We now go on to the evolution of massive stars, defined as those with masses above $\approx 8-10$ $M_\odot$. We will see that the ultimate fate of these stars is quite different than that of their lower mass brethren.

I. Post-main sequence evolution

A. Mass Loss

One important effect that distinguishes the evolution of massive stars from that of lower mass stars is the importance of mass loss, both on the main sequence and thereafter. Low mass stars do not experience significant mass loss before the AGB phase, but massive stars can lose mass while still on the main sequence, and can lose even more after they leave it.

Like other aspects of stellar mass loss, the exact mechanisms are not fully understood. Some of the mass loss is certainly due to interactions with other stars – it turns out that massive stars are very frequently in close binary systems, and as massive stars evolve on the main sequence their radii tend to grow, as we will discuss below. If they have a close companion, this growth can cause material toward the edges of the star to cease to be bound to the parent star, and instead to be accreted onto the companion. However, even for massive stars that do not have close companions, there is considerable evidence that they can lose a significant amount of mass, particularly for stars ~ 100 $M_\odot$ in size.

The most observationally-spectacular example of this that we have is a star known as η Carinae. This star is bright enough to be visible to the naked eye, and it was catalogued by both European and Chinese astronomers by the 1600s, but not much attention was paid to it. However, in the 1827 astronomer William Burchell noticed that it was much brighter than it had been; in modern notation, it had gone from being a magnitude 3-4 star to a magnitude 1 star. In 1837 it got even brighter, becoming at one point the second-brightest star in the night sky. During this period it was recorded in the oral histories of the Boorong people, whose traditional home is about 400 km northwest of Melbourne, near Lake Tyrrell. η Carinae stayed bright for a total of about 18 years before fading. It has shown repeated eruptions of this sort since then, though not quite as bright as the 1830-40s event, known as the Great Eruption.

Modern observations of η Carinae, as shown in Figure 1, show that it is a very massive star that is in the process of ejecting much of its envelope. The huge shell of dust and gas visible around it, known as the Homunculus Nebula, dates from the Great Eruption. The mass in the nebula is not entirely certain, but modern estimates mostly place it at $\approx 10-15$ $M_\odot$, meaning that the star must have ejected almost 1 $M_\odot$ per year of material during the Great Eruption. The mass of η Carinae itself is not known, but is thought to be of order 100 $M_\odot$, meaning that this star managed to lose $\sim 10\%$ of its mass in a span of ~ 20 years.

As mass loss processes like those occurring in η Carinae, erode a star's atmosphere, its outer layers become less and less dominated by hydrogen, eventually reaching $X \approx 0.1$ or even less. We see these stars as somewhat lower mass (but still very massive) stars whose atmospheres are dominated by helium rather than hydrogen. These are called Wolf-Rayet stars, and they are effectively the bare cores of massive

