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# Theoretical and Observational Evidence for Coriolis Effects in Coronal Magnetic Fields via Direct Current Driven Flaring Events

Darryl Z. Seligman<sup>1</sup><sup>(0)</sup>, Leslie A. Rogers<sup>2</sup><sup>(0)</sup>, Adina D. Feinstein<sup>2,9</sup><sup>(0)</sup>, Mark R. Krumholz<sup>3,4</sup><sup>(0)</sup>, James R. Beattie<sup>3</sup><sup>(0)</sup>,

Christoph Federrath<sup>3,4</sup>, Fred C. Adams<sup>5,6</sup>, Marco Fatuzzo<sup>7</sup>, and Maximilian N. Günther<sup>8,10</sup>

Department of the Geophysical Sciences, University of Chicago, Chicago, IL 60637, USA; dzseligman@uchicago.edu

<sup>2</sup> Department of Astronomy and Astrophysics, University of Chicago, Chicago, IL 60637, USA

<sup>3</sup> Research School of Astronomy and Astrophysics, Australian National University, Canberra, ACT 2611, Australia

ARC Centre of Excellence for Astronomy in Three Dimensions (ASTRO-3D), Canberra, ACT 2611, Australia

Physics Department, University of Michigan, Ann Arbor, MI 48109, USA

<sup>6</sup> Astronomy Department, University of Michigan, Ann Arbor, MI 48109, USA

Physics Department, Xavier University, Cincinnati, OH 45207, USA

<sup>8</sup> European Space Agency (ESA), European Space Research and Technology Centre (ESTEC), Keplerlaan 1, 2201 AZ Noordwijk, The Netherlands Received 2022 January 10; revised 2022 February 24; accepted 2022 March 4; published 2022 April 12

#### Abstract

All stars produce explosive surface events such as flares and coronal mass ejections. These events are driven by the release of energy stored in coronal magnetic fields, generated by the stellar dynamo. However, it remains unclear if the energy deposition in the magnetic fields is driven by direct or alternating currents. Recently, we presented observational measurements of the flare intensity distributions for a sample of  $\sim 10^5$  stars across the main sequence observed by TESS, all of which exhibited power-law distributions similar to those observed in the Sun, albeit with varying slopes. Here we investigate the mechanisms required to produce such a distribution of flaring events via direct current energy deposition, in which coronal magnetic fields braid, reconnect, and produce flares. We adopt a topological model for this process, which produces a power-law distribution of energetic flaring events. We expand this model to include the Coriolis effect, which we demonstrate produces a shallower distribution of flare energies in stars that rotate more rapidly (corresponding to a weaker decline in occurrence rates toward increasing flare energies). We present tentative evidence for the predicted rotation-power-law index correlation in the observations. We advocate for future observations of stellar flares that would improve our measurements of the power-law exponents, and yield key insights into the underlying dynamo mechanisms that underpin the self-similar flare intensity distributions.

Unified Astronomy Thesaurus concepts: Emerging flux tubes (458); Solar dynamo (2001); Solar magnetic reconnection (1504); Optical flares (1166)

## 1. Introduction

Since the first recorded observation of a Solar superflare in 1859 by Carrington (1859), the mechanisms driving explosive flaring in stars have been the subject of detailed inquiry from plasma physicists and astronomers alike. The origin of flares is closely related to the still-open problem of what mechanism is responsible for heating the Solar corona (Withbroe & Noyes 1977). In the working picture that has emerged since 1859, it has become clear that magnetic fields are an important component of the heating (Parker 1989; Golub & Pasachoff 1997; Schrijver & Zwaan 2000). The concentration and twisting of magnetic field lines can lead to the release of energy via the process of magnetic reconnection in flaring events (Sturrock et al. 1984; Kulsrud 1998; Priest & Forbes 2000). In this study, we are interested in understanding the role of the Coriolis force in the dynamo of other stars, and to accomplish this we apply theoretical insights about the

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heating of the Solar corona and the energy mechanisms driving Solar flares to other stars.

The role of the Coriolis force in the Solar dynamo produces observational signatures in the inclination, twist, and location of coronal loops and active regions. Loops tend to be inclined with respect to the latitudinal line (Hale et al. 1919; Choudhuri & Gilman 1987; Choudhuri & D'Silva 1990; D'Silva & Choudhuri 1993; Howard 1993; Longcope & Fisher 1996; Longcope et al. 1998; Weber et al. 2011, 2013). Magnetic fields in active regions tend to twist in the opposite direction in the Northern and Southern hemispheres (Seehafer 1990; Pevtsov et al. 1995; Abramenko et al. 1997; Bao & Zhang 1998; Longcope & Welsch 2000; Longcope & Pevtsov 2003; Seligman et al. 2014; Manek et al. 2018; Manek & Brummell 2021). Faster rotating sunspots are more likely to be toward the equator and slower rotating spots toward the pole (D'Silva & Howard 1995). Stars spin down as they age (Noves et al. 1984; Soderblom 2010; Notsu et al. 2013; Candelaresi et al. 2014; Doyle et al. 2018, 2019; Ilin et al. 2019; Doyle et al. 2020). Recent work has considered the role of the Coriolis force in the dynamo of other stars with different rotation rates (Solanki et al. 2006; Holzwarth 2007; Parker 2009; Rempel 2011; Kitchatinov & Olemskov 2015; Browning et al. 2016; Weber & Browning 2016). In this paper we consider an additional signature of the Coriolis force: its impact on the distribution of flares.

<sup>&</sup>lt;sup>9</sup> NSF Graduate Research Fellow.

<sup>&</sup>lt;sup>10</sup> ESA Research Fellow.

Observations at different wavelengths have demonstrated that the distribution of peak intensity of Solar flares follow power laws (Drake 1971; Datlowe et al. 1974; Lin et al. 1984; Dennis 1985), where

$$\frac{dN}{dE} \sim E^{-\alpha},\tag{1}$$

and *N* is the number of flares, *E* is the total energy released in the flare (where the flare magnitude  $m \propto \log E$ ), and the power-law exponent falls roughly in the range of  $\alpha \sim 1.25-1.5$ . Wheatland (2000) found that typical flare frequency distribution (FFD) indices did not vary for individual active regions on the Sun. However, it is important to note that converting from the intensity to energy requires assumptions about the geometry and physical conditions of the flaring region, the mechanisms producing extremely energetic photons (Lee et al. 1993; Brown et al. 1998), and the flare height as a function of amplitude (Mitra-Kraev & Benz 2001).

A useful parameter to describe the potential mechanisms driving coronal heating and the coupling of the convective interior and the corona is the plasma  $\beta$  parameter,

$$\beta = 2\frac{c_s^2}{v_A^2},\tag{2}$$

where  $c_s$  is the speed of sound and  $v_A$  is the Alfvén speed. When  $\beta > 1$  acoustic modes dominate the dynamics and transport of energy, and when  $\beta < 1$  Alfvénic modes dominate. This is useful for identifying the importance of electrodynamic coupling relative to mechanical coupling between the inner and outer atmosphere of the Sun. The electrodynamic coupling will dominate if  $\beta < 1$  in the outer atmosphere and  $\beta \sim 1$  in the inner atmosphere, where mechanical dynamics such as convection or differential rotation can couple to Alfvénic perturbations in the corona and drive the heating (Ionson 1985). This is generally believed to be the case for the Sun, and similarly for other stars.

Two broad mechanisms that have been proposed to explain the heating of the Solar corona (Withbroe & Noyes 1977) are direct current (DC) and alternating current (AC). Magnetic stress or DC heating dominates when large scale subsurface fluid motions have timescales that are much longer than the Alfvénic coronal crossing timescale,  $\tau_A = L_c/v_A$  (where  $L_c$  is the radial extent of the corona) (Ionson 1982), while AC heating dominates in the opposite limit.

