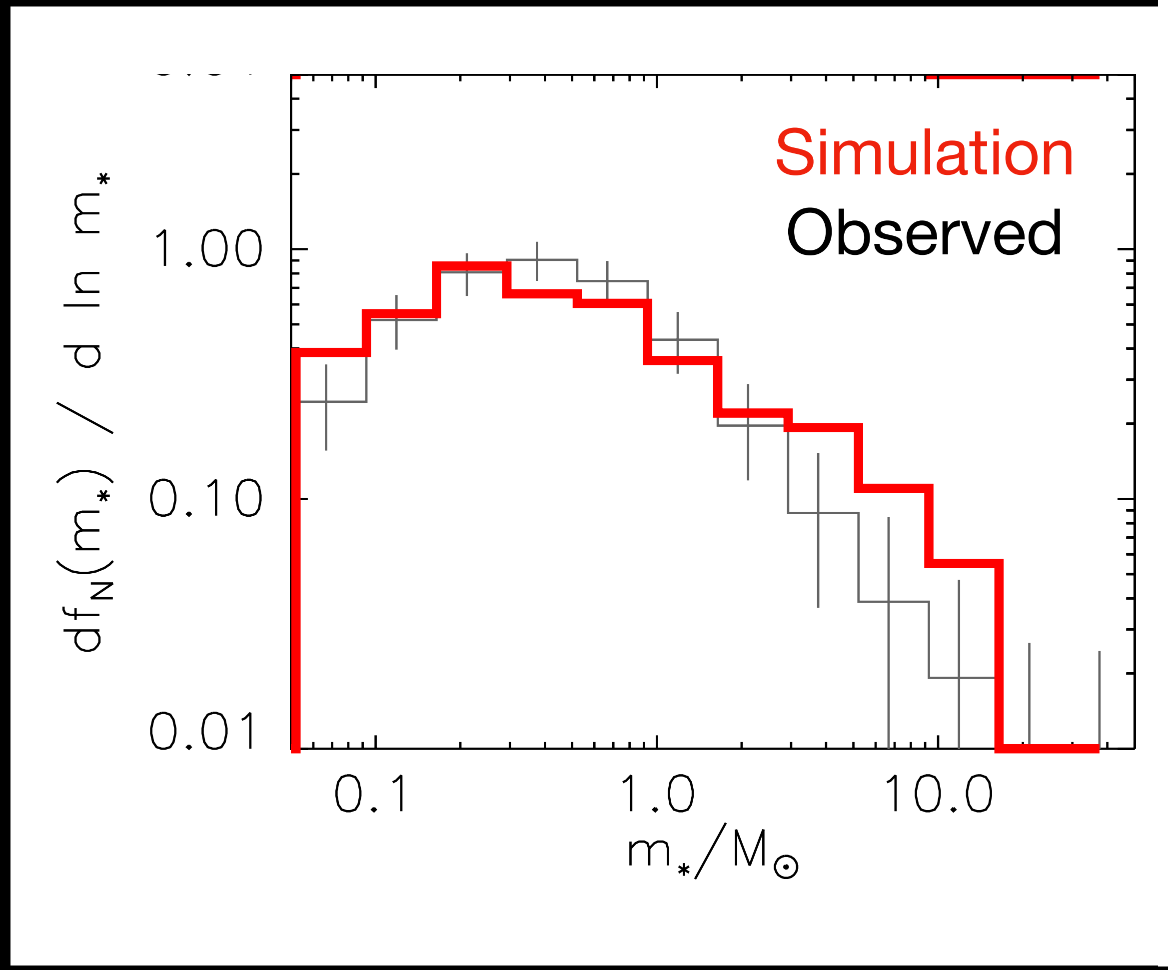
Key papers: Larson (2005), Krumholz+ (2006, 2011, 2012), Bate (2009, 2012)

