

LETTERS

The formation of stars by gravitational collapse rather than competitive accretion

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There are two dominant models of how stars form. Under gravitational collapse, star-forming molecular clumps, of typically hundreds to thousands of solar masses (M_{\odot}), fragment into gaseous cores that subsequently collapse to make individual stars or small multiple systems^{1–3}. In contrast, competitive accretion theory suggests that at birth all stars are much smaller than the typical stellar mass ($\sim 0.5M_{\odot}$), and that final stellar masses are determined by the subsequent accretion of unbound gas from the clump^{4–8}. Competitive accretion models interpret brown dwarfs and free-floating planets as protostars ejected from star-forming clumps before they have accreted much mass; key predictions of this model are that such objects should lack disks, have high velocity dispersions, form more frequently in denser clumps^{9–11}, and that the mean stellar mass should vary within the Galaxy⁸. Here we derive the rate of competitive accretion as a function of the star-forming environment, based partly on simulation¹², and determine in what types of environments competitive accretion can occur. We show that no observed star-forming region can undergo significant competitive accretion, and that the simulations that show competitive accretion do so because the assumed properties differ from those determined by observation. Our result shows that stars form by gravitational collapse, and explains why observations have failed to confirm predictions of the competitive accretion model.

In both theories, a star initially forms when a gravitationally bound gas core collapses. The crucial distinction between them is their prediction for what happens subsequently. In gravitational collapse, after a protostar has consumed or expelled all the gas in its initial core, it may continue accreting from its parent clump. However, it will not accrete enough to change its mass substantially^{13,14}. In contrast, competitive accretion requires that the amount accreted after the initial core is consumed be substantially larger than the protostellar mass. We define $f_m \equiv \dot{m} \cdot t_{\text{dyn}} / m_*$ as the fractional change in mass that a protostar of mass m_* undergoes each dynamical time t_{dyn} of its parent clump, starting after the initial core has been consumed. Gravitational collapse holds that $f_m \ll 1$, whereas competitive accretion requires $f_m \gg 1$.

We consider a protostar embedded in a molecular clump of mass M and mass-weighted one-dimensional velocity dispersion σ . Competitive accretion theories usually begin with seed protostars of mass $m_* \approx 0.1M_{\odot}$ (refs 4–7), so we adopt this as a typical value. We consider two possible geometries: spherical clumps of radius R and filaments of radius R and length L , where $L \gg R$. These extremes bracket real star-forming clumps, which have a range of aspect ratios. The virial mass for (spherical, filamentary) clumps is:

$$M_{\text{vir}} \equiv \left(\frac{5\sigma^2 R}{G}, \frac{2\sigma^2 L}{G} \right) \quad (1)$$

and the virial parameter is $\alpha_{\text{vir}} \equiv M_{\text{vir}}/M$ (refs 15, 16). The dynamical time is $t_{\text{dyn}} \equiv R/\sigma$.

First, we suppose that the gas that the protostar is accreting is not collected into bound structures on scales smaller than the entire clump. Because the gas is unbound, we may neglect its self-gravity and treat this as a problem of a non-self-gravitating gas accreting onto a point particle. This process is Bondi–Hoyle accretion in a turbulent medium, which gives an accretion rate¹²:

$$\dot{m}_* \approx 4\pi \phi_{\text{BH}} \bar{\rho} \frac{(Gm_*)^2}{(\sqrt{3}\sigma)^3} \quad (2)$$

where $\bar{\rho}$ is the mean density in the clump. The factor ϕ_{BH} represents the effects of turbulence, which we estimate in terms of parameters σ , m_* and R in the Supplementary Information¹². From equation (2) and the definitions of the virial parameter and the dynamical time, we find that accretion of unbound gas gives:

$$f_{\text{m-BH}} = \left(14.4, 3.08 \frac{L}{R} \right) \phi_{\text{BH}} \alpha_{\text{vir}}^{-2} \left(\frac{m_*}{M} \right) \quad (3)$$

for a (spherical, filamentary) star-forming region. From this result, we can immediately see that competitive accretion is most effective in low-mass clumps with virial parameters much smaller than unity.