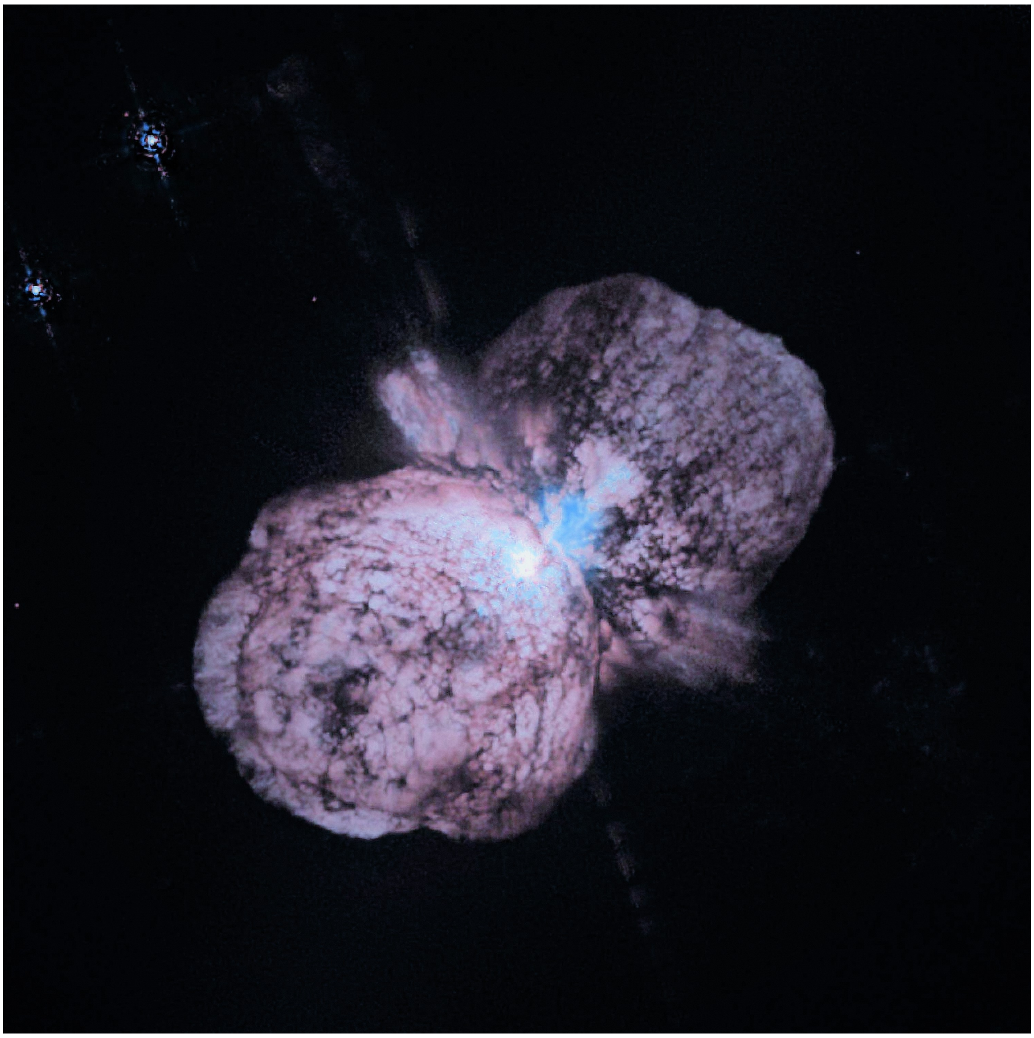

Figure 1: An image of η Carinae from the Hubble Space Telescope. Source: https://upload.wikimedia.org/wikipedia/commons/f/fc/Eta_Carinae.jpg.

Figure 2: An image of the nebula NGC 2359, known as Thor's Helmet. The nebula is a mix of mostly swept-up interstellar material and stellar ejecta, and is emitting due to illumination from the Wolf-Rayet star at its centre.

stars.

Stars become WR's while they are still on the main sequence, i.e., burning hydrogen in their centres. Stars in this case are called WN stars, because they are Wolf-Rayet stars that show large amounts of nitrogen on their surfaces. The nitrogen is the product of CNO cycle burning, which produces an equilibrium level of nitrogen above the amount that the star began its life with.

WR stars continue to lose mass rapidly, often producing spectacular nebulae that look like planetary nebulae. They shine for the same reason: the expelled gas is exposed to the high energy radiation of the star, and it flouresces in response. Figure 2 shows an example

Mass loss continues after the star exhausts H and begin burning He – at this point the surface composition changes and we begin to see signs of 3α burning. These are WC stars. The continuing mass loss removes the enhanced nitrogen from the CNO cycle, and convection brings to the surface the result of 3α burning, which is mostly carbon. Very rarely, we see WR stars where the carbon is being blown off, and the surface is dominated by oxygen.

Regardless of the evolutionary path, the effects of mass loss can be quite dramatic – 100 $M_\odot$ stars are thought to get down to nearly 30 $M_\odot$ by the time they evolve off the main sequence.

