1. We have computed the wavelength-dependence of the absorption and scattering coefficients for a single grain of fixed size, but in reality we know that there is a distribution of grain sizes. It is therefore useful to derive the wavelength-dependence for such a mixture.
   (a) Consider a population of spherical grains, where a is the grain radius. Write down an approximate expression for the scaling of the absorption and scattering coefficients C_{abs} and C_{sca} as a function of a and wavelength of light λ, which is valid in both the long wavelength ($\lambda \gg 2\pi a$) and short wavelength ($\lambda \ll 2\pi a$) limits. You can leave the overall normalisation unspecified – we just want to know the scalings – and you can just paste together approximations valid in the two limits; no need to do something fancy to handle the regime $\lambda \sim 2\pi a$.
   (b) Now suppose the grain population has a distribution of grain radii $dn/da \propto a^{-p}$ over a range of radii from a_0 to a_1 for some index $p > 0$. How does the scattering and absorption opacity of the grain population vary with wavelength λ? Again, we're just interested in the scaling with λ, not the overall normalisation.
   (c) For sufficiently large p, show that the opacities do not depend on a_0 or a_1 for wavelengths $2\pi a_0 \ll \lambda \ll 2\pi a_1$. In this limit, what is the scaling of the opacities with wavelength?

2. Consider dust grains being heated by a nearby star, which has surface temperature T_* and radius R_*. Approximate the stellar spectrum as a black body.
   (a) First consider grains of radius a so small that $2\pi a \ll hc/kT_*$. For such grains, estimate their equilibrium temperature.
   (b) Repeat the calculation for larger grains with $2\pi a \gg hc/kT_*$, but which still have $2\pi a \ll hc/kT_{\mathrm{gr}}$, where T_{gr} is the grain temperature.
   (c) How do the answers to the previous two parts differ, and why? Give a physical interpretation.