Class 16 Notes: Photoionised regions: dynamics

Our next topic is the dynamics of ionised gas. H II regions form around massive stars, which are born in dense, cold molecular gas at a temperature of ~ 10 K. The ionising radiation of the star quickly photodissociates the molecules and ionises the atoms. This increases the number of free particles by a factor of 4, and increases the temperature to $\sim 10^4$ K, as we showed last time. In this class we will study the behavior of this gas as it evolves dynamically. This will build on our understanding of the structure of ionisation fronts from the last class.

I. H II region expansion: the simplest case

We will first consider the simplest possible case of H II region expansion, in order to develop a basic understanding. We therefore consider a neutral medium of initial density ρ_0, within which at time 0 a source of ionising radiation of ionising luminosity Q_0 turns on. The only important force driving the expansion of the H II region is gas pressure, and where there are no radiative or magnetic forces to consider.

A. The R phase

Since the ionising flux per unit area around the star is formally infinite at $t = 0$ (or at least $Q_0/4\pi R_*^2$, which is very large), the front always starts as R type. The expansion in this case is supersonic with respect to both the neutral gas and the ionised gas, since the front speed greatly exceeds $u_R \approx 2c_i$, where c_i is the ionised gas sound speed. Thus to good approximation the gas does not have time to move or react hydrodynamically, and it remains fixed.

This is exactly the situation we considered in the last class when we assumed the gas density to remain constant and simply solved for the radius of the ionised region based on photon conservation. To remind you, the radius as a function of time is given by

$$R_i = R_S \left(1 - e^{-t/t_{\mathrm{rec}}}\right)^{1/3}, \tag{1}$$

where $t_{\mathrm{rec}} = 1/n_0\alpha_B$, $n_0 = \rho_0/\mu_{\mathrm{H}}$ and μ_{H} is the mean mass per hydrogen nucleus in the neutral gas. The expansion velocity of the ionisation front is therefore

$$V_i = \frac{dR_i}{dt} = \frac{R_S}{t_{\mathrm{rec}}} \frac{e^{-t/t_{\mathrm{rec}}}}{(1 - e^{-t/t_{\mathrm{rec}}})^{2/3}}. \tag{2}$$

B. Transition to D

As the time goes on, V_i clearly decreases, and eventually it drops to the point where $V_i = u_R \approx 2c_i$; this occurs at a time t such that

$$\frac{e^{-t/t_{\mathrm{rec}}}}{(1 - e^{-t/t_{\mathrm{rec}}})^{2/3}} \approx \frac{2c_i t_{\mathrm{rec}}}{R_S} \tag{3}$$

The time / radius at which this occurs clearly depends on the ratio on the right hand side, which is given by

$$\frac{2c_i t_{\rm rec}}{R_S} = 0.0038 n_{0,3}^{-1/3} Q_{0,49}^{-1/3} c_{i,10}, \tag{4}$$

where $n_{0,3} = n_0/10^3$ cm^{-3}, $Q_{0,49} = Q_0/10^{49}$ s^{-1}, and $c_{i,10} = c_i/10$ km s^{-1}. When the right hand side is small, $t/t_{\rm rec}$ must be significantly greater than unity; for these fiducial values, $t/t_{\rm rec} = 5.6$, so $R_i = 0.999 R_S$. Thus in general the expansion velocity does not drop to u_R until the R type front has reached almost all the way to the Strömgren radius.

Once this happens, the front must undergo a transition to D type, precipitated by a shock running ahead of the ionisation front.

C. The D phase

To see what happens next, we must consider the interior of the ionised region. An exact analytic solution for what happens during the transition between R and D type is not possible, but we can obtain a very good approximation that becomes increasingly good as time goes on by making some simple observations.

First, since the expansion is slow, the H II region interior must be very close to ionisation balance. In other words, the excess of photons reaching the front during a given time period that are available to ionise new material must be a small fraction of the total number emitted during that period. Thus to good approximation we must have ionisation balance in the H II region interior. Second, since we are transitioning to D type, the expansion velocity will be subsonic with respect to the ionised gas in the H II region interior. Thus to good approximation we may treat its density as uniform, with no variation as a function of distance from the source.