The fundamental idea behind DC models originates in the work of Parker (1972), who demonstrated that a large scale magnetic field could not be in hydrostatic equilibrium if field lines were topologically braided or knotted. The nonexistence of an equilibrium is robust to any pressure perturbations applied along the field line. Parker concluded that braided or knotted flux tubes produce rapid dissipation and merging of field lines into a one-dimensional topology. Parker (1983) connected this simple model to heating of the Solar corona via DC currents, whereby slow random walks in the footpoint positions generate magnetic braids that must reconnect and release energy. This braiding mechanism for heating of the Solar corona was extended by many authors (Mikic et al. 1989; Berger 1993; Longcope & Sudan 1994; Parker 1994; Galsgaard & Nordlund 1996; Berger & Asgari-Targhi 2009; Berger et al. 2015).

In the DC regime, a simplified view is that reconnection is triggered when the angle,  $\theta$ , between neighboring magnetic field vectors is greater than a critical angle  $\theta_c$ , which may trigger smaller events in neighboring field lines (Sturrock et al. 1984; Porter et al. 1987; Parker 1988; Sturrock et al. 1990; Berger 1993; Krucker & Benz 2000), sometimes referred to as nanoflares. These were recently observed by Antolin et al. (2021), who verified that these could be explained as reconnection events at small angles. Parker (1988) conjectured that  $\theta_c \sim 30^\circ$ , although Dahlburg et al. (2005) argued that  $\theta_c \sim 45^\circ$  with a more detailed analysis that included secondary instability. It is important to note, however, that models of coronal heating such as that presented by van Ballegooijen (1986) required reconnection events even at small  $\theta_c$  in a cascade of magnetic energy transport. This model was furthered by Cargill (1994), who presented a model of an active region as hundreds of small elemental flux loops randomly heated by nanoflares. In any case, when  $\theta < \theta_c$ , magnetic reconnection proceeds slowly, and magnetic energy is deposited in the form of braided fields quadratically in time (Parker 1983; Moffatt & Tsinober 1990; Berger 1993). The combination of twisting and braiding of field lines could explain the flare frequency spectrum observed in active regions (Zirker & Cleveland 1993a, 1993b).

For AC models, the timescale of the footpoint motions is of order or shorter than the coronal crossing timescale, so footpoint motions excite Alfvénic modes that travel back and forth along coronal magnetic field lines (van Ballegooijen et al. 2011; Asgari-Targhi & van Ballegooijen 2012; Asgari-Targhi et al. 2013; van Ballegooijen et al. 2014). This regime is driven by smaller scale surface convection mechanisms (Ionson 1985). Asgari-Targhi et al. (2014) demonstrated that the AC model was consistent with observations of nonthermal widths of coronal emission of Fe XII, Fe XIII, Fe XV, and Fe XVI from the Extreme-ultraviolet Imaging Spectrometer on board the Hinode spacecraft. However, it is not clear that AC models can explain the existence of exceptionally hot >5 MK coronal loops (Asgari-Targhi et al. 2015), and it has been speculated that DC events are important there.

The heating of the Solar corona has been linked to the theory of self-organized criticality (Bak et al. 1987, 1988), which describes dissipative dynamical systems that remain in a critical state with no intrinsic length or timescale. The theory requires a local instability mechanism that can trigger neighboring instabilities. This avalanching mechanism produces energetic events at all length scales (Kadanoff et al. 1989; Babcock & Westervelt 1990). Applications of the theory have been hypothesized in turbulence, percolation systems (Turcotte 1999), neuroscience (Ribeiro et al. 2010; Hesse & Gross 2014), landslides (Bak et al. 1990; Turcotte et al. 2002), atmospheric dynamics (Grieger 1992; Andrade et al. 1998), astrophysical accretion disks (Dendy et al. 1998), traffic patterns (Nagel & Herrmann 1993), evolution (Bak & Sneppen 1993), extinction events (Newman 1996), financial markets (Bak et al. 1997), earthquakes (Gutenberg & Richter 1956; Bak & Tang 1989; Carlson & Langer 1989; Sornette & Sornette 1989; Olami et al. 1992), and Conway's Game of Life (Bak et al. 1989). Lu & Hamilton (1991) proposed that the Solar coronal magnetic field is also in a self-organized critical state to explain the power law observed in the magnitude of Solar flares (Crosby et al. 1993; Lu et al. 1993; Aschwanden et al. 1998; Charbonneau et al. 2001;

de Arcangelis et al. 2006). This approach has been powerful in prediction of extreme flares (Morales & Santos 2020).

In the DC picture of coronal heating, the slow build-up of braided fields provides all of the requisites for a selforganized critical system. Twisted coronal fields are generated via dynamo mechanisms in the fluid-dominated interior (Charbonneau 2010), via convective and Coriolisdriven vortical subsurface plasma flows (Parker 1955; Moffatt 1978; Longcope et al. 1998; Seligman et al. 2014). Prior & MacTaggart (2016) suggested that active Solar regions could be formed via the injection of rising magnetic field topologies that were only braided, and not twisted. MacTaggart et al. (2021) demonstrated numerically that active regions are created by the emergence of a large flux tube of pre-twisted magnetic fields by examining the evolution of the topological quantity magnetic winding in the emergence of active regions. Berger & Asgari-Targhi (2009) and Berger et al. (2015) demonstrated that a topological model that included braiding and reconnection of coronal fields exhibited power-law distributions of energetic events.

It is important to note that while there exist physical connections between DC heating models and self-organized criticality, the theory is not the only way to create a power-law distribution of energetic events (Rosner & Vaiana 1978; Litvinenko 1996). For example, Newman & Sneppen (1996) demonstrated a dynamical system driven by coherent noise could arrive at a similar stationary state characterized by power-law distributions of avalanches, but without maintaining the *critical* state. MHD turbulence invoked in the AC regime can produce flare-like energy occurrence distributions without relying on the theory of self-organized criticality (Longcope & Sudan 1994; Einaudi et al. 1996; Galsgaard & Nordlund 1996; Dmitruk & Gómez 1997; Galtier & Pouquet 1998; Georgoulis et al. 1998; Einaudi & Velli 1999; Galtier 1999).

To date, most studies of heating and flaring mechanisms have focused on the Sun. While this is obviously the system for which we can obtain the richest and highest-quality data, it remains a sample of one. Studies of flaring in other stars can therefore offer a unique and valuable perspective. Flare-like X-ray emissions from other stars exhibit power-law energy distributions with similar indices to those seen in the Sun (Shakhovskaia 1989; Osten & Brown 1999; Audard et al. 2000). Aschwanden & Güdel (2021) found a power-law dependence of energies for optical flares observed with Kepler (Davenport 2016). The TESS mission provided 2 minute cadence light curves for  $\sim$ 200,000 stars, which allowed for the identification of a statistically significant sample of flaring events (Günther et al. 2020). Feinstein et al. (2022) demonstrated that the these flares follow a power-law distribution of intensity (with slopes  $\alpha' \approx 0.9 - 1.5$ ) for all main-sequence stars.<sup>11</sup> These newly measured values are close to the median of previously measured slopes (Figure 3 in Shibayama et al. 2013; Howard et al. 2019; Ilin et al. 2019; Lin et al. 2019; Yang & Liu 2019; Feinstein et al. 2020b; Günther et al. 2020; Raetz et al. 2020; Aschwanden & Güdel 2021; Feinstein et al. 2022; Ilin et al. 2021). In this paper, we attempt to interpret these power-law distributions under the framework of the DC heating mechanism.

This paper is organized as follows. In Section 2, we review the DC braiding model of reconnection events presented by Berger & Asgari-Targhi (2009) and Berger et al. (2015). We expand the braiding model to include a Coriolis-driven bias in handedness of braids injected, and calculate the resulting distributions of energetic events. In Section 3, we present observations of flare frequency distributions (FFDs) for slow and fast rotating stars, and show that they are consistent with the analytic predictions. In Section 4, we conclude and outline future observational and theoretical work.

