The Physics of Radiation Hydrodynamics: A brief Introduction

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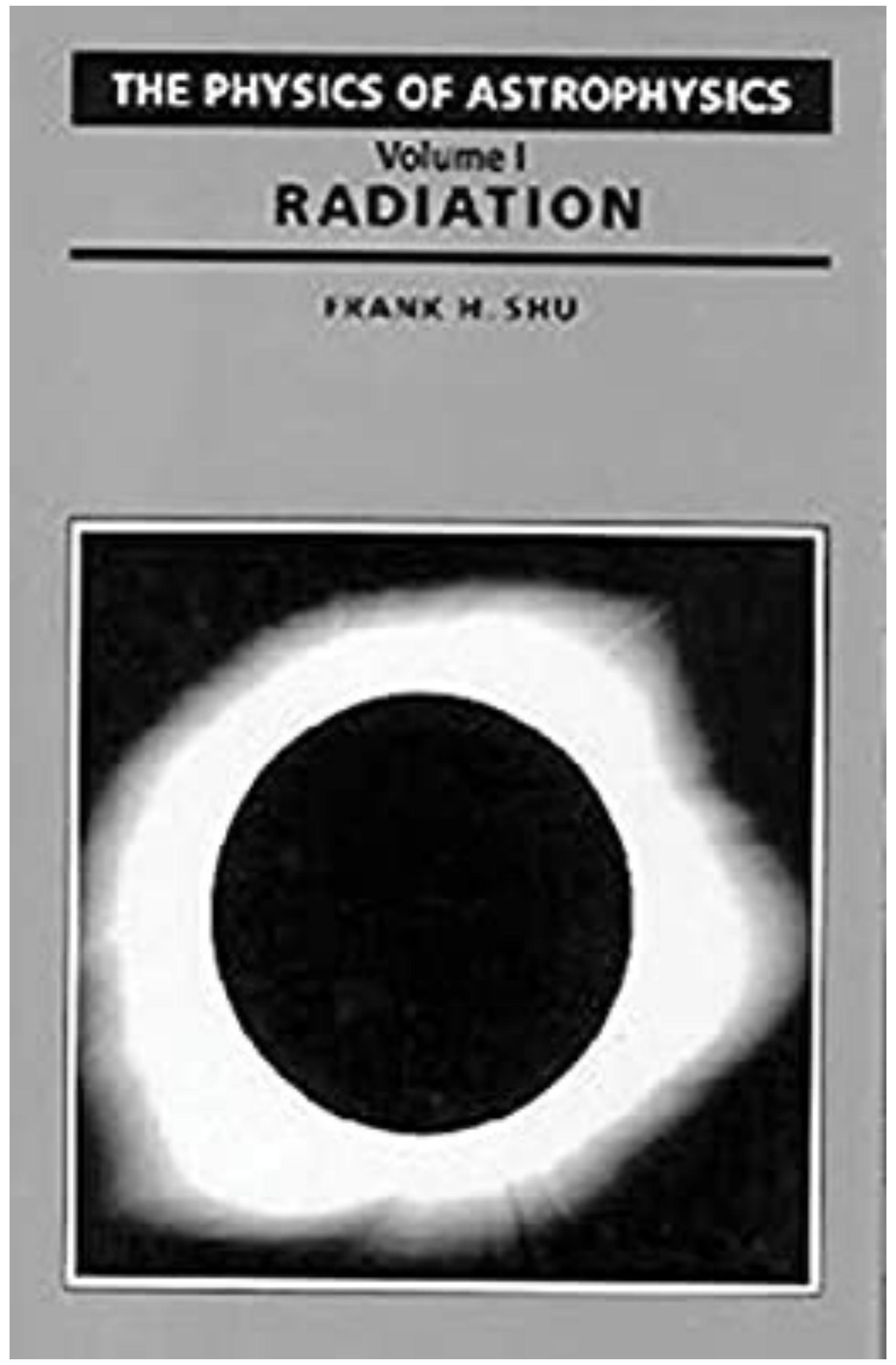
Outline