Tables 1 and 2 show a broad sample of observed star-forming regions. None of them have a value of $f_{\text{m-BH}}$ near unity, which is inconsistent with competitive accretion and in agreement with gravitational collapse. We note that the Bondi–Hoyle rate is an upper limit on the accretion. If the stars are sufficiently close-packed, their tidal radii will be smaller than their Bondi–Hoyle radii, and the accretion rate will be lower⁵. Also, radiation pressure will halt Bondi–Hoyle accretion onto stars larger than $\sim 10M_{\odot}$ (ref. 17).

The second possible way that a star could gain mass is by capturing and accreting other gravitationally bound cores. We can analyse this

Table 1 | Sample star-forming regions

Name	Ref.	Mass	Type	$M (M_{\odot})$	R (pc)	L (pc)	σ (km s ⁻¹)
L1495 I	30	Low	Sph.	410	2.1	-	0.58
L1495 II	30	Low	Sph.	950	2.4	-	0.67
L1709	16	Low	Fil.	140	0.23	3.6	0.48
L1755	16	Low	Fil.	171	0.15	6.3	0.53
W44	23	High	Sph.	16000	0.35	-	3.9
W75(OH)	23	High	Sph.	5600	0.25	-	3.5
f-fil	16	High	Fil.	5000	0.25	13	1.41
N-fil	16	High	Fil.	16000	2.3	88	1.54

Sph., spherical; Fil., filamentary; f-fil, Orion integral filament; N-fil, Orion North filament. For L1495 I and II, the data are from the ¹²CO observations of ref. 30, and the masses are M_{CO} from ref. 30. For W44 and W75(OH), the data are the CS $J = 5 \rightarrow 4$ observations of ref. 23, and the masses are M_n from ref. 23. (Here CS is carbon sulphide, and J is the rotational quantum number of the CS molecule.) CS $J = 5 \rightarrow 4$ is a very high-density tracer, so it biases the results to small virial parameters by excluding low-density parts of the clump.

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process by some simple approximations. First, when a star collides with a core it begins accreting gas from it, causing a drag force¹⁸. If drag dissipates enough energy, the two become bound. We can therefore compute a critical velocity below which any collision will lead to a capture and above which it will not. Second, cores and stars should inherit the velocity dispersion of the gas from which they form, so we assume they have Maxwellian velocity distributions with dispersion σ . The true functional form may be different, but this will only affect our estimates by a factor of order unity. Third, we neglect the range of core sizes, and assume that all cores have a generic radius R_{co} and mass M_{co} . Competitive accretion requires $M_{\text{co}} \leq m_*$, so we take $M_{\text{co}} = m_*$, which gives the highest possible capture rate. Finally, we make use of an important observational result: cores within a molecular clump have roughly the same surface density as the clump itself⁹, that is, $\Sigma = M(\pi R^2, 2RL)^{-1}$ for (spherical, filamentary) clumps. This enables us to compute the escape velocity from the surface of a core in terms of the properties of the clump:

$$v_{\text{esc}} = \left[\left(10, \sqrt{8\pi \frac{L}{R}} \right) \alpha_{\text{vir}}^{-1} \sqrt{\frac{M_{\text{co}}}{M}} \right]^{1/2} \sigma \quad (4)$$

With these approximations, it is straightforward to compute the amount of mass that a protostar can expect to gain by capturing other cores. In the Supplementary Information, we show that it is:

$$f_{\text{m-cap}} = (0.42, 0.36) \phi_{\text{co}} [4 + 2u_{\text{esc}}^2 - (4 + 7.32u_{\text{esc}}^2) \exp(-1.33u_{\text{esc}}^2)] \quad (5)$$

where ϕ_{co} is the fraction of the parent clump mass that is in bound cores and $u_{\text{esc}} \equiv v_{\text{esc}}/\sigma$. Surveys generally find core mass fractions of $\phi_{\text{co}} \approx 0.1$ (refs 20–22), so we adopt this as a typical value, giving the numerical values of $f_{\text{m-cap}}$ shown in Table 2. As with $f_{\text{m-BH}}$, all the estimated values are well below unity.