#### 2. Generalized Topological Braiding Model

Reconnection is at its base a phenomenon whereby (anomalous) resistivity dissipates the currents that sustain magnetic fields in a plasma, leading to a violation of flux freezing and allowing rearrangement of the magnetic topology. Regions where reconnection occurs must have a current flowing through them (since otherwise there would be nothing to dissipate), and thus necessarily have

$$\vec{\nabla} \times \vec{B} \mid \neq 0. \tag{3}$$

This result follows from consideration of Ampére's Law,

$$\vec{\nabla} \times \vec{B} = \frac{1}{c} \left( 4\pi \vec{J} + \frac{\partial \vec{E}}{\partial t} \right).$$
 (4)

In a plasma that is overall electrically neutral, the electric field in the rest frame vanishes,  $|\vec{E}| = 0$ , so that  $\partial \vec{E} / \partial t = 0$ . It is worth noting that in non-inertial frames,  $\vec{E} = \vec{B} \times \vec{V}/c$ , where  $\vec{V}$  is the plasma velocity and c is the speed of light. However,  $\partial \vec{E} / \partial t \ll 1$ , because in a non-relativistic plasma,  $|\vec{V}|/c \ll 1$ . The presence of a current,  $|\vec{J}| \neq 0$ , thus implies a nonvanishing curl in the magnetic field,  $|\vec{\nabla} \times \vec{B}| \neq 0$ . This condition, in turn, implies that the reconnection rate—and the flaring rate—must be connected to the distribution of the curl of the magnetic field. This insight motivates the analysis that follows.

In this section, we review the braiding model for DC coronal heating via reconnection events presented in Berger & Asgari-Targhi (2009) and further developed in Berger et al. (2015). We then generalize the braiding model to include a Coriolis-induced handedness bias. Before we begin, it is worth noting that other authors have investigated the role of helicity and braiding in active regions and found that both are important for supplying energy to the corona (Longcope et al. 2007; Liu et al. 2014).

## 2.1. Estimate of Free Magnetic Energy

We consider a small patch of a stellar atmosphere. We define height along the  $\hat{z}$  direction, defined such that z = 0 is the base of the fields and z = L represents some radial extent into the corona. At z = 0, which is nominally at the photosphere, although the exact position is irrelevant, subsurface convection provides stochastic forcing of the position for the magnetic flux tubes.

As was introduced in Section 2 of Berger & Asgari-Targhi (2009), we consider a braid consisting of N flux tubes or magnetic field lines that permeate the volume. The set of flux tubes is characterized by the number of crossing points, C, at which magnetic flux tubes cross. Each flux tube has a number

 $<sup>\</sup>frac{1}{1} \alpha'$  indicates the slope analogous to  $\alpha$  in Equation (1), for the normalized intensity distribution.



Figure 1. Schematic diagram of three braided flux tubes that cause an incoherent reconnection event—resulting in the merging of opposite signed sequences—that releases energy. In this setup, the two sequences on the left-hand side are braided in opposing directions. After the reconnection event, which removes the interchange, the two sequences unwind and release the magnetic energy density that was stored in transverse components of the fields. This figure is similar to Figure 4 in Berger & Asgari-Targhi (2009).

of crossings,  $n_{\rm C}$ , on average, given by

$$n_{\rm C} = \left(\frac{2C}{N}\right),\tag{5}$$

so the typical height between crossing junctures is

$$\delta z = \frac{L}{n_{\rm C}},\tag{6}$$

as was demonstrated in Equation (1) of Berger & Asgari-Targhi (2009). Let *D* be the typical diameter of a flux tube. Note that this quantity is related to typical separations between magnetic field lines, and is given by the resistivity of the ambient medium. If two of the tubes wrap around each other between heights of *z* and  $z + \delta z$ , the distance that they travel in the horizontal direction is roughly  $\delta \ell = \pi D/2$ . Therefore the ratio between the perpendicular and parallel components of the magnetic field with respect to the radial direction,  $B_{\perp}$  and  $B_{\parallel}$ , is approximately given by

$$\left(\frac{\delta\ell}{\delta z}\right) \sim \left(\frac{B_{\rm L}}{B_{\parallel}}\right) \sim \left(\frac{\pi D}{NL}\right) C.$$
 (7)

This step is important because it demonstrates that the perpendicular magnetic field is directly proportional to the number of crossings in a flux tube. Berger (1993) demonstrated that the free magnetic energy density is proportional to the square of the perpendicular or transverse magnetic field, so that the free energy per unit volume of the braided field obeys the scaling

$$E_{\rm Free} \propto C^2,$$
 (8)

as given by Equation (5) of Berger & Asgari-Targhi (2009).

#### 2.2. Evolution to Equilibrium

Berger & Asgari-Targhi (2009) considered a topological model of the braiding of flux tubes to explain the distribution of energy in reconnection events observed in the Solar corona. In the model, there are two defining topological quantities for a nest of individual magnetic flux tubes. The first is the *winding* number *w* of an individual sequence describing the braiding of two flux tubes around each other. This quantity encodes both the number of crossings, where |w| = C, and the handedness of the braiding itself. An *interchange* represents a region where a third flux tube crosses one of the two original flux tubes, and is inserted into the original braiding strand. These interchanges are the sites of reconnection events.

In Figure 1, we present a schematic diagram of the topological model, similar to Figure 4 in Berger & Asgari-Targhi (2009). The left-hand side shows a braid consisting of two sequences and one interchange. The primary flux tube is shown in red, and takes part in both of the sequences. The blue flux tube is braided around the red flux tube from the base where z = 0, and these two create a sequence with  $w_1 = -4$ , of negative handedness. Above the sequence, the blue flux tube is removed from the braid in an interchange with the yellow flux tube. The yellow flux tube then creates a sequence where  $w_2 = +4$  with the red flux tube above the interchange, of positive handedness.

Berger & Asgari-Targhi (2009) considered a model in which the coronal magnetic field consisted of many of these types of braided flux tubes, nested together. The defining feature of the model is that a reconnection event is modeled as the removal of an interchange. This process is demonstrated in the middle portion of Figure 1, where the interchange is removed and the blue and yellow flux tubes merge on the right-hand side. After the removal of the interchange, the two sequences above and below the interchange are able to merge. The final winding,  $w_f$  of the resulting single sequences is given by  $w_f = w_1 + w_2$ . In the configuration shown in the schematic figure, the equal and opposite windings above and below the interchange cancel each other out completely. In other words, the entire braid completely unwinds itself. This behavior is shown on the right-hand side of the figure, where the three flux tubes are now relaxed to an equilibrium where there is no braiding. Because of the results in the previous subsection, and namely Equation (8), the free magnetic energy that is released in the reconnection event encoded by the removal of the interchange is given by the number of winds that are undone.

Remarkably, Berger & Asgari-Targhi (2009) presented a method to evolve this simplistic model through time to an equilibrium state, which we summarize here. At some arbitrary initial time t=0, consider a neighborhood in the corona containing an integer number of braiding sequences,  $\Lambda$ , which by construction contains  $\Lambda - 1$  interchanges. The system then evolves through a series of idealized discrete time steps. During any given time step,

- 1. One sequence is added,
- 2. One interchange is added,
- 3. One reconnection event occurs, removing one preexisting interchange. Sequences on either side of the interchange merge.