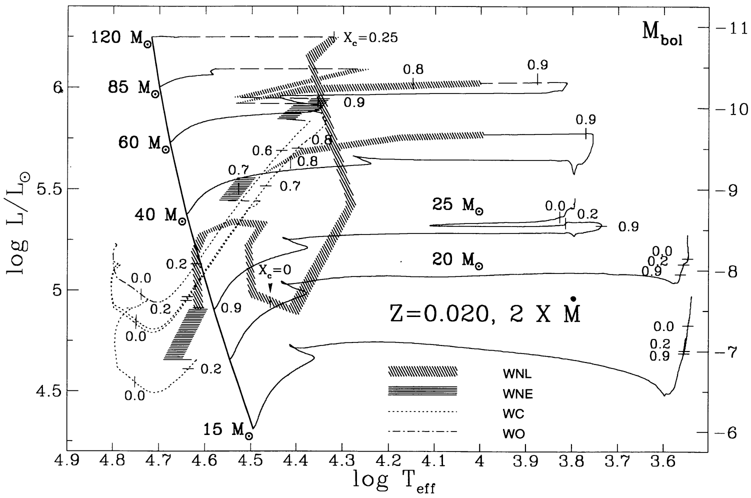

Figure 3: Evolutionary tracks for massive stars as calculated by the Geneva group. Source: Meynet et al. (1994, *Astronomy & Astrophysics*, 103, 87, http://adsabs.harvard.edu/abs/1994A%26AS..103...97M.

B. Movement on the HR diagram

While these stars show dramatic mass loss, their luminosities do not evolve all that much as they age. That is for the reason we mentioned last time in the context of low mass stars' luminosity evolution: the role of radiation pressure. The luminosity varies as $L \propto \mu^4 \beta^4$, and β is in turn given by the Eddington quartic:

$$0.003 \left(\frac{M}{M_\odot}\right)^2 \mu^4 \beta^4 + \beta - 1 = 0. \tag{1}$$

For very massive stars, the first term is dominant, so $\mu\beta$ is roughly constant, and L is too. This is simply a reflection of the fact that very massive stars are largely supported by radiation pressure. As a result, their luminosity is equal to the Eddington luminosity, which depends only on total mass, not on composition.

This non-evolution of the luminosity continues to apply even after these stars leave the main sequence. As the stars develop inert ash cores and burning shells like lower mass stars, they cannot increase in luminosity, but they can increase in radius and go to lower effective temperature. The net effect is that they move along nearly horizontal tracks on the HR diagram. Figure 3 shows numerical calculations of this phenomenon.

As the tracks show, the luminosities increase less and less for stars of higher and higher masses, and instead they evolve at constant luminosity. Thus massive stars never have a red giant phase, since that would require an increase in luminosity.

II. The stellar interior

In addition to showing major differences in their envelopes and external structure due to mass loss, massive stars' interiors are also quite different from those of low mass stars. The principal difference has to do with when and where nuclear burning stops.

A. The Chandresekhar limit

As we saw last time, the most massive of low mass stars evolve into AGB stars, which have cores of carbon and oxygen surrounded by an onion shell structure of helium burning and hydrogen burning regions. The core becomes supported by degeneracy pressure before getting hot enough to burn carbon and oxygen into heavier elements, and eventually the envelope is ejected. The final core mass that is left is of order 1 $M_\odot$. We do not understand the mass loss process well enough to predict in great detail the relationship between the final remnant mass and the initial mass of the star, but we can say that, not surprisingly, the more massive the star is at birth, the more massive the core it produces will be.

However, we will now demonstrate that there is an upper limit to the possible core mass, known as the Chandresekhar limit. To start, recall that a degenerate gas can be treated as polytrope, since it obeys and equation of state where the pressure depends on density alone. For a non-relativistic degenerate electron gas, we have

$$P = K_1' \left(\frac{\rho}{\mu_e}\right)^{5/3}, \tag{2}$$

so $\gamma_P = 5/3$, and $n = 1/(\gamma_P - 1) = 3/2$. The polytropic constant is $K_P = K_1'/\mu_e^{5/3}$.

Now recall the mass-radius relation for polytropes that we derived back in class 7:

$$\left[\frac{GM}{-\xi_1^2 (d\Theta/d\xi)_{\xi_1}}\right]^{n-1} \left(\frac{R}{\xi_1}\right)^{3-n} = \frac{[(n+1)K_P]^n}{4\pi G}. \tag{3}$$

Since K_P is a constant that is just determined by the composition of the stellar material and by fundamental constants, it is the same for all carbon-oxygen cores. If such a core is non-degenerate, then $n = 3/2$, and thus we

$$M^{1/2} R^{3/2} \propto \text{const} \qquad \Longrightarrow \qquad R \propto M^{-1/3}. \tag{4}$$

Thus as the mass of the degenerate core increases, its radius shrinks: a larger mass is able to compress the degenerate gas more than a smaller mass.