If we let $f_{\text{m}} = f_{\text{m-BH}} + f_{\text{m-cap}}$, then we can use our simple models to determine where in parameter space a star-forming clump must fall to have $f_{\text{m}} \geq 1$. For simplicity, we consider a spherical clump with fixed $\phi_{\text{BH}} = 5$ and $\phi_{\text{co}} = 0.1$ (typical values for observed regions), and a seed protostar of mass $m_* = 0.1$. In this case, both $f_{\text{m-BH}}$ and $f_{\text{m-cap}}$ are functions of $\alpha_{\text{vir}}^2 M$ alone, and we find $f_{\text{m}} \geq 1$ for $\alpha_{\text{vir}}^2 M < 8.4M_{\odot}$. The functional dependence is more complex if we include filamentary regions and allow ϕ_{BH} and ϕ_{co} to vary, but the qualitative result is unchanged. Observed star-forming regions have $\alpha_{\text{vir}} \approx 1$ and $M \approx 10^2 - 10^4 M_{\odot}$ (ref. 23), which produces $f_{\text{m}} \ll 1$. No known star-forming region has $\alpha_{\text{vir}}^2 M$ small enough for competitive accretion to work. Thus, the cores from which stars form must contain all the mass they will ever have, which is the gravitational collapse model.

Our simple estimate of f_{m} is consistent with simulations of competitive accretion as well, and explains why competitive accretion works in the simulations. All competitive accretion simulations have virial parameters $\alpha_{\text{vir}} \ll 1$. In some cases the simulations start in this condition^{5,6,24,25}, with $\alpha_{\text{vir}} \approx 0.01$ as a typical choice. In other cases, the virial parameter is initially of order unity, but as turbulence decays in the simulation it decreases to $\ll 1$ in roughly a crossing time^{7,9,10,26}. Once competitive accretion gets going, these simulations reach $\alpha_{\text{vir}} \ll 1$ as well. In addition, many of the simulations consider

star-forming clumps of masses considerably smaller than the $\sim 5,000M_{\odot}$ typical of most galactic star formation²³, with $M \leq 100M_{\odot}$ not uncommon. Consequently, the simulations have $\alpha_{\text{vir}}^2 M \leq 10M_{\odot}$, which explains why they find competitive accretion to be important. Note that simulations where turbulence decays will have $\phi_{\text{BH}} \approx 1$, rather than the typical value of $\phi_{\text{BH}} = 5$ we have used for real regions, but this does not substantially modify our conclusions.

Three other aspects of the simulations increase even further their estimate of f_{m} . First, the Bondi–Hoyle radius of a $0.1M_{\odot}$ seed protostar in a typical clump is only 5 AU (astronomical units), a smaller scale than any of the competitive accretion simulations resolve. This under-resolution may enhance accretion¹². Second, small virial parameters lead most of the mass to collapse to stars, giving $\phi_{\text{co}} \approx 0.5$ –1 after a dynamical time, and also tend to make the cloud fragment into smaller pieces, lowering M . Third, rapid collapse leaves no time for large cores to assemble. For example, one simulation of a $\sim 1,000M_{\odot}$ clump produces no cores larger than $1M_{\odot}$ (ref. 7), inconsistent with observations that find numerous cores more massive than this in similar regions^{22,27}. With no large cores, large stars can form only via competitive accretion.

Thus, our results are consistent with the simulations, but they show that the simulations are not modelling realistic star-forming clumps. One might argue that all clumps do enter a phase with $\alpha_{\text{vir}} \ll 1$ that occurs rapidly and has therefore never been observed, but that most stars are formed during this collapse phase. In this scenario, though, protostars associated with observed star-forming regions should have systematically lower masses than the field star population, because they were formed before the collapse phase in which competitive accretion might occur. We would expect to see a systematic variation in mean stellar mass with age in young clusters, corresponding to cluster evolution into a state more and more favourable to competitive accretion. We do not observe this.