Berger and Asgari-Targhi then investigate what steady-state distribution of windings w such an evolutionary sequence produces. At any point, the number of sequences where C = w is given by  $\xi(w)$ . Therefore,

$$\Lambda = \sum_{w=-\infty}^{\infty} \xi(w), \tag{9}$$

as defined with different notation in Equation (7) of Berger & Asgari-Targhi (2009). The probability that there will be a sequence of length w is given by the normalized probability function, f(w), which is defined in Section 5.2 of Berger & Asgari-Targhi (2009) as

$$f(w) = \frac{\xi(w)}{\Lambda}.$$
 (10)

Next, the probability of adding a new sequence of w at each time step is given by a function defined to be p(w). At each time step in the algorithm, the probability function evolves to a new value through the mapping

$$f(w) \to f(w) + \delta f(w), \tag{11}$$

defined by the three events that are allowed to happen in each time step itemized above. Therefore, this process can be described at each time step by the following equation:

$$\Lambda \delta f(w) = \delta \xi(w) = p(w) - 2f(w) + \int_{-\infty}^{\infty} f(\omega_1) d\omega_1 \int_{-\infty}^{\infty} f(\omega_2) \delta(w - (\omega_1 + \omega_2)) d\omega_2.$$
(12)

The three terms on the right-hand side denote (in order from left to right) the addition of a new sequence, the removal of the surrounding sequences, and the addition of the resulting merged sequence. In order to evolve this model to an equilibrium, it is sufficient to set Equation (12) equal to zero to indicate that a steady state has been reached. When the

equilibrium is reached, the mapping no longer changes the probability density function and  $\delta\xi(w) = 0$ . At this point, the equation reduces to

$$p(w) = 2f(w) - (f \star f)(w), \tag{13}$$

where the  $\star$  indicates a convolution, as in Equation (11) of Berger & Asgari-Targhi (2009). It is straightforward to demonstrate, as in Section 5.2 of Berger & Asgari-Targhi (2009), that this can be solved in Fourier space via Fourier transforms and inverse Fourier transforms. Assuming that the input of interchanges is a Poisson process such that the windings inserted in the sequences between interchanges follow an exponential distribution,

$$p(w) = \frac{\lambda}{2} e^{-\lambda |w|},\tag{14}$$

for some constant  $\lambda$ , then the equilibrium solution exhibits a probability distribution of windings characterized by the equation

$$f(w) = \frac{\lambda}{2} [L_{-1}(\lambda w) - I_{\rm I}(w\lambda)], \qquad (15)$$

where  $I_1$  is the Bessel-*I* function and  $L_{-1}$  is a Struve-*L* function. We note that this function is slightly different than Equation (17) in Berger & Asgari-Targhi (2009), although we have verified that our solution has the same form as Figure 5 in their paper. Importantly, for winding numbers with magnitude greater than unity, |w| > 1, the probability distribution has the form of a power law where  $f(w) \sim w^{-2}$ .

# 2.3. Energy Distribution of Flares

Consider the distribution of reconnection events in the previous subsection. Since we know that the magnetic free energy scales with the square of the crossing number of the set of flux tubes, this must also correspond to a distribution or power law of energetic flaring events once the equilibrium state has been reached, since  $E_{\rm free} \sim C^2$  as in Equation (8). Therefore, the difference in energy before and after a reconnection event,  $\Delta E$ , is given by the difference in the square of the initial and final values of *C*, i.e.,

$$\Delta E \sim C_0^2 - C_F^2. \tag{16}$$

Berger & Asgari-Targhi (2009) assumed that the reconnection occurs after the winding is greater than some critical value,  $C_{\rm crit}$ . They assumed the interchange removal merged two sequences with twist numbers  $\omega_1$  and  $\omega_2$ , where by convention  $|\omega_2| > |\omega_1|$ . Critically, they assume that the merged sequences have opposite sign, on the basis that mergers of sequences with the same winding direction will not create a release of energy in a flaring event. The merging after interchange produces a single sequence with  $\omega_f = |\omega_2| - |\omega_1|$ , and energy

$$\Delta E \sim C_{\text{crit}}^2 - (C_{\text{crit}} - 2|\omega_1|)^2.$$
(17)

If  $\omega_1 \ll C_{\text{crit}}$ , then to first order we have

$$E \sim 4C_{\rm crit}|\omega_1|,$$
 (18)

and we can deduce the distribution of flare energies, F(E) from the relationship between energies before and after reconnection



Figure 2. Schematic diagram of a coherent reconnection event. In this setup, the two sequences on the left-hand side both have positive winding. After the reconnection event that removes the interchange, the resulting braid on the left-hand side has a winding of w = +8.

and the distribution of winding numbers w:

$$F(E) \sim E^{-\alpha},\tag{19}$$

where  $\alpha = 2\gamma - 1$ , and  $\gamma$  is the exponent of the winding distribution  $f(w) \sim |w|^{-\gamma}$ , as demonstrated in Section 5.3 of Berger & Asgari-Targhi (2009).

# 2.4. Expansion of Braiding Model

The Berger & Asgari-Targhi (2009) model assumes that injection of braids with positive and negative winding are equally probable, which is a reasonable approach for a slowly rotating star like the Sun, where the Rossby number in the subsurface convection zone is relatively large (Greer et al. 2016). However, this assumption must begin to fail in more rapidly rotating stars with smaller Rossby numbers, and we must therefore generalize the model to include a bias in the sign of *w*.

To generalize this (but keeping the same distribution on the overall amplitude of w in the injected sequences) we just adjust the prefactor. We define a parameter  $\eta \in [0, 1]$  to be the probability that a sequence has a positive handed winding number. The probability distribution function for the input of sequences can be written as

$$p(w) = \begin{cases} \eta \lambda e^{-\lambda w} & \text{if } w > 0\\ (1 - \eta) \lambda e^{+\lambda w} & \text{if } w < 0. \end{cases}$$
(20)

In Equation (20), the value of  $\eta$  determines the percentage of sequences that are *locally* injected into the corona with the same sign or handedness in the twist. If there is a bias toward more sequences with the same sign, then one would expect that there should be a bias toward more coherent interchange removals, where the surrounding sequences add constructively.

In Figure 2, we show a schematic in which the two sequences have the same sign and number of windings. After

the interchange is removed, only one braid remains, but it has twice the number of windings as the original two braids, so no energy is released from the unwinding of braids. It is natural to expect that as  $\eta$  increases or decreases away from 1/2 (which represents an equal injection of positive and negative sequences), the steady-state winding distribution will be skewed toward higher numbers. We can demonstrate this trend analytically by taking the Fourier transform of Equation (20), which takes the form

$$\tilde{p}(k) = \left(\frac{\lambda^2 - i\lambda k(2\eta - 1)}{\lambda^2 + k^2}\right).$$
(21)

Using this result in Equation (13), the solution for the steadystate winding distribution function in Fourier space is

$$\tilde{f}(k) = 1 - \sqrt{\frac{k^2 + i\lambda k(2\eta - 1)}{\lambda^2 + k^2}},$$
(22)

and the corresponding real-space distribution is

$$f(w) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left[ 1 - \sqrt{\frac{k^2 + i\lambda k(2\eta - 1)}{\lambda^2 + k^2}} \right] e^{ikw} dk.$$
(23)

We verified analytically that Equation (23) is symmetric for positive and negative w, such that  $f_{\eta}(w) = f_{1-\eta}(-w)$ , and that it is independent of rescaling of  $\lambda$ , such that for  $x = \lambda w$ ,  $f(x) = f(w)/\lambda$ . In order to compute this numerically, we decompose Equation (22) into real and imaginary components,

$$\Re \mathfrak{e}[\tilde{f}(k)] = 1 - \sqrt{\frac{(\phi + k^2)}{2(k^2 + \lambda^2)}}$$
(24)

and

$$\Im\mathfrak{m}[\tilde{f}(k)] = -\frac{\lambda k(2\eta - 1)}{\sqrt{2(k^2 + \lambda^2)(\phi + k^2)}},$$
(25)

where

$$\phi = (k^4 + k^2 \lambda^2 (2\eta - 1)^2)^{1/2}.$$
(26)

It is important to note that  $\phi$  in Equation (26) is an even function of k. Therefore,  $\Re e[\tilde{f}(k)]$  (Equation (24)) is an even function of k and  $\Im m[\tilde{f}(k)]$  (Equation (25)) is an odd function of k. Equation (23) then becomes,

$$f(w) = \frac{1}{2\pi} \int_{-\infty}^{\infty} (\Re[\tilde{f}(k)] \cos(kw)) dk$$
$$- \frac{1}{2\pi} \int_{-\infty}^{\infty} (\Im[\tilde{f}(k)] \sin(kw)) dk, \qquad (27)$$

since the imaginary component is an odd function of k and integrates to 0.

We evaluate Equation (27) numerically and show the resulting distributions in Figure 3 for  $\eta = 1/2$ , 3/4, and 1. For the case of  $\eta = 1/2$ , the problem reduces to the analytic solution presented in the previous section and in Berger & Asgari-Targhi (2009). As  $\eta$  approaches 1 (or 0), and more coherent braids are injected into the corona, the FFD becomes shallower. This result is consistent with the findings of Berger & Asgari-Targhi (2009), who performed Monte Carlo realizations of the problem with an asymmetric injection of positive to negative handed sequences.



