ASTR4006/8006 2020: Galaxies Assignment 2.

Due Thursday June 4, 2020.

Please email your assignment to me (kenneth.freeman@anu.edu.au) as a single pdf

1. Johnson & Soderblom (1987: AJ 93, 864) present the formalism to derive the UVW velocity components and their errors for a star from its RA, Dec, radial velocities, proper motions and parallax (see lectures 94-96 in the Dynamics notes). Please describe each step of their process. There is no need to reproduce their mathematics, but I would like to be convinced that you understand what they have done. Explain what each variable is, and what each step in their process accomplishes.

distance = (1/parallax) is a biased estimator of the stellar distance. What does "biased estimator" mean ? Describe in a few lines what can be done to correct the bias. See Bailer-Jones et al (2018: AJ 156, 58). (10 marks)

2. The simple one-dimensional density distribution

$$\rho(z) = \rho_\circ \,\mathrm{sech}^2(z/2h_z)$$

is a self-consistent solution for a self-gravitating isothermal sheet of stars, discovered by Spitzer in 1942. It is a reasonably realistic first approximation to the vertical density distribution of a galactic disk and is widely used for analysing observations of galactic disks.

Here $\rho_\circ$ is the density of the disk at $z = 0$, and h_z, given by $h_z^2 = \sigma_z^2/(8\pi G\rho_\circ)$, is the scale height of the disk, where σ_z is the velocity dispersion of the isothermal sheet. The potential is $\Phi(z) = 8\pi G\rho_\circ h_z^2 \ln[\cosh^2(z/2h_z)]$ and the vertical force $-d\Phi/dz = -8\pi G\rho_\circ h_z \tanh(z/2h_z)$.

The main purpose of this question is to evaluate how the vertical (z) oscillation period of a star changes with its vertical amplitude, as measured by the maximum height z_{max} which the star reaches.

- Use the 1-D Jeans equation in z to show that the $\mathrm{sech}^2(z/2h_z)$ density distribution is a solution for an isothermal sheet of stars.
- Integrate orbits in the z-direction for about two full oscillations, starting with velocity $v_z = 0$ at $z = z_{max}$ for $z_{max} = [0.2, 0.3, 0.5, 1.0, 2.0, 3.0, 4.0, 5.0] * h_z$. Calculate the period (the time from z_{max} to $-z_{max}$ and back to z_{max}). I suggest you make your equation of motion dimensionless, using $G\rho_\circ$ and h_z as the dimensional quantities.
- Plot the period against z_{max}.
- How does the period scale with $\rho_\circ$?
- In a real disk, the (R, z) dependence of the density of the disk is approximately

  $$\rho(R, z) = \rho(0, 0)\exp(-R/h_R)\,\mathrm{sech}^2(z/2h_z)$$

  where h_R is the radial scale length. For an epicyclic orbit in the disk, the vertical period will then depend on the value of $\rho_\circ$ at its mean radius (guiding centre radius) and on its z-amplitude z_{max}. You have seen how the vertical period depends on z_{max} at a given R (i.e. at a given $\rho_\circ$). How would this distribution of vertical periods with z_{max} change with the mean radius of the orbit ? Sketch the expected shape of the contours of constant vertical period in the (mean radius, z_{max}) plane over the disk. (20 marks)

3. In the dynamics lectures (slides 126-127), we discussed the self-consistent solution for an isothermal sphere of stars, with the distribution function $f = f(E) = const. \exp(-E/\sigma^2)$, where E is the energy and σ is the isothermal velocity dispersion. This leads to a differential equation for the potential $\Phi(r)$ which has to be integrated numerically. Please integrate this differential equation, with the boundary conditions $\Phi = d\Phi/dR = 0$ at the center, out to a radius of 50 times the core radius r_c as given in the lecture notes. Again, write the differential equation in dimensionless form, using any two of the dimensional quantities $G\rho_\circ, \sigma$ and r_c.

Show that this system is indeed isothermal. Suggest using the spherical Jeans equation for an isotropic non-rotating system, and equations on page 126 of the lecture notes.

Calculate and plot the density as $\log\,\rho(r)$ against $\log\,(r)$. Calculate the potential gradient $d\Phi/dr$ and plot the rotation curve $V_c(r)$ in this potential. Comment on what you see in the plots. (20 marks)

4. Give a brief account ($\sim$ 500 words) of the origin and content of the circumgalactic medium of spiral galaxies, and how it may contribute to (i) the baryon budget and (ii) the ongoing star formation in these galaxies. (15 marks).

5. Summarize what you know ($\sim$ 500 words) about the various kinds of galactic rings, including current ideas about how they formed. (10 marks)

6. In a total of about 1000 words, describe:

- the different techniques used to measure the properties of the dark halos of galaxies.
- the scaling laws for the dark halos of spiral galaxies and dwarf galaxies.
- why the central densities of the dark halos in dwarf galaxies are so much higher than the central densities of giant spirals.
- why the dark halos of galaxies over a wide range of brightnesses appear to have a similar central surface density.
- what are the favourite current possibilities for the nature of the dark matter.

(20 marks)