This makes the problem of how the H II region evolves relatively easy. Since we are in ionisation balance and the number density n_i in the H II region interior is uniform, we must have

$$Q_0 = \frac{4}{3}\pi R_i^3 \alpha_B n_i^2, \tag{5}$$

just by the usual Strömgren argument. This implies that

$$n_i = \left(\frac{3Q_0}{4\pi\alpha_B R_i^3}\right)^{1/2} = n_0 \left(\frac{R_i}{R_S}\right)^{-3/2}, \tag{6}$$

where $n_0 = \rho_0/\mu_{\rm H}$ is the number density before expansion starts, $\mu_{\rm H}$ is the mean mass per hydrogen nucleus, and $R_S = (3Q_0/4\pi\alpha_B n_0^2)^{1/3}$ is the initial Strömgren radius. Since $\rho_i = n_i \mu_{\rm H} \propto R_i^{-3/2}$, and the total mass of ionised gas varies as

$$M_i \propto \rho_i R_i^3 \propto R_i^{3/2}. \tag{7}$$

In contrast, the total mass swept up by the H II region clearly varies as R_i^3. Thus the fraction of the swept-up mass in the ionised H II region interior declines as

$R_i^{-3/2}$; the remainder of the mass is in a neutral shell at its surface between the shock front and the ionisation front. Once the H II region has expanded any significant amount, this shell must contain the great majority of the swept-up mass.

In this limit, we can compute the evolution of the radius simply from momentum conservation. The total mass contained in the shell when its radius is R_i is just $(4/3)\pi\rho_0 R_i^3$, and thus the momentum of the shell is

$$p_{\rm sh} = \frac{4}{3}\pi\rho_0 R_i^3 \frac{dR_i}{dt}, \tag{8}$$

where we have neglected the small mass in the H II region interior. The pressure exerted by the ionised gas in the interior on this shell is

$$P = \rho_i c_i^2 = \rho_0 c_i^2 \left(\frac{R_i}{R_S}\right)^{-3/2}. \tag{9}$$

The total force exerted on the shell is simply $4\pi R_i^2 P$, assuming that the pressure within the H II is much larger than the pressure outside it.

Conservation of momentum requires that the rate of change in the shell momentum match the force applied by the ionised gas (since there are no other forces), and we therefore have

$$\frac{d}{dt}\left(\frac{4}{3}\pi\rho_0 R_i^3 \frac{dR_i}{dt}\right) = 4\pi R_i^2 \rho_0 c_i^2 \left(\frac{R_i}{R_S}\right)^{-3/2}. \tag{10}$$

The equation is easy to solve, particularly if we make a change of variables to non-dimensionalize things. The natural length scale of the problem is R_S, and the natural time scale is R_S/c_i, the sound crossing time of the original Strömgren radius. We therefore let $x = R_i/R_S$ and $\tau = t/(R_S/c_i)$, which changes the equation to

$$\frac{d}{d\tau}\left(x^3 \frac{dx}{d\tau}\right) = 3x^{1/2}. \tag{11}$$

Equations of this sort, in which both sides are proportional to x to some power and its derivatives, generally admit similarity solutions in which $x \propto \tau^p$ for some power p. This one is no exception. If one substitutes in $x = c\tau^p$, one finds that

$$x = \left(\frac{49}{12}\right)^{2/7} \tau^{4/7} \tag{12}$$

is a solution. Thus we have shown that $R_i/R_S \propto t^{4/7}$. Of course the zero point of time is arbitrary, so we are free to choose our time zero to be such that $R = R_S$ at $t = t_S$, where t_S is the time after which the H II region reaches a radius R_S and switches from R type to D type. Thus the solution is

$$R_i \approx R_S \left(\frac{t}{t_S}\right)^{4/7} \tag{13}$$

during the D type phase.

D. Stalling

This expansion cannot continue indefinitely. Instead, the expansion must slow down once the pressure in the H II region is no longer much larger than the pressure in the surrounding neutral gas. In this case we can no longer drop the term in the momentum equation describing the force applied to the shell by the external pressure. At what point do we need to consider this effect?

Suppose the ambient neutral material has a sound speed c_n, and thus a pressure $\rho_0 c_n^2$. The D phase begins to end once this is comparable to the pressure P in the H II region interior we computed earlier: $P = \rho_0 c_i^2 (R_i/R_S)^{-3/2}$. Combining this we have

$$\rho_0 c_n^2 \approx P \qquad \Longrightarrow \qquad R_i = R_S \left(\frac{c_i}{c_n}\right)^{4/3} \equiv R_{\mathrm{stall}}. \tag{14}$$

This quantity is referred to as the stalling radius of the H II region. Note that it depends on ρ_0 only indirectly, through R_S.