**Figure 3.** Distribution function for winding of sequences in an equilibrium state. We calculate these distributions by numerically performing the inverse Fourier transform defined by Equation (23) for  $\eta = 1/2$ , 3/4, and 1. The purple line shows the solution when  $\eta = 1/2$  for an equal probability of positive and negative twists, which has a power-law slope of  $\gamma = 2$ , consistent with the findings of Berger & Asgari-Targhi (2009). As  $\eta$  increases, and more coherent and same signed braids are injected into the corona, the slopes of the distribution functions get shallower and approach  $\gamma = 3/2$ . This is plotted using  $\lambda = 1$ , where  $|w| = 1/\lambda$  is the mean winding magnitude per injected sequence. Changing  $\lambda$  simply rescales w, leaving the power-law dependencies unchanged.

The distributions of f(w) in Figure 3 are symmetric about the y-axis for positive and negative winding numbers when  $\eta = 1/2$ . In Figure 4, we show the equilibrium distributions for positive and negative winding numbers for a range of  $\eta \in [0, 1]$ . When  $\eta \neq 1/2$ , the resulting equilibrium distribution is asymmetric about w = 0 and contains more sequences with the same sign that is preferentially input. As expected, the equilibrium distributions for  $\eta$  and  $1 - \eta$  (shown with solid and dashed lines) are symmetric about  $w = 0 \forall \eta \in [0, 1]$ . We note that the end-member case, where  $\eta = 1$  or 0, is formally not a physically plausible scenario. This is because, as shown



**Figure 4.** Distribution function for the winding of sequences in equilibrium. This is calculated in the same way as in Figure 3, and we show the distribution for positive and negative winding numbers with  $\eta = 0$ , 1/4, 4/10, 1/2, 6/10, 3/4, and 1. The distributions for  $\eta$  and  $1 - \eta$  are symmetric about w = 0. The equilibrium state reflects the same sign bias as the input of sequences in Equation (20).

in Figure 4, the distribution functions for these cases only have sequences with a single sign, which means the star would not be able to produce any energetic flares.

#### 2.5. Generalized Energy Distribution

It is important to note that the transformation from the distribution of winding to energy given by Equation (19) is no longer valid for asymmetric distributions where  $\eta \neq 1/2$ . Following the notation in Section 2.3, assuming that  $|w_2| > |w_1|$  and allowing for two cases where  $w_2 < 0$ ,  $w_1 > 0$ , and  $w_2 > 0$ ,  $w_1 < 0$ , the energy distribution, denoted by capital F(E), presented in Equation (20) in Berger & Asgari-Targhi (2009) can be generalized,

$$F(E) \sim \int_{0}^{\infty} \int_{-\infty}^{-w_{1}} (f(w_{1})f(w_{2})\delta(E-4|w_{1}|C_{\text{crit}}))dw_{2}dw_{1} + \int_{-\infty}^{0} \int_{|w_{1}|}^{\infty} (f(w_{1})f(w_{2})\delta(E-4|w_{1}|C_{\text{crit}}))dw_{2}dw_{1}.$$
 (28)

In this equation, the first term on the right-hand side corresponds to the reconnection events where  $w_1 > 0$ . Similarly, the second term on the right corresponds to reconnection events where  $w_1 < 0$ . Note that the  $\delta$  function in both integrands has an absolute value sign because of the form of Equation (18), where  $E \sim |w_1|$ . From Figures 3 and 4, the distribution for *w* can be approximated as a power law times a negative exponential. The approximate distribution function is given by

$$f(w) \sim \begin{cases} e^{-(2\eta - 1)w/w_{+}}|w|^{-\beta} & \text{if } w > 0\\ e^{(2\eta - 1)w/w_{-}}|w|^{-\psi} & \text{if } w < 0, \end{cases}$$
(29)

for some truncation winding numbers and power-law exponents for the positive and negative cases,  $w_+$  and  $w_-$ , and power-law exponents  $\beta$  and  $\psi$ . All four of these parameters depend on the value of  $\eta$ , and generally  $w_+ \gg w_-$  and  $\beta > \psi$ for  $\eta \in [1/2, 1]$ . Equation (29) was constructed to be symmetric for  $\eta$  and  $1 - \eta$ . For the remainder of this subsection, we solve for the energy distribution for the case of  $\eta \in [1/2, 1]$ , since the resulting distribution is the same for  $\eta$  and  $1 - \eta$ . By substituting Equation (29) into Equation (28), the distribution is given by

$$F(E) \sim \int_{0}^{\infty} \int_{-\infty}^{-w_{1}} (\delta(E - 4|w_{1}|C_{\text{crit}}) \times f(w_{1})(e^{(2\eta - 1)w_{2}/w_{-}}|w_{2}|^{-\psi}))dw_{2}dw_{1} + \int_{-\infty}^{0} \int_{|w_{1}|}^{\infty} (\delta(E - 4|w_{1}|C_{\text{crit}}) \times f(w_{1})(e^{-(2\eta - 1)w_{2}/w_{+}}|w_{2}|^{-\beta}))dw_{2}dw_{1}.$$
(30)

This can be solved analytically, by recalling the definition of the incomplete gamma function,

$$\Gamma(a, x) = \int_{x}^{\infty} t^{a-1} e^{-t} dt.$$
 (31)

We perform two change of variables to integrate Equation (30). For the first term on the right-hand side, we define the variable  $v = -(2\eta - 1)w_2/w_-$ , such that  $dw_2 = -w_-/(2\eta - 1)dv$ . Similarly, for the second term on the right-hand side, we define the variable  $u = (2\eta - 1)w_2/w_+$ , such that  $dw_2 = w_+/(2\eta - 1)du$ . After evaluating the first integral of the double integral in each term, Equation (30) reduces to

$$F(E) \sim \int_{0}^{\infty} \left[ \left( \frac{w_{-}}{2\eta - 1} \right)^{1 - \psi} f(w_{1}) \right] \\ \times \Gamma \left( 1 - \psi, \frac{2\eta - 1}{w_{-}} w_{1} \right) \delta(E - 4|w_{1}|C_{\text{crit}}) dw_{1} \\ + \int_{-\infty}^{0} \left[ \left( \frac{w_{+}}{2\eta - 1} \right)^{1 - \beta} f(w_{1}) \right] \\ \times \Gamma \left( 1 - \beta, \frac{2\eta - 1}{w_{+}} |w_{1}| \right) \delta(E - 4|w_{1}|C_{\text{crit}}) dw_{1}.$$
(32)

Using the composition properties of the Dirac delta function, the coefficient  $\delta(E - 4|w_1|C_{crit})$  can be written as

$$\delta(E - 4|w_1|C_{\text{crit}}) = \frac{1}{4C_{\text{crit}}} \left( \delta \left( w_1 - \frac{E}{4C_{\text{crit}}} \right) + \delta \left( w_1 + \frac{E}{4C_{\text{crit}}} \right) \right).$$
(33)

By substituting Equation (33) into Equation (32) and performing the integral, Equation (21) in Berger & Asgari-Targhi (2009) generalizes to the following energy distribution function:

$$F(E) \sim \left(\frac{1}{4C_{\text{crit}}}\right) \left(\frac{w_{-}}{2\eta - 1}\right)^{1-\psi} \\ \times \Gamma \left(1 - \psi, \frac{2\eta - 1}{w_{-}} \frac{E}{4C_{\text{crit}}}\right) f\left(\frac{E}{4C_{\text{crit}}}\right) \\ + \left(\frac{1}{4C_{\text{crit}}}\right) \left(\frac{w_{+}}{2\eta - 1}\right)^{1-\beta} \\ \times \Gamma \left(1 - \beta, \frac{2\eta - 1}{w_{+}} \frac{E}{4C_{\text{crit}}}\right) f\left(\frac{-E}{4C_{\text{crit}}}\right).$$
(34)

