Class 20 Notes: Molecular Gas: Giant Molecular Clouds and Star Formation

Now that we understand how H I turns into H_2 and C II to CO, we can study the regions where this transition has occurred and the gas is mostly molecular. In the Milky Way about a quarter of the gas mass inside the Solar circle is in this phase, although the volume occupied by this gas is essentially zero due to its high density. Most of this mass is in the form of giant molecular clouds with masses $\sim 10^4 - 10^6\ M_\odot$. It is these objects that we will discuss today.

I. Molecular cloud properties

A. Bulk properties

We can determine column densities of molecular clouds using the CO molecule, via the technique we described earlier in the class. Both the entire Milky Way disk and the disks of several other galaxies have been completely mapped in CO $J = 1 \to 0$, $J = 2 \to 1$, $J = 3 \to 2$, or in more than one of these lines. From these maps, we see that molecular gas tends to appear in the form of clouds that sit on top of regions of dense H I.

These clouds have masses ranging from $\sim 10^4 - 10^6\ M_\odot$, with a mass spectrum $dN/d\ln M \propto M^{-\gamma}$ with $\gamma \approx 0.5 - 0.8$ (Rosolowsky 2005). This means that most of the mass is in large clouds, though not by much: $\gamma = -1$ corresponds to equal mass per decade in cloud mass.

The mass scale of molecular clouds is thought to be related to the Toomre mass in a galaxy, though this is not certain. The Toomre mass is defined as $(\lambda_T/2)^2\Sigma$, where λ_T is the unstable wavelength for a marginally Toomre stable disk, $Q = 1$. Recall from our analysis of the Toomre instability that the most unstable wavenumber is

$$k = \frac{\pi G \Sigma}{\sigma^2}, \tag{1}$$

and when $Q = 1$ this is the only unstable wavelength. (Formally it is marginally stable, in the sense that $\omega = 0$ for this k, so perturbations are on the edge between growth and oscillation. Thus the Toomre mass is

$$M_T = \Sigma \left(\frac{\pi}{k}\right)^2 = \frac{\sigma^4}{G^2\Sigma} = 7.0 \times 10^6 \sigma_6^4 \Sigma_{10}^{-1}\ M_\odot, \tag{2}$$

where $\sigma_6 = \sigma/6$ km s^{-1} and $\Sigma_{10} = \Sigma/10\ M_\odot$ pc $^{-2}$. This is close to the mass of the most massive molecular clouds. Smaller ones, which are a minority of the mass, might form by fragmentation of the larger ones.

These clouds have column densities $\Sigma \approx 100\ M_\odot$ pc^{-2} independent of cloud mass. In contrast, the mean column density of the ISM is the Milky Way is about an order of magnitude lower, $\sim 10\ M_\odot\ \mathrm{pc}^{-2}$. The origin of this column density scale, which seems to be invariant from galaxy to galaxy (at least for the nearby galaxies within which we can resolve individual molecular clouds), is debated.

B. Temperature

One distinguishing characteristic of molecular clouds is that they are very cold, typically ~ 10 K. Observationally, we determine this using temperature-senstiive molecules like ammonia. This is for reasons related to both heating and cooling. On the heating side, recall that the grain photoelectric effect is the main source of heating in the diffuse ISM. In molecular clouds, however, the parts of the ISRF in optical and UV light are largely blocked, so there is no grain photoelectric heating. There is only cosmic ray heating, at a rate $\Gamma_{\mathrm{cr}} \approx 10^{-27} n_{\mathrm{H}}$ erg s^{-1}, about an order of magnitude below the rate of photoelectric heating in the diffuse ISM.

On the cooling side, CO is an extremely effective coolant. Figuring out the exact cooling rate is complicated by the fact that molecular clouds are extremely optically thick in the low J CO rotational lines. This reduces the ability of these lines to cool, since it means that most photons emitted in them do not escape the cloud, and instead simply deposit their energy elsewhere in a cloud's interior. Higher J lines are optically thin, but relatively few molecules are excited into these states, so they provide little cooling. Thus a detailed calculation of the cooling rate requires including radiative transfer effects, which of course depend on the cloud size and geometry. However, the result of these calculations is that the total cooling rate turns out to be dominated by the lowest rotational level J for which the $J \to J-1$ is not optically thick. For the conditions in molecular clouds, this turns out to be $J = 5$ most of the time.