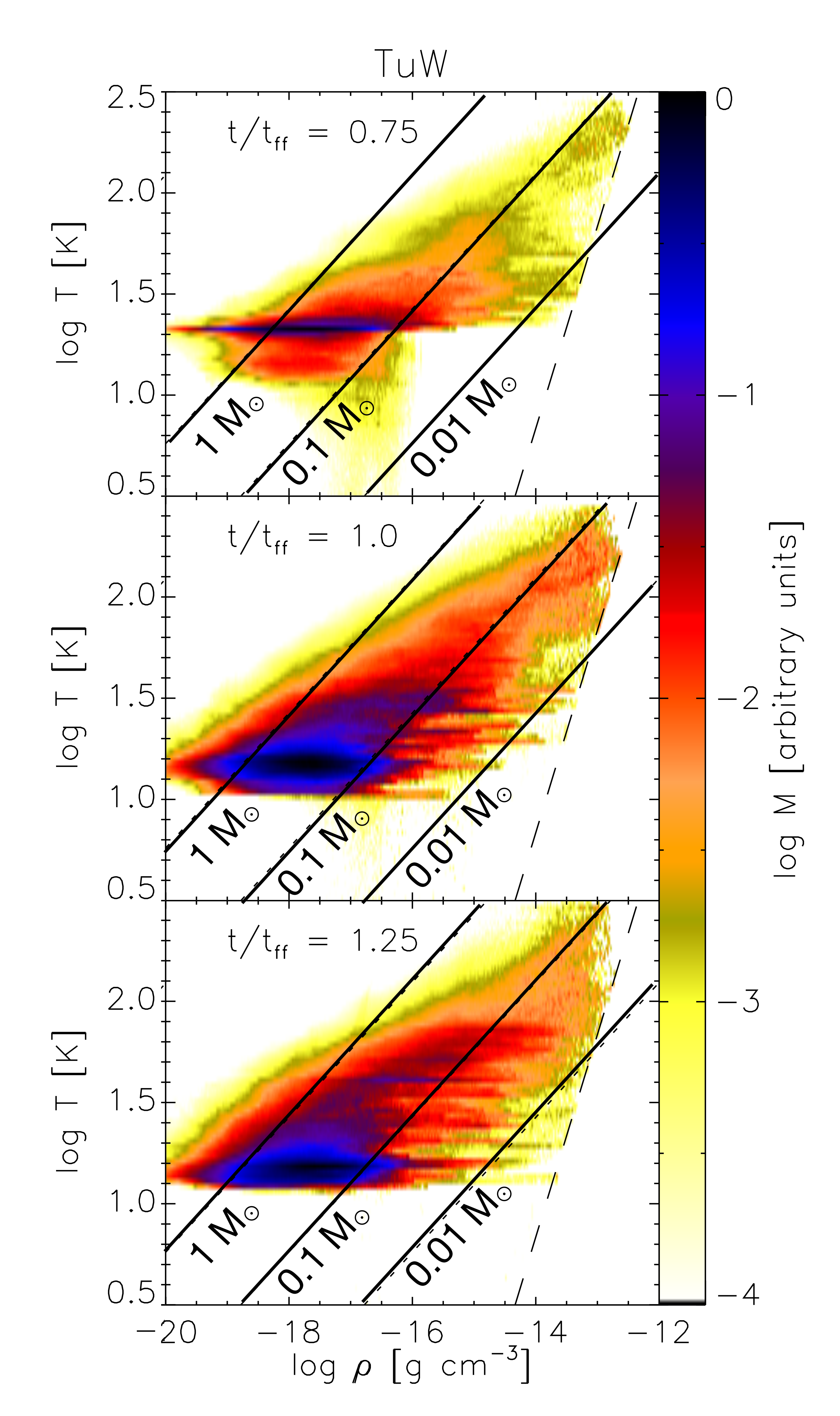
- Jeans mass depends on density and temperature as $M_J \sim (T^3/\rho)^{1/2}$, so as long as T ~ constant, Jeans mass decrease without limit as gas compresses
- However, if T starts to increase as ρ decreases, fragmentation can be slowed or halted (depending on how strongly T rises with ρ)
- Thus any physical process that makes T start increasing at some characteristic density ρ can "freeze in" a mass scale at the value of M_J evaluated at the (ρ, T) where the temperature increase starts

Origin of non-isothermality

Stellar radiation

- Main source of deviation from isothermal behaviour is radiation from forming stars
- Once a small star forms, accretion onto it releases energy, which comes out as radiation, and heats the surrounding gas
- Simulations including this effect reproduce observed IMF well





Krumholz+ 2012

Universality of the IMF from stellar radiation

Part I

- Consider a protostar with luminosity L; temperature of material a distance R from it will be approximately given by $L = 4\pi\, \sigma_{\rm SB}\, R^2\, T^4$

- Jeans mass $M_{\rm J} = [\, (k_{\rm B}T\, /\, \mu m_{\rm H}G)^3\, /\, \rho]^{1/2}$

- Consider spherical regions of increasing radius R around a seed star; as R increases, T and $M_{\rm J}$ fall and enclosed mass rises

- Hypothesis: characteristic mass where fragmentation suppressed set by condition that enclosed mass ≈ $M_{\rm J}$

- Result: $M = \left(\frac{1}{36\pi}\right)^{1/10} \left(\frac{k_B}{G\mu m_{\rm H}}\right)^{6/5} \left(\frac{L}{\sigma_{\rm SB}}\right)^{3/10} \rho^{-1/5}$

Universality of the IMF from stellar radiation

Part II

- Luminosity comes from accretion; we will show later in the course that energy yield from accretion ~constant for all protostars, $L \approx \psi\,(dM/dt)$, $\psi \approx 10^{14}$ erg/g

- Accretion time must be ~t_{ff}, so $dM / dt \sim M\,(G\rho)^{1/2}$

- Substitute in for L and solve for M:

$$M = \left(\frac{1}{36\pi}\right)^{1/7} \left(\frac{k_B}{G\mu m_{\mathrm{H}}}\right)^{12/7} \left(\frac{\psi}{\sigma_{\mathrm{SB}}}\right)^{3/7} \rho^{-1/14} \approx 0.3 \left(\frac{n}{100\ \mathrm{cm}^{-3}}\right)^{-1/14} M_{\odot}$$

- This is nearly independent of environmental conditions; basic reason: higher density favours fragmentation (lowers M_J) but also higher accretion rate and thus higher temperature (raises M_J), and effects almost perfectly cancel