By substituting Equation (29) into Equation (34), the final energy distribution is given by

$$F(E) \sim \left(\frac{1}{4C_{\text{crit}}}\right) \left(\frac{w_{-}}{2\eta - 1}\right)^{1-\psi} \Gamma\left(1 - \psi, \frac{2\eta - 1}{w_{-}} \frac{E}{4C_{\text{crit}}}\right)$$
$$\times \left(e^{-(2\eta - 1)E/(w_{+}4C_{\text{crit}})} \left|\frac{E}{4C_{\text{crit}}}\right|^{-\beta}\right)$$
$$+ \left(\frac{1}{4C_{\text{crit}}}\right) \left(\frac{w_{+}}{2\eta - 1}\right)^{1-\beta} \Gamma\left(1 - \beta, \frac{2\eta - 1}{w_{+}} \frac{E}{4C_{\text{crit}}}\right)$$
$$\times \left(e^{-(2\eta - 1)E/(w_{-}4C_{\text{crit}})} \left|\frac{E}{4C_{\text{crit}}}\right|^{-\psi}\right). \tag{35}$$

For large x, the incomplete Gamma function can be approximated as the asymptotic series (Temme 1975),

$$\Gamma(a, x) \approx x^{a-1}e^{-x} \times (1 + (a-1)x^{-1} + (a-1)(a-2)x^{-2} + ...),$$
(36)

for  $x \gg |a - n|$ , where *n* is the index of the term number in the series, or for a series truncated at the point where  $(a - n)/x \sim 1$ . Therefore, Equation (35) has the form of a sum of power laws times a negative exponential for large flare energies. We verified numerically that Equation (35) has the form of a power law with an exponent  $\alpha \approx \beta + \psi - 1$  and an exponential cutoff set by  $w_-$  (where  $w_- < w_+$ ), analogous to the simpler case found in Berger & Asgari-Targhi (2009), where  $\beta = \psi$  and  $\alpha = 2\beta - 1$ . For example, for  $w_- = 10$ , the inertial range extends from 0 < w < 100, while for  $w_- = 1$ , it extends from 0 < w < 10. As can be seen in Figure 4, although the negative exponential dominates for most of the domain for negative winding numbers,  $\psi$  slightly decreases as  $\eta$  increases. Therefore, as  $\eta$  changes,  $\alpha$  scales primarily with  $\beta$ .

To validate this analytic calculation, we computed the energy distribution  $F(\tilde{E})$  for a proxy for the flare energy,  $\tilde{E} \sim w/4C_{\rm crit}$ . We computed this by performing a numerical integration of Equation (28) directly, for the distribution functions presented in Figure 4. We show the energy distributions in Figure 5, for a range of  $\eta$ . As expected, the energy distribution becomes shallower as winding parity violation is increased.

Our main conclusion is that the slope decreases as  $\eta \rightarrow 1$ . This result can be interpreted physically as follows. As more coherent sequences are injected into the corona, the removal of



**Figure 5.** Energy distribution of flaring events for a range of  $\eta$ . These distributions were computed by performing numerical integrations of Equation (28) for the distributions presented in Figure 4. The energy distribution becomes shallower as  $\eta$  increases.

interchanges yields a larger number of coherent additions of sequences, as shown in Figure 2. Compared to the symmetric case ( $\eta = 1/2$ ), this has the effect of allowing larger values of w| to build up, since the injection of a new braid is now more likely to increase |w| than to decrease it. The winding distribution f(w) thus becomes flatter. Since the flare energy index  $\alpha$  is directly related to the winding number index, a flatter distribution of winding numbers immediately produces a flatter energy distribution. This change in slope is compensated for by a reduction in the overall number of flaring events at  $\eta \rightarrow 1$ , so that the total amount of energy dissipated remains the same.

# 3. Connection with TESS Observations

Studying stellar activity on a statistical level requires the observation and identification of a large number of flaring events. Historically, such a catalog has been difficult to compile due to the long observational baselines with high temporal cadences required for each star to capture short-lived events. It is also a nontrivial task to identify and characterize flares. Recently, surveys designed to discover extrasolar planets such as Kepler/K2 (Borucki et al. 2010; Howell et al. 2014) and the Transiting Exoplanet Survey Satellite (TESS; Ricker et al. 2015) have provided a wealth of observations that can be used for compiling a catalog of flares (see also Section 1).

# 3.1. Sample and Flare Identification

Our calculations suggest that stars with strong Coriolis forces should exhibit a higher proportion of high-energy flares relative to low-energy ones. To verify this empirically, we consider the subset of stars that were observed by TESS at 2 minutes cadence, are included in our sample from Feinstein et al. (2022), and which have rotation periods available in the literature. The rotation periods used here were measured from TESS and Evryscope light curves, and were taken directly from Feinstein et al. (2020b) and Howard et al. (2020), which yields a sample of 1380 stars ranging from  $T_{\rm eff} = 2300-9300$  K.



**Figure 6.** Gaia DR2  $B_p - R_p$  color of every star in our sample vs. the measured rotation periods from the literature (Feinstein et al. 2020b; Howard et al. 2020). Each point is colored by the flare rate, taken from (M. N. Günther et al. 2022, in preparation). All of these stars were observed by TESS at a 2 minute cadence. Our sample is biased toward stars with rotation periods of <13 days due to the observing strategy of TESS.

A summary of this subsample is presented in Figure 6. Although requiring a measured period significantly decreases the number of stars in our sample, we still have enough stars to check for signatures of the Coriolis force.

The flares were detected using the convolutional neural network models (CNNs) presented in Feinstein et al. (2020b). These models were specifically designed to find flares in TESS 2 minute cadence light curves and assign a *probability* to each flare of being real (1) or not (0). We run the 10 CNNs recommended by Feinstein et al. (2020b) and average the prediction outputs for each light curve. The average prediction is used as our probability that each flare is real.

Feinstein et al. (2020b) included all flares with a threshold of  $\geq 0.5$  to be potential true flares, meaning the flares in their sample have a 50% probability of being a true flare. However, we adopt a more conservative threshold of 0.9 to ensure that at least 90% of the samples are classified as true flares.

Given the limited size of the training set for the Feinstein et al. (2020b) CNNs, we applied additional false-positive filters from (M. N. Günther et al. 2022, in preparation), which are summarized in Feinstein et al. (2022). The filters remove flares with peak intensities that are within  $3 \times \text{rms}$  of the light curve, flares with equivalent durations of  $\leq 4$  minutes, and flares that are associated with periodic events (e.g., eclipsing binaries or variable stars). Once these filters are applied, we are left with a sample of 16,184 flares with true-flare probabilities  $\geq 0.9$  from 869 stars.

# 3.2. Observational Results

In Figure 7, we show the intensity distributions of flaring events for stars that we identify as fast ( $P_{\text{rot}} < 3$  days) and slow rotators ( $P_{\text{rot}} \ge 3$  days). In these plots, the flare amplitude, *A* (expressed as a percentage of the star's normalized flux), is divided into bins of equal width in log-space from  $-2.0 \le \log_{10}(A) \le 1$  (with 5 bins per dex, and 30 bins in total), and the vertical axis shows the number of flares, weighted by their true-flare probability, observed per star per day within each bin. The bins represent the flare rate for all flares with a threshold of  $\ge 0.9$ . The error bars on the flare rate



**Figure 7.** FFDs for the flares in our sample, binned by the intensity of the flare in percentage of the star's normalized flux. The flare amplitude, *A*, is divided into bins of equal width in log-space (with 5 bins per dex) ranging from  $-2.0 \le \log_{10}(A) \le 1$ , and the vertical axis plots the number of flares observed per star per day in each amplitude bin. The top panel shows the distribution of flares on stars with rotation periods,  $P_{\rm rot}$ , of <3 days (522 stars with 11,614 flares total), and the bottom shows the distribution of flares on stars with *P*<sub>rot</sub>  $\ge$  3 days (347 stars with 4570 flares total). While our sample of slower rotators is incomplete, there is evidence that the more rapidly rotating objects have shallower slopes characterized by more energetic events, which implies stronger winding parity violation.

in each bin are defined as follows. Our upper estimate for the flare rate includes all flares with a threshold  $\ge 0.5$ . Our lower estimate is for all flares with a threshold  $\ge 0.99$ . We have visually verified that even the very large amplitude flares we measure are real, and not the result of a failure in the detection algorithm. In Figure 8, we show the light curves of five of the highest amplitude flares in the fast rotators to reinforce this conclusion.