Given this result, we can estimate the equilibrium temperature by computing the cooling rate due to CO collisional excitation. We will do so by approximate that, since the level we are interested in is on the edge of being optically thick, we will approximate that its population is not far from the value we would expect if it were in LTE. However, we will also adopt an escape probability of unity, since the line is marginally optically thin. Thus the cooling rate per unit volume from CO molecules transitioning from state J to state $J-1$ is

$$\Lambda_{\mathrm{CO}} = n_{\mathrm{CO}} A_{J,J-1} \frac{e^{-E_J/kT}}{Z} \tag{3}$$

where n_{CO} is the number density of CO molecules, $A_{J,J-1}$ is the Einstein A coefficient for this transition, E_J is the energy of level J, and Z is the partition function of the CO molecule.

To remind you, for a heteronuclear diatomic molecule the energies and degeneracies of the rotational levels and the Einstein A's between them are

$$
\begin{aligned}
E_J &= B_v J(J+1) && (4)\\
g_J &= 2J+1 && (5)\\
A_{J,J-1} &= \frac{128\pi^3}{3\hbar}\left(\frac{B_v}{hc}\right)^3 \mu^2 \frac{J^4}{J+1/2}, && (6)
\end{aligned}
$$

where B_v is the rotation constant for the molecule and μ is its dipole moment. For CO, $B_v/h = 57$ GHz for the ground vibrational state, and $\mu = 0.11$ Debye ($= 1.1\times 10^{-11}$ esu·Å). Plugging these into the formula for the cooling rate and simplifying a bit, for the CO $J \to J-1$ transition we obtain

$$
\Lambda_{\mathrm{CO}} = 5.3\times 10^{-23} n_{\mathrm{CO}} \frac{J^5}{2J-1}\left(\frac{B_v}{kT}\right)\exp\left[-J(J+1)\left(\frac{B_v}{kT}\right)\right] \text{ erg s}^{-1}. \tag{7}
$$

Equating the rates of heating and cooling for $J = 5$, we find that

$$
\frac{B_v}{kT}\exp\left(-30\frac{B_v}{kT}\right) = 1.8\times 10^{-4}\frac{\Gamma_{\mathrm{CR},-27}}{x(\mathrm{CO})_{-4}}, \tag{8}
$$

where $x(\mathrm{CO}) = n_{\mathrm{CO}}/n_{\mathrm{H}}$, $x(\mathrm{CO})_{-4} = x(\mathrm{CO})/10^{-4}$, and $\Gamma_{\mathrm{CR},-27} = \Gamma_{\mathrm{CR}}/(10^{-27} n_{\mathrm{H}})$. The solution to this transcendental equation for the fiducial parameters is $T =$ 11.4 K. Due to the -30 in the exponential factor, the result is extremely insensitive to either $x(\mathrm{CO})$ or Γ_{CR}; changing the ratio by a factor of 10 leads to only $\sim 50\%$ variations in T.

Thus we always expect molecular clouds to have temperatures around 10 K unless there is some strong local heat source, e.g. a star putting out a lot of infrared light that heats up the dust.

C. Turbulence and self-gravity

One might think that molecular clouds' low temperatures would give them small linewidths, but that is not the case. Instead, linewidths of molecular clouds typically indicate velocity dispersions of several km s^{-1}, whereas the sound speed in 10 K molecular gas is $c_s = \sqrt{kT/\mu} = 0.18$ km s^{-1}, using a mean particle mass $\mu = 2.3 m_{\mathrm{H}}$, appropriate for the standard He abundance when all the H is molecular. This means that the gas in molecular clouds is highly supersonic, with Mach numbers $\gtrsim 10$.

The velocity dispersion seems to depend on the size of the cloud being measured:

$$
\sigma \propto L^p, \tag{9}
$$

with $p \approx 1/2$. Solomon et al. (1987) find for galactic GMCs

$$
\sigma \approx 0.72 (R/\mathrm{pc})^{0.5} \text{ km s}^{-1}, \tag{10}
$$

where R is the cloud radius. (Note: *Draine* gives a slightly different relation from Larson (1981); it is different in part because it is based on earlier data, but also because Bruce uses 3d velocity dispersions, while I am using the 1D velocity dispersions more commonly used in observational work. They differ by a factor of $\sqrt{3}$.)

Typical GMCs are tens of pc in radius and have velocity dispersions of several km s^{-1}, but smaller clouds, or smaller regions within clouds, have velocity dispersions that are smaller. Relations of this form are known as linewidth-size relations, and are expected for either subsonic or supersonic turbulence, for reasons that you will have to take the star formation class to hear about.