However, there is a limit to this. If the radius is getting smaller at fixed mass, then the mean density must be rising, $\rho \propto M/R^3 \propto M^{-2}$. Also recall that, as the density and pressure in a degenerate gas rise, the mean electron energy must rise too, since more electrons crowded into the same space have to occupy higher and higher energy states. Eventually the mean electron energy is high enough for the electrons to be relativistic, and the gas switches from being degenerate and non-relativistic to degenerate and relativistic. We showed back in class 4 that the threshold density at which this transition occurs is $\rho_{\rm cr} \approx 3 \times 10^6 \mu_e$ g cm^{-3}.

To get a sense of whether this is relevant for stars, as and example consider a degenerate core with a mass of 2 $M_\odot$ composed mostly of C and O, which will give $\mu_e \approx 2$ (because C and O both have 2 nucleons per electron). Plugging in $K_1' = 1.0 \times 10^{13}$ dyne cm^{-2} (g cm^{-3})$^{-5/3}$, and using $n = 3/2$, which gives $\xi_1 = 3.654$ and $-\xi_1^2 (d\Theta/d\xi)_{\xi_1} = 2.714$, we obtain $R = 8900$ km. The corresponding mean density is 7×10^5 g cm^{-3} – and recall that this is the mean density. The central density must be even higher. Clearly for objects above ~ 1 $M_\odot$, we are going to be getting into the relativistic regime.

Thus let us consider the case for the opposite limit, an extreme relativistic degenerate electron gas. In this case the equation of state is

$$P = K_2' \left(\frac{\rho}{\mu_e}\right)^{4/3}, \tag{5}$$

and we have an $n = 3$ polytrope with $K_P = K_2'/\mu_e^{4/3}$. Using the mass-radius relation again, we see that for $n = 3$ the term that depends on R disappears, and we get something that is a function of M only. Specifically, we get

$$M = M_{\rm ch} = -\frac{4}{\sqrt{\pi}}\xi_1^2 \left(\frac{d\Theta}{d\xi_i}\right)_{\xi_1} \left(\frac{K_P}{G}\right)^{3/2} \tag{6}$$

$$= \frac{\sqrt{3}}{2^{3/2}\pi}\left[-\xi_1^2\left(\frac{d\Theta}{d\xi_i}\right)_{\xi_1}\right]\left(\frac{hc}{Gm_{\rm H}}\right)^{3/2}\frac{1}{\mu_e^2} \tag{7}$$

$$= \frac{5.83}{\mu_e^2} M_\odot, \tag{8}$$

where in the second step we replaced K_2' with its formula in terms of elementary constants. For $\mu_e = 2$, this corresponds to a mass of 1.46 $M_\odot$.

The physical implication is clear. Imagine considering CO cores of larger and larger mass. At masses well below 1 $M_\odot$, the core can be held up by electron degeneracy pressure with no difficulty. As the mass increases, the radius shrinks and the density and pressure go up. As it passes 1.2 or 1.3 $M_\odot$, the electrons start to become relativistic, and by 1.46 $M_\odot$, they are completely relativistic. However, there is nowhere to go beyond this. A core that is larger than this mass cannot be supported by electron degeneracy pressure, because even in the ultra-relativistic limit, the largest mass that can be supported is 1.46 $M_\odot$. Thus if the core is above this limit, electron degeneracy will fail to support it, and if there is no other source of support, the core will begin to collapse.

A historical note: this limit is named after Subrahmanyan Chandrasekhar. As a young man living in India, he was awarded a scholarship from the (at the time British-controlled) government of India to study at Cambridge, and in 1930 he boarded a ship to make the long trip from his home in Madras to England. The trip would take 2-3 weeks. He derived the basic idea behind what we now call the Chandrasekhar limit during that ship journey... and he was just 20 years old.