Combining this with our calculation of R_i above, we can also identify a characteristic stalling time at which the H II region reaches this radius. Plugging in $R_i \approx R_S (t/t_S)^{4/7}$, we have

$$t_{\mathrm{stall}} = t_S \left(\frac{c_i}{c_n}\right)^{7/3}. \tag{15}$$

To get a sense of typical values here, it is helpful to plug in some numbers. Doing so gives

$$R_{\mathrm{stall}} = 13.9 c_{i,10}^{4/3} c_{n,1}^{-4/3} n_{0,3}^{-2/3} Q_{0,49}^{1/3}\ \mathrm{pc} \tag{16}$$

$$t_{\mathrm{stall}} = 13.6 c_{i,10}^{4/3} c_{n,1}^{-7/3} n_{0,3}^{-2/3} Q_{0,49}^{1/3}\ \mathrm{Myr}, \tag{17}$$

where $c_{n,1} = c_n/1\ \mathrm{km\ s^{-1}}$. Note that this is significantly longer than the lifetime of a typical massive star, so unless the ambient gas is significantly denser the value of 10^3 we've adopted here, or the driving star is a fairly wimpy B star instead of an O star (which lowers Q_0 and also raises the stellar lifetime), most H II regions will not stall before their driving stars evolve off the main sequence and stop providing ionising photons.

II. H II region expansion beyond the simplest case

A. Radiation pressure

In our treatment of H II region dynamics up to this point, we neglected radiation pressure. We can, however, include it in the D type phase fairly easily simply by adding the radiation pressure force to our equation of momentum conservation. Suppose that the source has a bolometric luminosity L, and that each photon it emits is absorbed in the shell or the H II region interior, and then escapes – this is a reasonably good approximation. The force exerted by the radiation on the shell is

$$F_{\mathrm{rad}} = \frac{L}{c} \tag{18}$$

In this case the equation of momentum conservation becomes

$$\frac{d}{dt}\left(\frac{4}{3}\pi\rho_0 R_i^3 \frac{dR_i}{dt}\right) = 4\pi R_i^2 \rho_0 c_i^2 \left(\frac{R_i}{R_S}\right)^{-3/2} + \frac{L}{c}, \tag{19}$$

In the limit where the radiation pressure force term is much greater than the gas pressure term, we can drop the first term on the right hand side, and the equation simply becomes

$$\frac{d}{dt}\left(R_i^3 \frac{dR_i}{dt}\right) = \frac{3L}{4\pi\rho_0 c}. \tag{20}$$

Not surprisingly, this equation too admits a similarity solution: $R_i \propto t^{1/2}$.

This solution cannot hold indefinitely, however. The gas pressure force varies with radius as $R_i^{1/2}$, while the radiation force is independent of radius. Thus there is always some radius at which the radiation force becomes smaller than the gas pressure force. Equating the two terms gives an expression for the characteristic radius at which the two forces are equal

$$4\pi R_{\mathrm{ch}}^2 \rho_0 c_i^2 \left(\frac{R_{\mathrm{ch}}}{R_S}\right)^{-3/2} = \frac{L}{c}. \tag{21}$$

Note that this expression is actually independent of the density ρ_0, because $R_S \propto \rho_0^{-2/3}$, so density cancels out of the problem. Solving for R_{ch} and doing some re-arranging, we obtain

$$R_{\mathrm{ch}} = \frac{\alpha_B}{12\pi}\left(\frac{I_{\mathrm{H}}}{kT}\right)^2 \chi^2 \frac{Q_0}{c^2} = 5.8 \times 10^{-3} T_4^{-2} \chi^2 Q_{0,49} \text{ pc} \tag{22}$$

where I_{H} is the ionisation potential of hydrogen, $T \approx 10^4$ K is the gas temperature inside the H II region, and $\chi = L/(Q_0 I_{\mathrm{H}})$, which depends on the stellar spectrum and for hot stars is typically a few. Since this is fairly small, usually smaller than the initial Strömgren radius, gas pressure generally dominates H II region expansion. The exception is for extremely luminous clusters of many stars, which can reach $Q_{0,49} \sim 100$ in some cases. Such a cluster can have $R_{\mathrm{ch}} \sim 1 - 10$ pc, and radiation pressure will dominate its initial expansion.

One can find direct observational evidence for this in some H II regions, where it is possible to estimate the radiation pressure from photometry and the gas pressure from emission measures determined in the radio.