The existence of the common surface density and the linewidth-size relation has an important implication for the role of gravity in GMCs. From the virial theorem one can show that a uniform-density sphere of radius R and mass M that is in virial balance between internal pressure (turbulent or thermal) and gravity has a velocity dispersion

$$\sigma^2 = \frac{GM}{5R}. \tag{11}$$

We define the virial parameter of a cloud by

$$\alpha = \frac{5\sigma^2 R}{GM}. \tag{12}$$

A value $\alpha = 1$ corresponds to a cloud that is confined entirely by self-gravity; larger values of α require either that the cloud be out of equilibrium, or that it be confined by external pressure.

If we adopt a linewidth size relation $\sigma = \sigma_0 (R/R_0)^{1/2}$, as the observations suggest, then we have

$$\alpha = \frac{5\sigma_0^2 R^2}{G R_0 M} = \frac{5\sigma_0^2}{\pi G R_0 \Sigma}, \tag{13}$$

where $\Sigma = M/\pi R^2$ is the cloud surface density. Since σ_0, R_0, and Σ are the same from cloud to cloud, this means that α is as well. Moreover, if we plug in $\Sigma = 100$ $M_\odot$ pc^{-2}, $\sigma_0 = 0.72$ km s^{-1}, and $R_0 = 1$ pc, as observed, then we obtain $\alpha = 1.9$, meaning that, within the errors, molecular clouds are confined by self-gravity.

These three results – that molecular clouds have constant surface densities, that they obey a linewidth-size relation with an index near 1/2, and that they are self-gravitating – are known as Larson's Laws. He first proposed them in a paper in 1983. Only two are independent, and the third can always be deduced from the other two.

D. Pressure balance

The self-gravitating nature of GMCs also has important implications for their place in the ISM. Recall that we showed that for isothermal gas in a slab geometry, the midplane pressure is related to the gas surface density by

$$P = \frac{\pi}{2} G \Sigma^2. \tag{14}$$

In spherical geometry the coefficient is not $\pi/2$, but the same basic relation applies $P \approx G\Sigma^2$, with a coefficient of order unity.

Molecular cloud surface densities are a factor of $\sim$ 10 greater than the surface densities of the ISM as a whole, and this means that their internal pressures are a factor of 100 greater. The typical pressure in a GMC is $P/k \approx 3 \times 10^5$ K cm^{-3}, compared to around 3000 K cm^{-3} in as the mean of the ISM.

The implication of this result is that molecular clouds are not in pressure balance with the rest of the ISM. In this sense they are more like stars than like other components of the ISM, and for exactly the same reason. Stars are gaseous entities too, and they have surface pressures, but they do not contribute to the pressure balance of the ISM because they are self-gravitating, and their pressure is entirely offset by their gravity. Our result indicates that the same general statement applies to molecular clouds. They contribute gravity but not pressure, and they act like collisionless particles in the larger dynamics of the galactic disk.

E. Magnetic fields

In addition to their high pressures and Mach numbers, molecular clouds have strong magnetic fields, as measured by Zeeman splitting of Zeeman-sensitive molecules such as OH and CN. Magnetic field strengths range from tens of μG in relatively low density regions to mG in dense regions. The magnetic field strength correlates with the gas density and velocity dispersion as

$$B \propto \sigma\sqrt{\rho} \tag{15}$$

The best fit, from Basu (2000) based on data from Crutcher (1999), is

$$B = 6.1\sigma_1 n_2^{1/2}\,\mu\text{G}, \tag{16}$$

where $\sigma_1 = \sigma/1$ km s^{-1} and $n_2 = n_{\rm H}/100$ cm^{-3}.

The implication of this becomes clear if we compute the Alfven speed this implies:

$$v_A = \frac{B}{\sqrt{4\pi\rho}} = \frac{B_0/\sigma_0}{\sqrt{4\pi\rho_0}}\sigma. \tag{17}$$

Thus we see that the observed correlation implies that the Alfvenic Mach number of molecular clouds is roughly constant:

$$\mathcal{M}_A = \frac{\sigma}{v_A} \approx \frac{B_0/\sigma_0}{\sqrt{4\pi\rho_0}} = 1.1. \tag{18}$$

Thus we see that the observed turbulence in molecular clouds is trans-Alfvenic.

In fact, this can be understood naturally from the physics of MHD turbulence. Suppose one started with a turbulent medium where the turbulence was highly super-Alfvenic. In this case the magnetic pressure is weak compared to the ram pressure of the turbulent flows. In this case, however, the turbulence has no problem twisting and tangling the magnetic field lines. When this happens, though, the magnetic field strength is amplified. The result is that an initially weak field can be amplified – this is called a turbulent dynamo. The amplification process stops when the magnetic pressure and tension are sufficient to resist further twisting of the field, and this is equivalent to the condition that $\mathcal{M}_A \sim 1$.

