ASTR4006/8006 2020: Galaxies Assignment 1.

Due Monday April 20, 2020. Please email it to me (kenneth.freeman@anu.edu.au) as a single pdf or put hard copy in my mailbox at Stromlo.

1. Give a simple order of magnitude argument to show why the relaxation time for 2-body encounters in a stellar system $T_R \sim \sigma^3/m^2 n$, where σ is the velocity dispersion, m the typical stellar mass and n the number density of stars. (10 marks)

2.. The spherical isochrone potential is defined by

$$\Phi(r) = -\frac{GM}{b + \sqrt{(b^2 + r^2)}}$$

where M is the mass and b is a length.

- Show that the density distribution is

$$\rho(r) = M \frac{3(b+a)a^2 - r^2(b+3a)}{4\pi(b+a)^3 a^3}$$

  where $a^2 = (b^2 + r^2)$. What is the form of $\rho(r)$ for $r \ll b$ and for $r \gg b$?

- Show that the circular velocity is

$$V_c^2(r) = \frac{GMr^2}{(b+a)^2 a}.$$

  What can you say about $V_c(r)$ for $r \ll b$ and for $r \gg b$?

- Integrate an orbit in this potential. Write your equations of motion in dimensionless form, so the length unit is b, the velocity unit is $\sqrt{(GM/b)}$ and the time unit is $\sqrt{(b^3/GM)}$. Start the orbit at $r = b$ with a radial velocity of $0.20\sqrt{(GM/b)}$ and a tangential velocity of $0.25\sqrt{(GM/b)}$.
- Estimate its apo- and perigalactic radii analytically from the equation for its energy and angular momentum.
- Integrate the orbit until it reaches its third apogalacticon.
- Plot the orbit in its plane.
- Plot r(t) and estimate the radial period.
- Estimate the apo- and perigalactic radii from your integration. Does it agree with the analytic estimate ?
- Estimate the azimuthal distance between apogalactica.
- Now integrate another orbit, again starting at $r = b$. This time, change the initial conditions so that the tangential velocity is $0.30\sqrt{(GM/b)}$ but the energy is the same as for the first orbit. Integrate this new orbit until it reaches its second apogalacticon and estimate the radial period. What do you find ? Why do you think this potential is called the isochrone potential ? (25 marks)

3. In a galaxy with a potential

$$\Phi(R) = V^2 \ln(R)$$

with $V = 200$ km s^{-1}, consider a 2-dimensional star cluster lying in the galactic plane, with the cluster centre at $R = 10$ kpc and all stars randomly (uniformly) distributed within a circle of radius 500 pc. The cluster centre is moving in a circular orbit around the galactic centre, with the circular velocity V. Relative to the centre of the cluster, the stars have a gaussian distribution of radial (R) and tangential (ϕ) velocities, with a mean of zero and a dispersion of 5 km s^{-1} in each direction.

- Integrate the orbits of 100 stars for 10 galactic rotation periods at the centre of the cluster, assuming that the cluster stars are test particles in the gravitational field of the galaxy.
- plot the positions of the stars on the galactic plane.

What has happened to the star cluster ? What difference would it make if the stars initially had zero random velocity relative to the cluster centre ? (20 marks)

4. In a spherical galaxy with potential

$$\Phi(r) = V^2 \ln(r),$$

the *infall time* is the time taken for a particle initially at rest at radius r to fall in to the center of the galaxy. What is the ratio of the infall time to the rotation period at radius r ? (Hint: you can do this analytically). (10 marks).

5. This question is to be done analytically. A spherical galaxy has a density distribution

$$\rho(r) = \rho_o[1 + r^2/a^2]^{-1}.$$

- Show that the enclosed mass $M(r) \propto r^3$ for $r \ll a$ (*i.e.* in the core) and $M(r) \propto r$ for $r \gg a$ (*i.e.* in the outer halo).
- Now take a population of massless test particles in the potential of this galaxy. Assume that this population is spherical, non-rotating, isothermal and isotropic, with velocity dispersion σ in each velocity component. What is the radial density distribution of this test particle population ? (Hint: use the spherical Jeans' equation)
- At large r, this density distribution simplifies and its form depends on a dimensionless number. Give a physical interpretation of this dimensionless number. What is the condition that the density distributions of the test particle population and the galaxy itself have similar forms at large r ? (15 marks)

6. This is an essay question ($\sim$ 500 words) on the angular momentum (J/M) - mass (M) relation for galaxies. Please discuss:

- the possible causes of the relation
- the possible reasons for the difference between spirals and ellipticals

- how the J/M - M relation relates to the baryonic Tully-Fisher relation for spirals
- observational issues in measuring the J/M - M relation.

For an idealised galaxy with an exponential disk of stars and a flat rotation curve, work out the radial (R) distribution of the angular momentum per unit mass

$$h(R) = 2\pi R \times \text{surface density}(R) \times \text{circular velocity}(R) \times R$$

Around what radius does most of the angular momentum of the stellar disk reside ?
(20 marks)