B. Magnetic fields

A second complication to this story is magnetic fields. The ISM is magnetised, and magnetic fields will resist compression. We can therefore ask when magnetic forces are strong enough to modify the expansion of H II regions. Consider the same setup as before: a uniform density region within an ionising source, but now add a uniform magnetic field B_0 in the initial condition.

First let us consider some basic ratios. The dynamic importance of the magnetic field can be characterised by the Alfven speed $v_A = B_0/\sqrt{4\pi\rho_0}$: large values correspond to strong magnetic fields and small ones to weak fields. The magnetic pressure force is ρv_A^2. For typical ISM magnetic field strengths and densities, we will usually have $v_A \ll c_i$, so when an H II region starts expanding, the gas pressure $\rho_0 c_i^2$ will greatly exceed the magnetic pressure $\rho_0 v_A^2$, and the field will not significantly interfere with expansion.

As the gas expands, we can ask how the field lines move. It is helpful to ask how many field lines pass through the interior of the H II region, and how many pass through the shell that bounds it. Since the field is frozen into the matter, this is equivalent to asking where gas started out. Gas that was initially near enough to the ionising source, within some radius r_0, will be in the shell interior, and gas that started out at a distance from r_0 to R_i will be in the shell. In order to compute r_0, recall that we showed earlier that the mass of ionised material as $M_i \propto R_i^{3/2}$. Thus when the shell is at radius R_i, the ionised mass is

$$M_i = \frac{4}{3}\pi\rho_0 R_S^3 \left(\frac{R_i}{R_S}\right)^{3/2}. \tag{23}$$

If we equate this to the material that began within a distance r_0 of the center, which has mass $(4/3)\pi\rho_0 r_0^3$, then we immediately find that

$$\frac{4}{3}\pi\rho_0 R_S^3 \left(\frac{R_i}{R_S}\right)^{3/2} = \frac{4}{3}\pi\rho_0 r_0^3 \qquad \Longrightarrow \qquad r_0 = \sqrt{R_i R_S}. \tag{24}$$

Flux freezing means that the magnetic flux passing through the interior of the H II region is equal to that which started within a radius r_0 of the center, thus we have

$$\Phi_i = \pi B_0 r_0^2 = \pi B_0 R_i R_S. \tag{25}$$

The flux passing through the dense shell must be equal to all the swept up flux minus this contribution, that is

$$\Phi_{\rm sh} = \pi B_0 R_i^2 - \Phi_i = \pi B_0 R_i (R_i - R_S) \approx \pi B_0 R_i^2 \tag{26}$$

when $R_i \gg R_S$.

Thus in the initial expansion phase, all the magnetic flux is in the shell. This phase will end once the magnetic pressure is able to significantly restrain the expansion. The condition for this to happen is that the external magnetic pressure become comparable to the internal thermal pressure, i.e.

$$\rho_i c_i^2 \approx \rho_0 v_A^2. \tag{27}$$

Recall that $\rho_i = \rho_0 (R_i/R_S)^{-3/2}$ due to ionisation balance. Substituting this in and re-arranging, we find that magnetic effects become important when

$$R_i \approx \left(\frac{c_i}{v_A}\right)^{4/3} R_S \equiv R_m, \tag{28}$$

where R_m is the magnetic critical radius. After this point the expansion of the shell must become aspherical.

We can plug in some numbers to get a sense of when magnetic effects are important. First, we can check the assertion that $v_A \ll c_i$. Typical magnetic field strengths in the vicinity of H II regions are tens to hundreds of μG, and

$$v_A = \frac{B}{\sqrt{4\pi\rho_0}} = 5.8 n_{0,3}^{-1/2} B_2 \text{ km s}^{-1} \tag{29}$$

where $B_2 = B/100\ \mu$G. This is indeed less than $c_i \approx 10$ km s^{-1} for $B = 10 - 100$ μG. The magnetic critical radius is

$$R_m = 1.3 Q_0^{1/3} c_i^{4/3} B^{4/3} \text{ pc.} \tag{30}$$

Note that density drops out because of a cancellation between the Alfven speed and the Stromgren radius. Physically this makes sense: the magnetic critical radius is determined by the balance between magnetic forces trying to restrain expansion and pressure forces trying to cause them. The magnetic force does not depend on the gas density, and the pressure force depends only on the density inside the H II region, which is set by ionisation balance and does not depend at all on the ambient density.