II. Molecular cloud collapse and star formation

Probably the most interesting aspect of molecular gas is that it is the phase of the ISM associated with star formation. We will only very briefly touch on this phenomenon here, in order to understand why GMCs are the sites of star formation.

A. The virial theorem

Since molecular clouds are gravitationally bound structures, as we showed last time, their evolution depends on the contest between self-gravity and other forces that oppose collapse. This can be described in many ways, but perhaps the most powerful is via the virial theorem, a sort of integrated form of the equations of motion. We will not prove the theorem in this class, but you will see the proof if the take the star formation class, so we simply assert the result here:

$$\frac{1}{2}\ddot{I} = 2(\mathcal{T} - \mathcal{T}_S) + \mathcal{B} + \mathcal{W} - \frac{1}{2}\frac{d}{dt}\int_S (\rho \mathbf{v} r^2) \cdot d\mathbf{S}. \tag{19}$$

This is the Eulerian form of the theorem. so I is the moment of inertia of the material within some fixed surface S, $\mathcal{T}$ its its total kinetic plus thermal energy, $\mathcal{T}_S$ is the integrated pressure on its surface, $\mathcal{B}$ is the magnetic energy in the volume, $\mathcal{W}$ is the gravitational potential energy, and the last term represents the change in moment of inertia due to fluxes of material across the edge of the volume.

Looking at this equation, we can see that there are two terms that tend to make $\ddot{I}$ positive, and thus promote expansion, two terms that make it negative and therefore promote collapse, and one term that could go either way. The expansionary terms are $\mathcal{T}$ and $\mathcal{B}$, while the compressive ones are $\mathcal{T}_S$ and $\mathcal{W}$ (since gravitational potential energy is negative). The mass flux term could be either depending on whether mass is entering or leaving the volume.

Physically, the two expansionary terms represent the influence of thermal and ram pressure, and magnetic pressure, while the two compressive ones represent external pressure and self-gravity. The fact that our analysis from last time indicates the molecular clouds are self-gravitating implies that the gravitational term is the more important of these two, although formally we must include both. We will compare the gravitational term to the other terms as a way of estimating when a molecular cloud can be stable against collapse.

B. Thermal pressure support

Let us first compare the gravitational term to the kinetic one, first assuming only thermal pressure. To make this definite, we will consider a spherical region of mass M and radius R, bounded by an external pressure p_0. The gravitational self-energy of a sphere is

$$\mathcal{W} = -\frac{3}{5}a\frac{GM^2}{R}, \tag{20}$$

where a is a constant that depends on the internal density distribution; for a uniform sphere $a = 1$.

The kinetic term is

$$\mathcal{T} = \frac{3}{2}\int P\,dV = \frac{3}{2}c_s^2\int \rho\,dV = \frac{3}{2}Mc_s^2, \tag{21}$$

where we have taken the gas to have a constant sound speed c_s.

Finally, the surface term is

$$\mathcal{T}_S = \frac{1}{2}\int p_0\mathbf{r}\cdot d\mathbf{S} = 2\pi R^3 p_0. \tag{22}$$

Plugging these in, the condition for stability, $\ddot{I} = 0$, implies that

$$0 = 3Mc_s^2 - 4\pi p_0 R^3 - \frac{3}{5}a\frac{GM^2}{R} \qquad \Longrightarrow \qquad p_0 = \frac{1}{4\pi R^3}\left(3Mc_s^2 - \frac{3}{5}a\frac{GM^2}{R}\right). \tag{23}$$

Now consider what this tells us about the relationship between p_0 and R for a cloud of fixed mass. Clearly $p_0 = 0$ if $R = R_{\rm min} = aGM/5c_s^2$, and for larger R the external pressure p_0 has a positive values. However, in the limit $R \to \infty$, we also have $p_0 \to 0$. Thus p_0 much reach a maximum for some value of R between $R_{\rm min}$ and infinity. Solving for this maximum by taking $dp_0/dR = 0$, we can find this maximum:

$$p_{0,\rm max} = \frac{3^4 5^3}{4^5\pi}\frac{c_s^8}{a^3G^3M^2}. \tag{24}$$

We can now turn this argument around: for a given external pressure p_0, there exists a maximum mass M for which the cloud can be in equilibrium – for larger masses we would have $p_0 > p_{0,\rm max}$. Plugging this in, we have

$$M_{\rm max} = \frac{225}{32\sqrt{5\pi}}\frac{c_s^4}{(aG)^{3/2}}\frac{1}{\sqrt{p_0}} = 0.26\left(\frac{T}{10\text{ K}}\right)^2\left(\frac{10^6\text{ cm}^{-3}\text{ K}}{p_0/k}\right)^{1/2} M_\odot, \tag{25}$$

where in the numerical evaluation we have used $a = 1.67$, the numerical result from a more rigorous calculation. This is known as the Bonnor-Ebert mass. Thus at the typical pressures and temperatures in a GMC, any blob of gas larger than $\sim 0.25\ M_\odot$ cannot be held up against gravitational collapse by thermal pressure.