In order to evaluate the slope of the distribution of flare amplitudes, and test whether there is a significant difference between these two samples, we fit a truncated power-law distribution  $dp/dA \propto A^{-\alpha'}e^{-A/A_*}$  for amplitudes  $A > A_{min}$ , where  $A_{min} = 10^{-2.0}$  is the smallest amplitude flare for which our data are complete—there is a visible turn-down in the flare frequency distribution below this limit. We perform this fit separately to each of the two subsamples (fast and slow rotators). Note that the form of this equation is similar to Equation (1), although we include the possibility of a truncation at high amplitude, and denote the slope here  $\alpha'$ rather than  $\alpha$  to indicate that we are fitting the index for flare amplitude, not energy. We fit the slopes using a Markov chain Monte Carlo (MCMC) method implemented with the emcee package (Goodman & Weare 2010; Foreman-Mackey et al. 2013), using the log-likelihood function

$$\log \mathcal{L} = \sum_{i} \log \left( \mathcal{N} A_i^{-\alpha'} e^{-A/A_*} \right), \tag{37}$$

where  $A_i$  is the amplitude of the *i*th flare in the sample,  $A_*$  is a flare amplitude cutoff parameter to be fit, and  $\mathcal{N}$  is a normalization factor chosen to ensure that  $\int_{A_{\min}}^{\infty} (dp/dA)dA = 1$ . The MCMC fit has two free parameters— $A_*$  and  $\alpha'$ —and we adopt priors that are flat in  $\log A_*$  and  $\alpha'$ .



**Figure 8.** Five of the highest amplitude flares (highlighted in blue) from five different stars in our sample. The inset panels are of the full light curve, displaying various forms of starspot driven modulation, used to measure the rotation periods of these stars. Each of these stars are fast rotators, and contribute to the top panel of Figure 7.

We initialized our MCMC fit with 100 walkers and iterated for 2500 steps; we discard the first 1000 steps for burn-in, and verify visually that the chains are well converged. We show the median fits obtained by this method, as well as the confidence intervals around them, overlaid on the data in Figures 7; we show the joint and marginal posterior probability distribution functions (PDFs) for the slope  $\alpha'$  and cutoff parameter  $A_*$  in Figure 9. As the plots show, there is a marginally significant difference in the posterior PDFs for the short- and long-period samples the  $2\sigma$  regions do not overlap, but are close, and the marginal parameter values do overlap at the  $2\sigma$  level. Repeating the fits using different breaks between fast and slow rotators of 1 day or 2 days yields qualitatively similar results. Formally, we find  $\alpha' = 1.866 \pm$ 0.035 and  $\log A_* = 0.004^{+0.131}_{-0.106}$  for the short-period sample, and  $\alpha' = 1.967^{+0.068}_{-0.070}$  and  $\log A_* = -0.017^{+0.336}_{-0.223}$  for the long-period sample; the quoted uncertainties here indicate the 5th-95th percentile range. To the extent that the differences are real, and not simply statistical noise, we find the fast rotators showing a flatter slope but with a cutoff at somewhat smaller amplitude, and the slow rotators showing a steeper slope but more gradual cutoff. A shallower slope for fast rotators would be consistent with that seen in the literature (Hartmann & Noyes 1987; Maggio et al. 1987; Doyle et al. 2020), but we emphasize that our present sample does not provide a statistically significant detection.

It has been established that stellar properties other than rotation period, such as mass and surface temperature, affect



**Figure 9.** Distributions of the best-fitting power-law exponent and amplitude cutoff parameter for the slow and fast rotators, from the MCMC fitting process described in Section 3.2. This shows the full 2D posterior PDF, including both slopes and cutoffs, with histograms showing the marginal 1D PDFs. In the central panel showing the 2D PDF, the shaded regions are the 68%, 90%, and 95% CIs, which visually demonstrate that the CIs are overlapping at 68% confidence. This suggests that the distributions of flares are marginally different for fast and slow rotators.

the dynamo. The Rossby number, Ro, incorporates the rotational period, mass, and temperature, and is defined as  $\text{Ro} = P/\tau$ , where  $\tau$  is the convective turnover time (Noyes et al. 1984). For a given stellar type and  $\tau$ , the Rossby number is a direct proxy for rotational period. X-ray luminosity and overall activity increases with rotational period for a range of stellar types, including Sun-like stars (Pallavicini et al. 1981; Wright et al. 2011; Candelaresi et al. 2014) and low mass, fully convective stars (Wright & Drake 2016). However, this relationship does not apply for very fast rotators, and the X-ray luminosity saturates for Ro < 0.13, independent of spectral type (Vilhu 1984; Wright & Drake 2016).

Motivated by this work, we repeat the same analysis but group both subsamples based on the Rossby number instead of just the rotational period. We convert stellar mass to the convective turnover time,  $\tau$ , using Equation (11) in Wright et al. (2011),

$$\log_{10}(\tau) = 1.16 - 1.49 \log_{10}\left(\frac{M}{M_{\odot}}\right) - 0.54 \log_{10}^{2}\left(\frac{M}{M_{\odot}}\right).$$
 (38)

By binning the stars based on the saturation break of Rossby number, the two resulting subsamples include 458 stars with 11,148 flares total where Ro < 0.13, and 349 stars with 3,507 flares total with Ro  $\ge 0.13$ . The resulting distributions are shown in Figure 10, with the best-fit distributions overplotted. The fits were calculated in the same manner as the fits for subsamples grouped by rotational period. In Figure 11, we show the joint and marginal posterior probability distributions functions for slope  $\alpha'$  and cutoff parameter  $A_*$ , with the two subsamples binned by Rossby number. We find that for Ro < 0.13,  $\alpha' = 1.847^{+0.032}_{-0.033}$  and  $\log(A_*) =$  $-0.012^{+0.117}_{-0.102}$ . For Ro > 0.13,  $\alpha' = 2.155^{+0.112}_{-0.115}$  and  $\log(A_*) =$  $-0.111^{+1.996}_{-0.348}$ . The results of all fits are presented in Table 1. It appears that the differences in the distributions are more pronounced when grouped by Rossby number. Since the



**Figure 10.** Same as Figure 7, with the two subsamples grouped by Rossby number. The top panel shows the distribution of flares on stars with Rossby number, Ro, of <0.13 (458 stars with 11,1148 flares total), and the bottom panel shows the distribution of flares on stars with Ro  $\ge 0.13$  (349 stars with 3,507 flares total).



**Figure 11.** Same as Figure 9, with the two subsamples grouped by Rossby number. For Ro < 0.13, the fits produced a slope  $\alpha' = 1.847^{+0.032}_{-0.033}$  and log  $(A_*) = -0.012^{+0.112}_{-0.112}$ . For Ro > 0.13 the fits produced  $\alpha' = 2.155^{+0.112}_{-0.115}$  and  $\log(A_*) = -0.111^{+1.996}_{-0.348}$ .

 Table 1

 Summary of the Parameter Fits for the Flare Frequency Distributions Binned by Rotation Period and Rossby Number

Sample	Slope, $\alpha'$	$log(A_*)$
$P_{\rm rot} < 3$ days $P_{\rm rot} \ge 3$ days	$\frac{1.866 \pm 0.035}{1.967^{+0.068}_{-0.070}}$	$\begin{array}{r} 0.004^{+0.131}_{-0.106} \\ -0.017^{+0.336}_{-0.223} \end{array}$
Ro < 0.13 Ro ≥ 0.13	$\frac{1.847^{+0.032}_{-0.033}}{2.155^{+0.112}_{-0.115}}$	$-0.012\substack{+0.117\\-0.102}\\-0.111\substack{+1.996\\-0.348}$

Rossby number scales with the rotational period, it is plausible that the Coriolis effect investigated in this paper is responsible for the differences in the slopes of these FFDs.