We hypothesize that the primary problem with the simulations—the reason they evolve to $\alpha_{\text{vir}} \ll 1$ —is that they omit feedback from star formation. Recent observations of protostellar outflow cavities show that outflows inject enough energy to sustain the turbulence and prevent the virial parameter from declining to values much less than unity²⁸. Another possible problem in the simulations is that they simulate isolated clumps containing too little material. Real clumps are embedded in molecular clouds, and large-scale turbulent motions in the clouds may cascade down to the clump scale and prevent the turbulence from decaying. A third possibility is that turbulence decays too quickly in the simulations because they do not include magnetic fields and their initial velocity fields, unlike in real clumps, are balanced rather than imbalanced between left- and right-propagating modes²⁹.

One implication of our work is that brown dwarfs need not have been ejected from their natal clump, so their velocity dispersions should be at most slightly greater than those of stars, and their frequency need not change as a function of clump density. This also removes a discrepancy between observations showing that brown dwarfs have disks¹¹ and theoretical models of their origins. We also conclude that the mean stellar mass need not vary from one star-forming region to another as competitive accretion predicts, removing a discrepancy between theory⁸ and observations that have thus far failed to find any substantial variation in typical stellar mass with the star-forming environment. In the gravitational collapse scenario, the mean stellar mass may be roughly constant in the Galaxy, but may vary with the background radiation field in starburst regions and in the early Universe³.

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1. Shu, F. H., Adams, F. C. & Lizano, S. Star formation in molecular clouds—observation and theory. *Annu. Rev. Astron. Astrophys.* **25**, 23–81 (1987).
2. Padoan, P. & Nordlund, Å. The stellar initial mass function from turbulent fragmentation. *Astrophys. J.* **576**, 870–879 (2002).

Table 2 | Computed properties of sample star-forming regions

Name	α_{vir}	ϕ_{BH}^{12}	$f_{\text{m-BH}}$	u_{esc}	$f_{\text{m-cap}}$
L1495 I	2.0	2.4	0.0022	0.28	0.0015
L1495 II	1.3	3.0	0.0026	0.28	0.0015
L1709	2.8	0.93	0.0042	0.44	0.0072
L1755	4.8	0.54	0.0017	0.40	0.0052
W44	0.39	6.4	0.0038	0.25	0.0010
W75(OH)	0.63	5.2	0.0034	0.26	0.0011
f-fil	2.4	4.1	0.0023	0.26	0.00097
N-fil	6.2	5.7	0.0001	0.11	0.00003