C. Champagne flows and rocket nozzles

A third complication is that H II regions rarely occur in uniform media. Instead, they tend to occur in clumpy, non-uniform regions where the ionised gas is not fully confined, and instead is able to blow out and escape.

As an idealised model of what this looks like, consider an ionising source placed at the edge of a dense cloud. To be precise, suppose that we have an ionising source of ionising luminosity Q_0 located at the origin, and the initial neutral density is n_0 for $x < 0$ and 0 for $x > 0$. As the initially dense, neutral gas is ionised, it will flow away into the low-density vacuum, rather than remaining confined in a sphere. How does this change our picture of H II region expansion?

In full detail this problem cannot be solved analytically, but we can come up with a rough analytic solution by supposing that the expanding ionisation front remains approximately hemispherical. The characteristic density of the material that has been ionised off the expanding front, but not yet escaped into the vacuum, will again be determined by photoionisation equilibrium:

$$\frac{1}{2} Q_0 = \frac{2\pi}{3} \pi R_S^3 \alpha_B n_i^2, \tag{31}$$

where the factor of 1/2 on the left hand side is coming from the fact that only half of the emitted photons go into the hemisphere facing the dense medium, and the factor of $2\pi/3$ rather than $4\pi/3$ on the right coming from the fact that the ionised gas occupies a roughly hemispherical volume, rather than a full sphere.

This is clearly an idealisation of the geometry, but it is probably not off by more than a factor of two or so. The key point here is that the factors of 1/2 cancel, so we are left with the same basic scaling between density and radius as in the spherical case:

$$n_i = n_0 \left(\frac{R_i}{R_S}\right)^{-3/2}. \tag{32}$$

Next we can again consider momentum balance. The mass and momentum of the swept-up shell is the same as in the spherical case, just with an extra factor of 1/2:

$$p_{\rm sh} = \frac{2}{3}\pi\rho_0 R_i^3 \left(\frac{R_i}{R_S}\right)^{-3/2}. \tag{33}$$

To estimate the pressure provided by the ionised gas, we have to realise that the system we have set up is effectively a rocket: we heat up a gas that is confined on one side, and it freely expands into a vacuum. As it does so, it pushes on the dense confining walls, imparting momentum to them. In terms of momentum conservation, the governing equation is

$$\frac{d}{dt}p_{\rm sh} = A\rho_i \left[c_i^2 + u_i\left(u_i - \frac{dR_i}{dt}\right)\right]. \tag{34}$$

Here $A \approx 2\pi R_i^2$ is the area on which the escape gas is pushing and u_i is the speed with which the ionised gas leaves the ionisation front. The first term in square brackets is the pressure exerted by the ionised gas, and the second is the force per unit area exerted by the recoil of the ionised gas – the rocket effect. This term goes to zero if either the escaping gas does not move away ($u_i = 0$) or if the gas is moving at the same speed as the front and thus does not push on it ($u_i - dR_i/dt = 0$), but is non-zero otherwise.

So what is u_i? To answer that question, recall our discussion of D type ionisation fronts. Since there is nothing confining the ionised gas, we expect the density to be very low, and in this case we showed that the rate at which gas flows away from the ionisation front just approaches the ionised gas sound speed. We therefore have $u_i \approx c_i$. At late times, when the front is expanding slowly compared to the ionised gas sound speed, $dR_i/dt \ll c_i$, we therefore have

$$\frac{d}{dt}p_{\rm sh} =\approx 2A\rho_i c_i^2 = 4\pi R_i^2 \rho_i c_i^2. \tag{35}$$

Inserting this into the equation of momentum conservation, we obtain

$$\frac{d}{dt}\left(\frac{2}{3}\pi\rho_0 R_i^3 \frac{dR_i}{dt}\right) = 4\pi R_i^2 \rho_0 c_i^2 \left(\frac{R_i}{R_S}\right)^{-3/2}. \tag{36}$$

Note that this is nearly but not quite identical to Equation 10: the right hand side is the same, but the left-hand side is a factor of 2 smaller. The LHS is smaller because the mass and momentum of a hemisphere is half that of a sphere. On the RHS, on the other hand, the factor of 2 smaller working surface of a hemisphere is compensated for by a factor of 2 increase in the force per unit area – we get one factor of $\rho_i c_i^2$ from the gas pressure, and another factor of $\rho_i c_i^2$ because the gas rockets away from the surface rather than staying trapped near it. As a result, the H II region expands somewhat faster. When you work through the similarity solution, the result is also nearly identical to the spherical case, just with a slightly larger coefficient.