B. Implications of the Chandrasekhar limit: the onion structure

The existence of the Chandrasekhar limit means that stars that form carbon-oxygen cores larger than about 1.4 $M_\odot$ cannot evolve into white dwarfs. Instead, electron degeneracy pressure will be unable to support the core, and once it exceeds the Schönberg-Chandrasekhar limit for its host star (i.e., once the core is too large a fraction of the entire star's mass), it must collapse further.

As it collapse it will heat up, and will eventually reach the ignition temperature for the components of the carbon-oxygen core to burn. The set of nuclear reactions gets fairly complex, but we can list some major ones:

$$^{12}_{6}\mathrm{C} + ^{12}_{6}\mathrm{C} \rightarrow ^{20}_{10}\mathrm{Ne} + ^{4}_{2}\mathrm{He} \tag{9}$$

$$^{12}_{6}\mathrm{C} + ^{12}_{6}\mathrm{C} \rightarrow ^{23}_{11}\mathrm{Na} + ^{1}_{1}\mathrm{H} \tag{10}$$

$$^{16}_{8}\mathrm{O} + ^{16}_{8}\mathrm{O} \rightarrow ^{28}_{14}\mathrm{Si} + ^{4}_{2}\mathrm{He} \tag{11}$$

$$^{16}_{8}\mathrm{O} + ^{16}_{8}\mathrm{O} \rightarrow ^{31}_{15}\mathrm{Si} + ^{1}_{1}\mathrm{H} \tag{12}$$

Following our rule that the temperature required to start up a reaction scales with the Coulomb barrier, for carbon to burn requires temperatures of 5×10^8 K or more, while oxygen burning requires a temperature of $\approx 2 \times 10^9$ K. However, since the Chandrasekhar limit means that cores cannot become supported by electron degeneracy if they are too massive, even this high temperature must eventually be achieved.

Nuclear burning stabilises the core temporarily, but the reprieve gets shorter and shorter as we march up the periodic table. This is for two reasons, one obvious and one subtle. The obvious one is the curve of binding energy: heavy nuclei release much less energy per unit mass when they fuse than lighter nuclei. Burning hydrogen to helium releases 7 MeV per nucleon, and burning helium to oxygen releases about 1 MeV per nucleon. In contrast, burning O to Si releases only 0.3 MeV per nucleon.

The subtle reason is neutrino losses. As the temperature increases, more and more of the photons inside the star have energies larger than the rest energy of two electrons. Such photons can create electron-positron pairs when they interact with atomic nuclei:

$$\gamma \rightarrow e^- + e^+. \tag{13}$$

The positrons immediately annihilate with electrons, and the great majority then go back into being photons. However, some small fraction decay via

$$e^+ + e^- \rightarrow \nu_e + \overline{\nu}_e, \tag{14}$$

forming an electron neutrino - anti-electron neutrino pair. The neutrinos then escape them the star, carrying away their energy. As the core temperature rises, neutrino losses get more and more severe, lessening the time for which a given nuclear reaction can support the star.

As each element is used up in the core, a degenerate ash region builds up at the centre of the star, surrounding by actively-burning regions above it. The ash core then collapses until it is hot enough to ignite the next stage of nuclear burning. The resulting stellar structure is an onion-skin, consisting of a core with one element burning and a series of shells with ever lighter elements burning above it, all the way through a very thin hydrogen burning shell on top of the star. Figure 4 shows an example.

A. The final stages

This process of burning successively more massive elements sounds like it can't go on forever, and it can't. We now consider the final fate of massive stars.

B. The iron core

Once the temperature in the core reaches around 3×10^9 K, photons have enough energy to start disintegrating nuclei, and this happens to some silicon atoms:

$$^{28}_{14}\mathrm{Si} + \gamma \rightarrow {}^{24}_{12}\mathrm{Mg} + {}^{4}_{2}\mathrm{He}. \tag{15}$$