There are four potential confounding factors that might influence our results. First and most obviously, the sample is relatively small: only  $\approx 3000$  flares in total with amplitude  $A > 10^{-0.5}$  where we are reasonably complete. It may simply be that the sample is too small to yield statistically significant results. Second, the measured rotation periods are biased toward <12.5 days due to the observing strategy of TESS. Thus our sample of slow rotators is perhaps better described as a sample of moderator rotators; truly slow rotators like the Sun are largely excluded. Third, our sample has not been corrected for detection biases. Methods such as injection-recovery tests have been used to adjust observed flare rates in previous studies, as in Figure 11 in Kővári et al. (2020). We do not perform injection-recovery tests on this sample for two reasons. For one, Feinstein et al. (2020b) demonstrated that injectionrecovery tests did not produce accurate results for their CNN, due to the differences in the shape of the flares themselves. Additionally, we only start at 0.32% amplitudes here and therefore ignore the smallest flares (which are most affected by this bias). A final confounding factor is that, while rotation period is a reasonable proxy for the importance of Coriolis forces in the dynamo, it is certainly not the only factor that might affect flare distributions. In particular, stars of different ages and effective temperature almost certainly have different surface magnetic field strengths, and our fast and slow rotator samples mix together a wide range of stellar age and effective temperature. Similar to our marginal agreement here, Feinstein et al. (2020b) measured the FFD slope for stars with age  $\leq$  50 and >50 Myr and found they were in a 1 $\sigma$  agreement with each other. For the specific details of this analysis, we refer the reader to Figures 13-14 and Section 4.1 in Feinstein et al. (2020b). While the results from our analysis incorporating the Rossby number are highly suggestive, it is possible that an analysis that separates all of these contributing factors-age, rotation period, and effective temperature-might yield stronger results.

# 4. Conclusions

In this paper, we have investigated the role of the Coriolis force in stellar dynamos. More specifically, we considered the DC model of coronal heating (Parker 1972) along with the model of magnetic braiding and reconnection (proposed by Berger & Asgari-Targhi 2009; see also Berger et al. 2015). We expanded the braiding model to incorporate the effects of the Coriolis force, which should be more dominant in faster rotating stars. Specifically, we incorporated a bias in handedness of injected braids, which should be present in stars with dynamos that are strongly affected by the Coriolis force. As increasingly coherent braids are injected into the corona, the slopes of the resulting power-law distributions of energetic flaring events decreases in magnitude (corresponding to a weaker decline in occurrence rates toward increasing flare energies). We search for this effect in the flare frequency distributions for stars observed by TESS that have measured rotation periods; however, while the results are suggestively consistent with the theoretical prediction, the sample is too small to yield a definitive statistical conclusion.

While the results presented in this study are suggestive, there are many opportunities for future theoretical work. It would be informative to perform numerical experiments of reconnection events for both the DC and AC regimes of coronal heating. Specifically, numerical simulations of avalanching reconnection events similar to those presented by Lu & Hamilton (1991) may also exhibit shallower slopes with driving that mimics effects of the Coriolis force. Being able to directly probe the relation between the magnitude of the Coriolis force and the  $\eta$ (handedness) parameter would be an invaluable measurement to test the role of rotation on the topology of the flux tubes and hence the energy of the flaring events. This type of calculation would also yield insights into the connection between the theory of self-organized criticality and the DC braiding and reconnection picture of coronal heating.

Alternatively, it is possible that the power-law distributions of stellar flares are simply realizations of forced MHD turbulence, along the framework of the AC regime. Therefore, numerical simulations of forced coronal plasmas that track reconnection events may reveal an alternative explanation for the differences in the observed slopes.

Several possible adjustments to the observational data could further validate the hypothesis presented here. We have included a relatively small ( $\approx$ 3000 stars) sample of stars with measured rotation periods. However, there are 10<sup>5</sup> stars with high-cadence observations from TESS. By measuring the rotation periods for more stars with known flaring events, we would be able to vastly increase the statistical sample in this study. This analysis would also be improved by the proposed increase from a 10–3 minute cadence of TESS Full-Frame Images for the second extended mission. This improvement would increase our sample of stars by an order of magnitude, which would improve the slope measurements.

It will also be necessary to develop a reliable method for measuring long rotation periods (P > 13 days) in TESS (e.g., Lu et al. 2020; Breton et al. 2021; Claytor et al. 2022). In the current state, we are limited both in the baseline for the TESS sector observing strategy ( $\sim 27$  days) and the orbital gap halfway through each sector. Traditional methods of measuring rotation periods often identify the systematics that are associated with the beginning or end of each orbit. We are thus limited to only reliably being able to measure short rotation periods. Alternatively, indirect estimates of stellar rotation periods may be useful for expanding our sample (Mamajek & Hillenbrand 2008).

It would also be useful to improve the convolutional neural networks presented in Feinstein et al. (2020b) to detect flares on rapid rotating stars (P < 1 day). When these methods were developed, there were relatively few examples of these light curves, so that the original models were trained on a limited number of high-energy events on rapidly rotating stars. As a result, the sharp rotational features are often confused for flare events. Now that more high-energy events have been observed, we have the opportunity to improve our method for flare identification using machine learning.

Finally, it would be of significant interest to test the impact of stellar age versus rotation period on the flare rates. We were not able to do this analysis with our relatively small sample. However, populations within young stellar clusters, such as Pleiades, Hyades, or Praesepe, have well constrained masses and ages. The activity in young clusters has already been examined using K2 long-cadence data (Ilin et al. 2019, 2021), although this cadence is inadequate for detecting lowamplitude flares. Examining activity in young stellar clusters using new high-cadence TESS data may yield key insights into the relationship between rotation, dynamo, and spin-down. We thank Mitch Berger, Mahboubeh Asgari-Targhi, Eugene Chiang, Amir Siraj, Jared Siegel, Dimitar Sasselov, Konstantin Batygin, Jacob Bean, Juliette Becker, Fred Ciesla, and Harriet Lau for useful conversations. We thank the two anonymous reviewers for insightful comments and constructive suggestions that strengthened the scientific content of this manuscript, especially for suggesting searching for differences in the power-law distribution based on the Rossby number.

A.D.F. acknowledges support from the National Science Foundation Graduate Research Fellowship Program under grant No. DGE-1746045. J.R.B. acknowledges financial support from the Australian National University, via the Deakin PhD and Dean's Higher Degree Research (theoretical physics) Scholarships and the Australian Government via the Australian Government Research Training Program Fee-Offset Scholarship. M.R.K. acknowledges support from the Australian Research Council through its Future Fellowships scheme, award FT180100375. L.A.R. gratefully acknowledges support from the Research Corporation for Science Advancement through a Cottrell Scholar Award. C.F. acknowledges funding provided by the Australian Research Council (Future Fellowship FT180100495), and the Australia-Germany Joint Research Cooperation Scheme (UA-DAAD). M.F. is grateful for the support he received from the Hauck Foundation.

*Facilities:* TESS (Ricker et al. 2015), Kepler (Borucki et al. 2010; Howell et al. 2014).

*Software:* lightkurve (Lightkurve Collaboration et al. 2018), matplotlib (Hunter 2007), astropy (Astropy Collaboration et al. 2013; Price-Whelan et al. 2018), astroquery(Ginsburg et al. 2019), numpy (Van Der Walt et al. 2011; Harris et al. 2020), stella (Feinstein et al. 2020a), emcee (Foreman-Mackey et al. 2013).

#### **ORCID** iDs

Darryl Z. Seligman https://orcid.org/0000-0002-0726-6480 Leslie A. Rogers https://orcid.org/0000-0003-0638-3455 Adina D. Feinstein https://orcid.org/0000-0002-9464-8101 Mark R. Krumholz https://orcid.org/0000-0003-3893-854X James R. Beattie https://orcid.org/0000-0001-9199-7771 Christoph Federrath https://orcid.org/0000-0002-0706-2306 Fred C. Adams https://orcid.org/0000-0002-8167-1767 Marco Fatuzzo https://orcid.org/0000-0002-1116-6140 Maximilian N. Günther https://orcid.org/0000-0002-3164-9086

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