1. **The Ammonia Thermometer. [20 points]**
Ammonia (NH_3) is often used as a temperature indicator in molecular regions. Consider a gas cloud with molecular hydrogen number density n_{H_2} and gas kinetic temperature T, with no ambient radiation field except the CMB. The cloud contains a small density of ammonia molecules, which are excited by collisions with H_2. In order to calculate the emission from these molecules, you can either write your own program to solve the equations, or you can use a package like `DESPOTIC` or `RADEX`, as you prefer. If you choose to write your own code, you can obtain the collision rate coefficients, energy level data, and Einstein coefficients for NH_3 from the Leiden Atomic and Molecular Database.

   (a) Compute the luminosity per NH_3 molecule radiated in the $(1, 1)$, $(2, 2)$, and $(4, 4)$ inversion transitions of para-NH_3 as a function of n_{H_2} and T, assuming the gas is in statistical (not thermodynamic) equilibrium and that optical depths in thelines are negligible. Plot the luminosity per NH_3 molecule in each of these lines as a function of T for $n_{H_2} = 10^2, 10^3, 10^4, 10^5$, and 10^6 cm^{-3}.

   (b) Plot the following line ratios as a function of T for the same densities as in part (a): $L(2, 2)/L(1, 1)$, $L(4, 4)/L(2, 2)$, and $L(4, 4)/L(1, 1)$. Based on this plot, explain why NH_3 is a good thermometer.

2. **Cooling in the C^+-dominated regime. [30 points]**
The book discusses the equilibrium temperature and cooling time for molecular clouds where cooling is dominated by CO. However, at low metallicity there may be substantial regions of H_2 where the carbon is in the form of C^+ rather than CO, and in this problem we will show that the same qualitative conclusions apply in this regime. To good approximation we can treat cooling by C^+ as arising through a single collisionally-excited fine-structure line at a wavelength of $\lambda = 157.74$ μm; the lower state of this transition is the ground state. You can find rate coefficients and all other required atomic data in the Leiden Atomic and Molecular Database.

   (a) Consider a molecular cloud with H_2 number density n_{H_2} and temperature T, within which there are x_{C^+} free C^+ atoms per H_2. Assuming that n_{H_2} is much smaller than the critical density, and that the bulk composition is pure H_2 (i.e., neglect He, atomic H, free electrons, etc.), give an expression for the cooling rate due to this line in terms of these quantities together with whatever rate coefficients (for collisional or radiative transitions) are required.

   (b) Suppose that the cloud has a constant heating rate $\Gamma_{CR} = 2 \times 10^{-27}$ erg s^{-1} per H nucleus as a result of cosmic rays, and that the C^+ abundance $x_{C^+} = 10^{-5}$, typical of a metal-poor dwarf galaxy. Plot the equilibrium temperature as a function of density for $n_{H_2} = 10^2 - 10^6$ cm^{-3}; for this purpose you can neglect the (weak) temperature-dependence of the collision rate, and just adopt a reasonable average value.

   (c) Plot the cooling time due to C^+ cooling as a function of n_{H_2} for gas at the equilibrium temperature found in the previous part, using the same method as found in chapter 3 in the textbook. Based on the results in this part and the previous part, is it reasonable to treat gas cooled by C^+ as approximately isothermal? If so, over what density range?