The resulting helium nuclei can then be added to other silicon atoms to build up

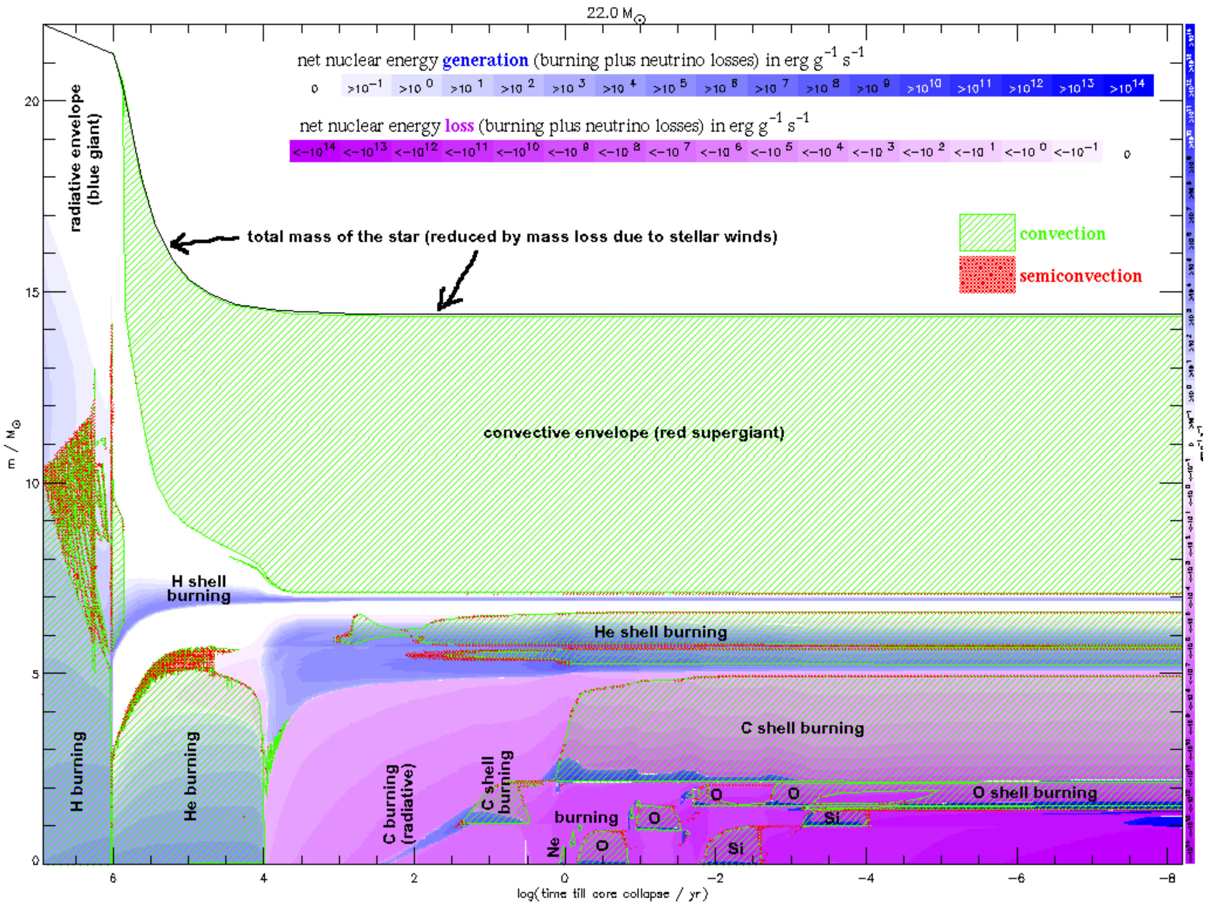

Figure 4: The structural evolution of a 22 $M_\odot$ star, displayed in a style known as a Kippenhahn diagram. The x axis is time until core collapse, measured on a logarithmic scale, so that the fast stages at the end of the star's life are spread out and visible. The y axis shows mass coordinate within the star. Colours indicate the rate of nuclear energy generation or loss, with blue indicating net nuclear energy generation (darker for more rapid energy generation) and purple indicating net loss (mostly due to neutrino loss; darker is a higher loss rate). Hatched regions indicate parts of the star that are convective. Source: Alex Heger's website at Monash, http://2sn.org/stellarevolution/explain.gif.