C. Magnetic support

OK, if thermal pressure can't support molecular clouds, how about magnetic fields? We will consider a spherical cloud of radius R and mass M, now threaded by a uniform magnetic field $\mathbf{B}$. For simplicity we will assume that the magnetic field outside the cloud is negligible compared to that inside it, and we will assume perfect flux freezing, which is a good assumption except on small scales. In this case the magnetic term in the virial theorem is

$$\mathcal{B} = \frac{1}{8\pi}\int_V B^2\, dV + \int_S \mathbf{x}\cdot\mathbf{T}_M\cdot d\mathbf{S}, \tag{26}$$

where

$$\mathbf{T}_M = \frac{1}{4\pi}\left(\mathbf{BB} - \frac{B^2}{2}\mathbf{I}\right) \tag{27}$$

is the Maxwell stress tensor. The $\mathbf{BB}$ term is the tensor product of $\mathbf{B}$ with itself, and $\mathbf{I}$ is the identity tensor.

If we approximate that the field inside the volume is uniform,

$$\frac{1}{8\pi}\int_V B^2\, dV = \frac{B^2R^3}{6} = \frac{\Phi_B^2}{6\pi^2 R}, \tag{28}$$

where $\Phi_B = \pi B R^2$ is the magnetic flux threading the cloud. The second term in $\mathcal{B}$ is negligible if the field at the virial surface is much weaker than that inside the cloud. The reason for writing things in terms of the magnetic flux is that it remains constant as the cloud expands or contracts, since we have perfect flux freezing.

Plugging this into the virial theorem, and neglecting all terms other than the magnetic and gravitational ones, we find that the condition for virial equilibrium is

$$0 = \mathcal{B} + \mathcal{W} = \frac{\Phi_B^2}{6\pi^2 R} - \frac{3}{5}\frac{GM^2}{R} \equiv \frac{3}{5}\frac{G}{R}\left(M_\Phi^2 - M^2\right), \tag{29}$$

where we haven't bothered with the factor of a since our treatment of B is inexact, and we have defined

$$M_\Phi = \sqrt{\frac{5}{2}}\left(\frac{\Phi_B}{3\pi G^{1/2}}\right) \tag{30}$$

to be the magnetic critical mass. A more precise calculation, including the fact that the cloud density is non-uniform (Tomisaka et al. 1998), gives

$$M_\Phi = 0.12\frac{\Phi_B}{G^{1/2}}, \tag{31}$$

as opposed to the 0.17 we got in the uniform case.

If $M > M_\Phi$, then $\mathcal{B} + \mathcal{W} < 0$ and the cloud will contract. Moreover, since M and M_Φ are invariant as the cloud expands or contracts, then $\mathcal{B} + \mathcal{W}$ will remain negative forever, and the field will never halt collapse. Conversely, if $M < M_\Phi$ then the cloud will never collapse.

Thus we see that magnetic fields do provide a potential mechanism to stabilize molecular clouds, if they are strong enough. Whether they are or not is an empirical question. The consensus answer seems to be that they are not, although not by a huge amount. However, this is something that is still hotly debated in the literature. This is a *very* difficult observations to make.

If magnetic fields are sufficient to prevent collapse, then star formation will occur only in those locations where the assumption of flux-freezing is violated as a result of ambipolar drift or other non-ideal MHD effects.

D. Turbulent support

A third possible source of support comes from the turbulent part of the kinetic term. We showed last time that $\alpha \approx 1$, which is equivalent to the statement that $2\mathcal{T} + \mathcal{W} \approx 0$. Thus turbulence is at least potentially able to prevent cloud collapse.

This doesn't mean that there is no star formation at all, simply that the cloud as a whole is globally stable. The linewidth-size relation implies that most of the power in the turbulence is on large scales, and large-scale shocks are compressive, at least in certain locations. In these regions the colliding flow will reduce $\mathcal{T}$, and locally the region may be way out virial equilibrium and may collapse. This will produce star formation, but at a low rate – which turns out to be about what observations of the star formation rate require.