1. It is sometimes convenient to write down a simple analytic expression for the rate of cooling in the neutral ISM.

   (a) We will start with C^+ cooling. Make the following assumptions: (1) only collisions between C^+ and H are significant; (2) the collisional *de*-excitation rate coefficient for such collisions, $k_{C^+,\mathrm{de}}$ is a constant that is independent of temperature; (3) the density is much less than the critical density of C^+; (4) all carbon is C^+. Under these assumptions, give a simple analytic formula for the C^+ cooling rate in gas with H number density n_H and C/H ratio δ_C.

   (b) Plot the cooling function you have just derived as a function of temperature. Use $\delta_\mathrm{C} = 10^{-4}$, and you can get a numerical value for $k_{C^+,\mathrm{de}}$ from the Leiden Atomic and Molecular Database.

   (c) Produce a similar analytic expression and plot for cooling by O, but for O assume that the de-excitation rate coefficients for the two upper states scale with temperature as $k_{\mathrm{O,de}} \propto k_{\mathrm{O,de},0}\sqrt{T/T_0}$.

2. One additional feature we can add to our model of vertical hydrostatic equilibrium is a layer of dense material near the midplane, which might for example represent dense molecular clouds. Suppose that we have a layer of atomic hydrogen with total surface density Σ_g and constant velocity dispersion σ_g, and also an infinitesimally thin layer of material with surface density Σ_mp located at the midplane.

   (a) Write down the equation of hydrostatic balance including the midplane material.

   (b) Make the same transformation from ρ_g to s_g as your variable that we did in class.

   (c) Solve in the limits $\Sigma_\mathrm{mp} \ll \Sigma_g$ and $\Sigma_\mathrm{mp} \gg g$.

   (d) Show that, for $z \to \infty$, the solution is the same in both limits, as long as $\Sigma_\mathrm{g} + \Sigma_\mathrm{mp}$ remains constant.