heavier elements:

$$
\begin{aligned}
{}^{28}_{14}\mathrm{Si} + {}^{4}_{2}\mathrm{He} &\rightarrow {}^{32}_{16}\mathrm{S} && (16)\\
{}^{32}_{16}\mathrm{S} + {}^{4}_{2}\mathrm{He} &\rightarrow {}^{36}_{18}\mathrm{Ar} && (17)\\
{}^{36}_{18}\mathrm{Ar} + {}^{4}_{2}\mathrm{He} &\rightarrow {}^{40}_{20}\mathrm{Ca} && (18)\\
{}^{40}_{20}\mathrm{Ca} + {}^{4}_{2}\mathrm{He} &\rightarrow {}^{44}_{22}\mathrm{Ti} && (19)\\
{}^{44}_{22}\mathrm{Ti} + {}^{4}_{2}\mathrm{He} &\rightarrow {}^{48}_{24}\mathrm{Cr} && (20)\\
{}^{48}_{24}\mathrm{Cr} + {}^{4}_{2}\mathrm{He} &\rightarrow {}^{52}_{26}\mathrm{Fe} && (21)\\
{}^{52}_{24}\mathrm{Fe} + {}^{4}_{2}\mathrm{He} &\rightarrow {}^{56}_{28}\mathrm{Ni} && (22)
\end{aligned}
$$

Each of these reactions is exothermic, and as long as the He nuclei produced by photodisintegration are eventually reassembled into other elements, the entire process is net exothermic as well.

However, this is the end of the line, because ${}^{56}_{26}\mathrm{Ni}$ is right near the peak of the binding energy curve, known as the iron peak. Adding another helium nucleus to make ${}^{60}_{30}\mathrm{Zn}$ is an endothermic process, not an exothermic one. The ${}^{56}_{28}\mathrm{Ni}$ can eventually decay to ${}^{56}_{26}\mathrm{Fe}$ and release a little bit more energy, but the energy release is small and comes mostly in the form of neutrinos that will not be captured. Moreover, this requires weak reactions that take weeks. This is time the star does not have. The silicon burning phase exhausts the available silicon in only ~ 1 day, mostly due to the extremely large neutrino losses incurred. The result is a core that consists solely of iron peak elements, Fe and Ni.

C. Collapse

Once a core consisting of iron and nickel is produced, there is no more energy generating capacity available. The core will continue to collapse and heat up. At first this collapse proceeds on a Kelvin-Helmholtz timescale, limited by the rate at which energy can leak out of the core, but this very rapidly converts into a dynamical collapse due to a series of instabilities.

The first is photodisintegration. At a temperature of $6 - 7 \times 10^9$ K, photons at the tail of the energy distribution have enough energy to reverse the burning process of adding He nuclei to produce heavier elements. Instead, photons stars photodisintegrating nuclei, converting them to less massive elements plus helium. The ${}^{52}_{26}\mathrm{Fe}$ and ${}^{56}_{26}\mathrm{Fe}$ are converted back into ${}^{4}_{2}\mathrm{He}$, essentially taking back all of the energy that was released by nuclear burning. Each time one of these nuclei is reduced to its constituent He atoms, 2 MeV per nucleon is removed from the radiation field. At slightly higher temperatures, even the He can be photodisintegrated into is component protons and neutrons, and this absorbs another 7 MeV per nucleon.

Second, at the high pressures found in the core, heavy nuclei can undergo reactions of the form

$$\mathcal{I}(\mathcal{A}, \mathcal{Z}) + e^- \rightarrow \mathcal{J}(\mathcal{A}, \mathcal{Z} - 1) + \nu_e,$$

i.e., nucleus $\mathcal{I}$ captures a free electron, which converts one of its protons into a neutron. We'll discuss why these reactions happen in a moment. However, for now notice the effect: reactions of this sort reduces the number of electrons, and thus the degeneracy pressure.

The combination of photodisintegration and electron capture change the number of free particles, and thus we are in a regime much like the one we considered within the ionisation layers of stars. Any work done on the core (in this case by gravity) goes not into increasing the temperature, but into changing the chemical state of the gas. As a result, the adiabatic index is effectively $\gamma_a < 4/3$, which renders the star unstable. As a result, the star goes into a runaway collapse.