3. Larson, R. B. Thermal physics, cloud geometry and the stellar initial mass function. *Mon. Not. R. Astron. Soc.* **359**, 211–222 (2005).
4. Bonnell, I. A., Bate, M. R. & Zinnecker, H. On the formation of massive stars. *Mon. Not. R. Astron. Soc.* **298**, 93–102 (1998).
5. Bonnell, I. A., Bate, M. R., Clarke, C. J. & Pringle, J. E. Competitive accretion in embedded stellar clusters. *Mon. Not. R. Astron. Soc.* **323**, 785–794 (2001).
6. Bonnell, I. A., Clarke, C. J., Bate, M. R. & Pringle, J. E. Accretion in stellar clusters and the initial mass function. *Mon. Not. R. Astron. Soc.* **324**, 573–579 (2001).
7. Bonnell, I. A., Vine, S. G. & Bate, M. R. Massive star formation: nurture, not nature. *Mon. Not. R. Astron. Soc.* **349**, 735–741 (2004).
8. Bate, M. R. & Bonnell, I. A. The origin of the initial mass function and its dependence on the mean Jeans mass in molecular clouds. *Mon. Not. R. Astron. Soc.* **356**, 1201–1221 (2005).
9. Bate, M. R., Bonnell, I. A. & Bromm, V. The formation mechanism of brown dwarfs. *Mon. Not. R. Astron. Soc.* **332**, L65–L68 (2002).
10. Bate, M. R., Bonnell, I. A. & Bromm, V. The formation of a star cluster: predicting the properties of stars and brown dwarfs. *Mon. Not. R. Astron. Soc.* **339**, 577–599 (2003).
11. Mohanty, S., Jayawardhana, R. & Basri, G. The T Tauri phase down to nearly planetary masses: echelle spectra of 82 very low mass stars and brown dwarfs. *Astrophys. J.* **626**, 498–522 (2005).
12. Krumholz, M. R., McKee, C. F. & Klein, R. I. Bondi-Hoyle accretion in a turbulent medium. *Astrophys. J.* (in the press); preprint at (<http://arXiv.org/astro-ph/0510410>) (2005).
13. McKee, C. F. & Tan, J. C. The formation of massive stars from turbulent cores. *Astrophys. J.* **585**, 850–871 (2003).
14. Padoan, P., Kritsuk, A., Norman, M. L. & Nordlund, Å. A solution to the pre-main-sequence accretion problem. *Astrophys. J. Lett.* **622**, L61–L64 (2005).
15. Bertoldi, F. & McKee, C. F. Pressure-confined clumps in magnetized molecular clouds. *Astrophys. J.* **395**, 140–157 (1992).
16. Fiege, J. D. & Pudritz, R. E. Helical fields and filamentary molecular clouds—I. *Mon. Not. R. Astron. Soc.* **311**, 85–104 (2000).
17. Edgar, R. & Clarke, C. The effect of radiative feedback on Bondi-Hoyle flow around a massive star. *Mon. Not. R. Astron. Soc.* **349**, 678–686 (2004).
18. Ruffert, M. & Arnett, D. Three-dimensional hydrodynamic Bondi-Hoyle accretion. 2: Homogeneous medium at mach 3 with $\gamma = 5/3$. *Astrophys. J.* **427**, 351–376 (1994).
19. Larson, R. B. Turbulence and star formation in molecular clouds. *Mon. Not. R. Astron. Soc.* **194**, 809–826 (1981).
20. Motte, F., Andre, P. & Neri, R. The initial conditions of star formation in the ρ Ophiuchi main cloud: wide-field millimeter continuum mapping. *Astron. Astrophys.* **336**, 150–172 (1998).
21. Testi, L. & Sargent, A. I. Star formation in clusters: A survey of compact millimeter-wave sources in the Serpens core. *Astrophys. J. Lett.* **508**, L91–L94 (1998).
22. Johnstone, D., Fich, M., Mitchell, G. F. & Moriarty-Schieven, G. Large area mapping at 850 microns. III. Analysis of the clump distribution in the Orion B molecular cloud. *Astrophys. J.* **559**, 307–317 (2001).
23. Plume, R., Jaffe, D. T., Evans, N. J., Martin-Pintado, J. & Gomez-Gonzalez, J. Dense gas and star formation: Characteristics of cloud cores associated with water masers. *Astrophys. J.* **476**, 730–749 (1997).
24. Klessen, R. S. & Burkert, A. The formation of stellar clusters: gaussian cloud conditions I. *Astrophys. J. Suppl.* **128**, 287–319 (2000).
25. Klessen, R. S. & Burkert, A. The formation of stellar clusters: gaussian cloud conditions II. *Astrophys. J.* **549**, 386–401 (2001).
26. Bate, M. R., Bonnell, I. A. & Bromm, V. The formation of close binary systems by dynamical interactions and orbital decay. *Mon. Not. R. Astron. Soc.* **336**, 705–713 (2002).
27. Beuther, H. & Schilke, P. Fragmentation in massive star formation. *Science* **303**, 1167–1169 (2004).
28. Quillen, A. C. *et al.* Turbulence driven by outflow-blown cavities in the molecular cloud of NGC 1333. *Astrophys. J.* (in the press); preprint at (<http://arXiv.org/astro-ph/0503167>) (2005).
29. Cho, J. & Lazarian, A. Compressible magnetohydrodynamic turbulence: mode coupling, scaling relations, anisotropy, viscosity-damped regime and astrophysical implications. *Mon. Not. R. Astron. Soc.* **345**, 325–339 (2003).
30. Kramer, C. & Winnewisser, G. A molecular survey of the dark cloud L 1495 in Taurus. *Astron. Astrophys. Suppl.* **89**, 421–428 (1991).

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