1. Although all ionised gas emits brehmsstralung, in practice the emission tends to be dominated by the coolest phases that are still ionised.

   (a) As a first step, consider a region of pure, ionised hydrogen within which the pressure is P. Rewrite the free-free emissivity and absorption coefficient in terms of P and T, i.e., eliminate n_e and n_i.

   (b) Most of the gas at the Galactic midplane is at about the same pressure ($P/k_B \approx 3000$ K cm^{-3}), dictated the weight of the Galactic disc. How do the free-free emissivity and absorption coefficient scale with temperature at fixed pressure? Can you see why the coolest phases dominate?

   (c) Suppose that there is a total mass M_i of ionised H in the disc of the Milky Way, and that this gas is arranged in a disc with a thickness $L \sim 500$ pc, with pressure $P/k_B \approx 3000$ K cm^{-3}. What is the approximate total free-free luminosity of the disc? For the purposes of this problem, you can approximate $\overline{g}_{\rm ff}$ as constant, and neglect the effects of opacity.

2. When a free electron recombines with a proton, this produces radiation, which leaves the gas. This is therefore a cooling mechanism.

   (a) Make an order of magnitude estimate the rate of cooling per unit volume due to recombinations in a gas of pure, fully-ionised hydrogen with density n and temperature T. Important hint: cooling means loss of *kinetic* energy, not binding energy – so the 13.6 eV binding energy of the H that is radiated away doesn't count as cooling, only the associated kinetic energy that is lost.

   (b) Compare this cooling rate to the rate at which the gas loses energy by free-free cooling – you can estimate the latter just by integrating the free-emissivity over all frequencies, and neglecting opacity effects. Again, approximate $\overline{g}_{\rm ff}$ as constant. Which is a more important cooling process, free-free or recombination?