D. Neutronisation

As the star collapses, its temperature rises, and ever more of its nuclei are photodisintegrated into their constituent protons and neutrons, yet another chemical change occurs: neutronisation. The physics is as follows. Protons are slightly lower mass than neutrons, and for this reason neutrons are unstable outside an atomic nucleus – this is why the universe contains protons without neutrons (in the form of hydrogen atoms), but not the other way around. Free neutrons spontaneously decay via

$$n \rightarrow p + e^- + \overline{\nu}_e \tag{23}$$

with a lifetime of 614 seconds. The energy release is

$$\Delta E = (m_n - m_p)c^2 = 1.3 \text{ MeV}. \tag{24}$$

However, we are not in free space. We are considering a gas that is full of an electron gas that is not incredibly relativistic and incredibly degenerate. Consider the energetic implications. If a new electron is created by neutron decay, it cannot go into a ground state, because the gas is degenerate, and the ground state is occupied. It must instead go into the lowest energy unoccupied state.

We showed in our discussion of the equation of state that, for a degenerate gas, the lowest energy unoccupied state has a momentum equal to the Fermi momentum,

$$p_F = \left(\frac{3h^3 n}{8\pi}\right)^{1/3}, \tag{25}$$

where n is the number density. Since the electrons are extremely relativistic, the corresponding energy is

$$E_F = p_F c = \left(\frac{3h^3 n}{8\pi}\right)^{1/3}. \tag{26}$$

If we set this equal to the energy difference ΔE between a proton and a neutron, we find that the Fermi energy is larger once

$$n > \frac{8\pi}{3}\left(\frac{\Delta E}{hc}\right)^3, \tag{27}$$

or, in terms of mass density,

$$\rho > \frac{1}{\mu_e} n m_{\mathrm{H}} = \frac{4\pi}{3\mu_e}\left(\frac{\Delta E}{hc}\right)^3 = 1.6 \times 10^7/\mu_e \text{ g cm}^{-3}. \tag{28}$$

As the core collapses, it will exceed this threshold density, and as a result the free neutrons produced when He is photodisintegrated cannot turn into protons. However, we must also consider the reverse process:

$$p + e^- \rightarrow n + \nu_e. \tag{29}$$

From one standpoint, this reaction is highly endothermic, and for it to occur requires electrons with > 1.3 MeV of energy. However, we have just demonstrated that, thanks to the degeneracy of the gas in the collapsing core, there are plenty of such electrons available.

From another standpoint this reaction is highly exothermic: each time it occurs, it consumes 1.3 MeV, but by reducing the number of free electrons it reduces the energy of the Fermi gas of degenerate electrons by more than 1.3 MeV. Can can treat this as a chemical potential, and if we do so, then we can say that the total free energy of the system is decreased rather than decreased each time a proton combines with an electron to make a neutron, so the reaction can occur spontaneously. The excess energy is carried off by the neutrino.

Note that this same argument is the reason why electron capture onto nuclei tends to happen in iron cores prior to all the nuclei being photodisintegrated. Electron captures that would be energetically unfavourable in free space, with zero chemical potential, become energetically favourable thanks to the high chemical potential associated with the degenerate electron gas.

In any event, the net result of this process is that the core of the star, as it collapses, is converted mostly to neutrons. Outer layers of the star, where the pressure and density are less, remain ordinary nuclei.

E. Neutron degeneracy

What happens next is somewhat hazy. As the collapse continues, on certain thing is that eventually the neutrons become degenerate, and they provide a source of pressure. Whether this is enough to support the core of the star depends on how massive it is, but also on complex nuclear physics that is only poorly understood.

A degenerate neutron gas is significantly more complex than a degenerate electron one. In a degenerate electron gas, there is degeneracy pressure, and that is the only force. A gas of pure electrons would repel each other by Coulomb force, but in a star they are mixed with an equal number of protons so the gas is overall electrically neutral, and we can neglect the electric force. In a neutron gas, on the other hand, there are strong nuclear forces that, when the neutrons are packed together at high density, are non-negligible.

As a result, the equation of state of the matter post-neutronisation is significantly uncertain, and we cannot easily calculate the resulting stellar structures. However, we have significant empirical constrains, and we know that, in at least some cases, the equation of state is very stiff, so that the neutronised matter all of a sudden becomes very resistant to compression above some threshold density. When this happens, the core of the star can stop collapsing suddenly, and the material above the core that was falling will "bounce". When this happens, the star can explode. We will return to this topic in the next class.