In this tutorial we will explore the demographics of degenerate stellar end states in the Milky Way Galaxy, in order to understand how many of various types of objects we should be able to see. For the purposes of this calculation, we will make use of two empirical facts. The first is that the rate of star formation in our Galaxy is observed to be about 2 $M_\odot$ / yr of new stars. The second is that the distribution of the masses of those stars that form, known as the initial mass function (IMF). Empirically, the observed IMF can be approximately described by

$$\frac{dN}{dM} = \mathcal{N} \begin{cases} (M/M_\odot)^{-1.3}, & 0.08\, M_\odot < M < 0.5\, M_\odot \\ 0.5^{-1.3}(M/M_\odot)^{-2.3}, & 0.5\, M_\odot < M < 120\, M_\odot \end{cases}, \tag{1}$$

where $\mathcal{N}$ is a normalisation factor. The way to interpret this distribution is that the number of stars that form that have masses in the range from M_1 to M_2 is given by

$$N = \int_{M_1}^{M_2} \frac{dN}{dM}\, dM. \tag{2}$$

Exercise 1. *Just so we have an idea what this distribution looks like, make a plot of it. Use logarithmic axes, and ignore the arbitrary normalisation factor $\mathcal{N}$.*

Exercise 2. *From the IMF, compute the mean mass of a star that is born in the Milky Way. That is, what mass does the average star have? Using this information, compute the number of stars born in the Milky Way per year.*

Now that we know how many stars per year are born in the Milky Way, we are in a position to do some demographics on the population. Let's start with the most visible end state of stars, supernovae. With modern technology these should be visible anywhere in the Galaxy, unless we get really unlucky and one occurs on the far side of the Galactic nucleus, where dust would prevent us from seeing it. We will ignore this relatively small correction, and just assume that supernovae are visible no matter where in the Galaxy they occur.

Exercise 3. *Suppose that all stars with initial masses $M > 9\ M_\odot$ end their lives as supernovae. How many supernovae per century should occur in the Galaxy? For the purposes of this problem, you may assume that the lifetimes of the stars that produce supernovae are negligibly short.*

In reality, the last supernova to occur in our Galaxy that was observed on Earth was Kepler's Supernova in 1064. (It is named after Johannes Kepler, of Kepler's Laws, who made the most complete records of it that we have today, and wrote a book about it.) There were probably two more recent ones, around 1680 and 1870, but they were not detected at the time, probably due to dust obscuration that made them less bright in visible light than they would otherwise have been.

Exercise 4. *Suppose that the most recent supernova was in fact in 1604, and thus was 413 years ago. Given the supernova rate you computed in exercise 3, what is the probability of going 413 years without a supernovae? How about the probability of going ≈ 150 years? Is there a problem with our rate calculation in either case? Hint: you can compute the probability of going a certain amount of time without a supernova by assuming that supernovae, like all random processes, follow the Poisson distribution.*

Now we can start to work out the demographics of the degenerate remnants.

Exercise 5. *Assume that stars with initial masses between 9 and 20 $M_\odot$ produce neutron stars, and that star formation in the Galaxy has been occurring at a steady rate for the last 10*

Gyr. How many neutron stars are there in the Galaxy? Assuming that the neutron stars are distributed uniformly in a cylinder that is 10 *kpc in radius and* 1 *kpc thick, how close is the closest neutron star to Earth?*

Now we want to repeat this calculation for white dwarfs. They are a bit trickier, because the lifetimes of stars that produce white dwarfs are not necessarily short compared to the age of the Galaxy. As a very rough fit, assume that the time it takes a star with an initial mass M to evolve into a white dwarf is

$$t_{\rm evol} = 10 \text{ Gyr} \left(\frac{M}{M_\odot}\right)^{-2.5}, \tag{3}$$

and that all stars with initial masses below 9 $M_\odot$ will eventually produce white dwarfs.

Exercise 6. *Again assuming that star formation in the Milky Way has been ongoing at a constant rate for the last 10 Gyr, compute the number of white dwarfs in the Galaxy, and the expected distance between Earth and the nearest white dwarf. Hint: use the expression for the stellar evolution time to figure out, for progenitors of a given mass M, how much of the 10 Gyr lifetime of the galaxy was long enough ago that those stars have had time to produce white dwarfs. Once you have that result, you'll have to combine it with the IMF to make your estimate.*