1. **Modernising Kennicutt. [15 points]**
In this problem we will work with the data set of global star formation rates in nearby galaxies compiled by Kennicutt (1998, ApJ, 498, 541). We will update the data using more recent estimates for the IMF (Kennicutt uses a Salpeter IMF) and $\alpha_{\rm CO}$ (Kennicutt treats this as constant), then examine the resulting relationship between gas and star formation surface densities.

   (a) Extract the data from Tables 1 and 2 in Kennicutt (1998), and make a plot of the two forms of the Kennicutt-Schmidt relation using the original data; that is, plot $\log \Sigma_{\rm SFR}$ versus $\log \Sigma_{\rm gas}$ and $\log \Sigma_{\rm SFR}$ versus $\log \Sigma_{\rm gas}/t_{\rm dyn}$. Here $\Sigma_{\rm gas}$ should include both H I and H_2, where both are available.

   (b) Next we will correct the estimated star formation rates and H_2 surface densities using more recent estimates of the conversions from Hα / IR luminosity to star formation rate, and from CO luminosity to H_2 mass. For star formation rates, update the estimated SFRs given in Kennicutt (1998)'s tables using the revised conversions between Hα and IR luminosity and SFR given in Kennicutt & Evans (2012, ARA&A, 50, 531). For H_2, update the tabulated surface densities using the $\alpha_{\rm CO}$ value suggested by Bolatto et al. (2013, ARA&A, 51, 207). For this purpose, assume that all galaxies in the sample have Solar metallicity (approximately true), that GMCs have a characteristic surface density of 100 $M_\odot$ pc^{-2}, and use the gas surface density in place of the total surface density for simplicity. Repeat the plots from part (a) for your updated gas and stellar surface density estimates.

   (c) Fit a simple linear relations to $\log \Sigma_{\rm SFR}$ versus $\log \Sigma_{\rm gas}$ and $\log \Sigma_{\rm SFR}$ versus $\log \Sigma_{\rm gas}/t_{\rm dyn}$, using both the original data from part (a) and the corrected data from part (b). You may use the fitting technique of your choice. Give the best-fit parameters. How different are the fits? Explain qualitatively why any differences arise.

2. **A simple, wrong IMF model. [15 points]**
In this problem we will work through a very simple model for the origin of the lognormal part of the IMF, based on the statistics of turbulent density fields. We consider a region of isothermal gas with mean density $\bar{\rho}$, whose density distribution is described by a lognormal with variance σ_s. Let $M_{J,0}$ be the Jeans mass computed at the mean density $\bar{\rho}$.

   (a) As a first step, compute the fraction $f_J(< M_J)$ for which the Jeans mass is $< M_J$.

   (b) As a simple hypothesis, suppose that the masses of stars made in the cloud mirrors the distribution of Jeans masses – that is, the fraction $f_*(< m_*)$ of stellar mass found in stars with mass $< m_*$ is equal to the fraction of cloud mass for which the Jeans mass is $< m_*$, so $f_*(< m_*) = f_J(< m_*)$. Calculate the resulting stellar IMF $dn/d\ln m_*$; you can omit the normalisation factor, and just get the dependence on stellar mass. (Hint: think about how the IMF $dn/d\ln m_*$ is related to $f(< m_*)$.)

   (c) Plot the function you just obtained on top of the Chabrier (2005) IMF (equation 2.3 of the textbook) using $M_{J,0} = 0.4\ M_\odot$ and $\sigma_s = 1.1$. You should find that the resulting function is a strikingly-good match to the IMF over the mass range $0.01 - 1$ $M_\odot$. Argue that, nonetheless, this is not a particularly good model for the origin of the IMF. (Hint: are these values of $M_{J,0}$ and σ_s reasonable parameter choices for all molecular clouds?)