- Introduction: Radiation Hydrodynamics — What and Why?
- Some useful definitions and concepts
- Moving to the Equations of Radiation Hydrodynamics and challenges in solving them
- Astrophysical Phenomena where RHD is crucial

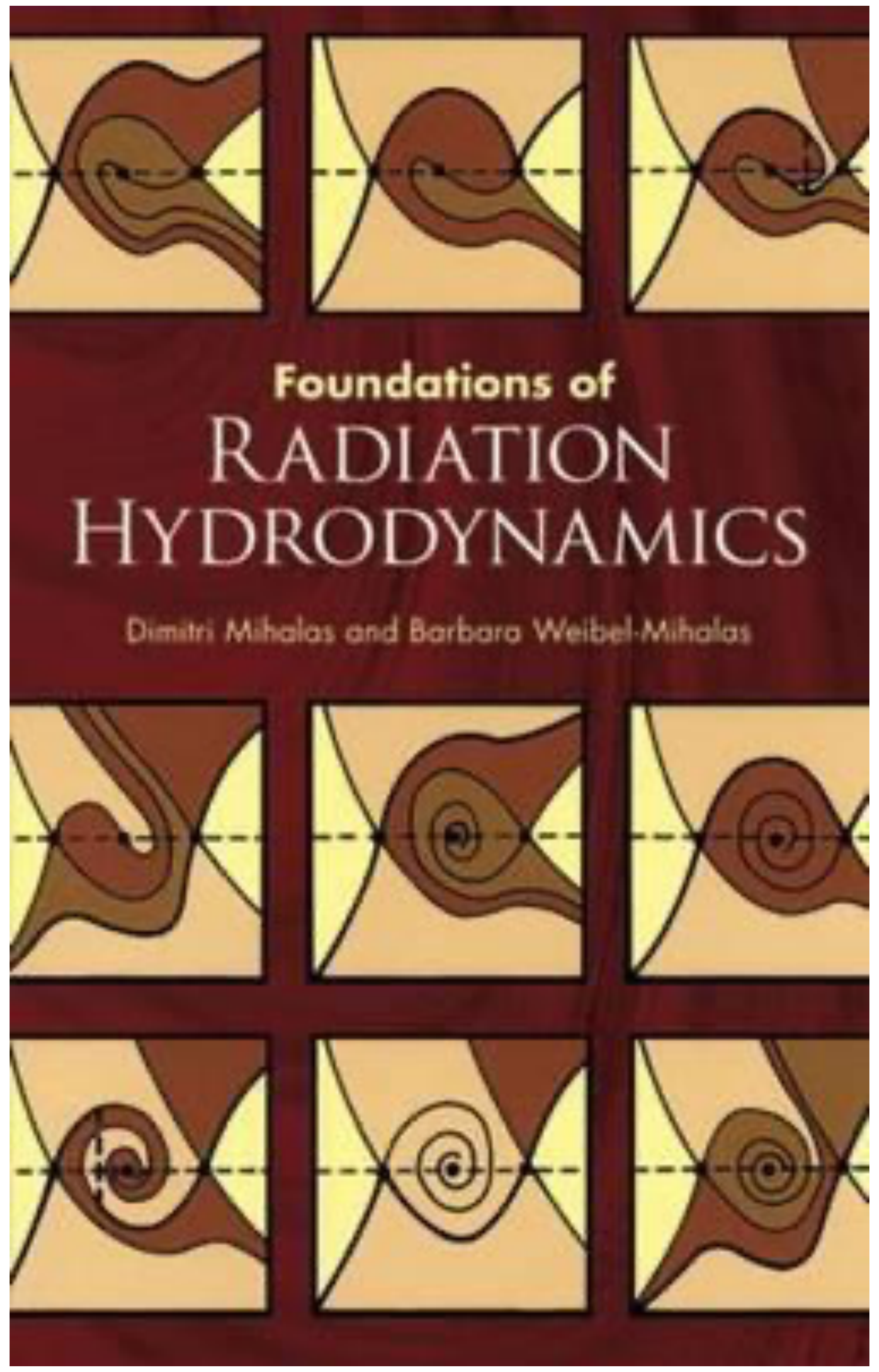
Introduction

The What and Why of Radiation Hydrodynamics

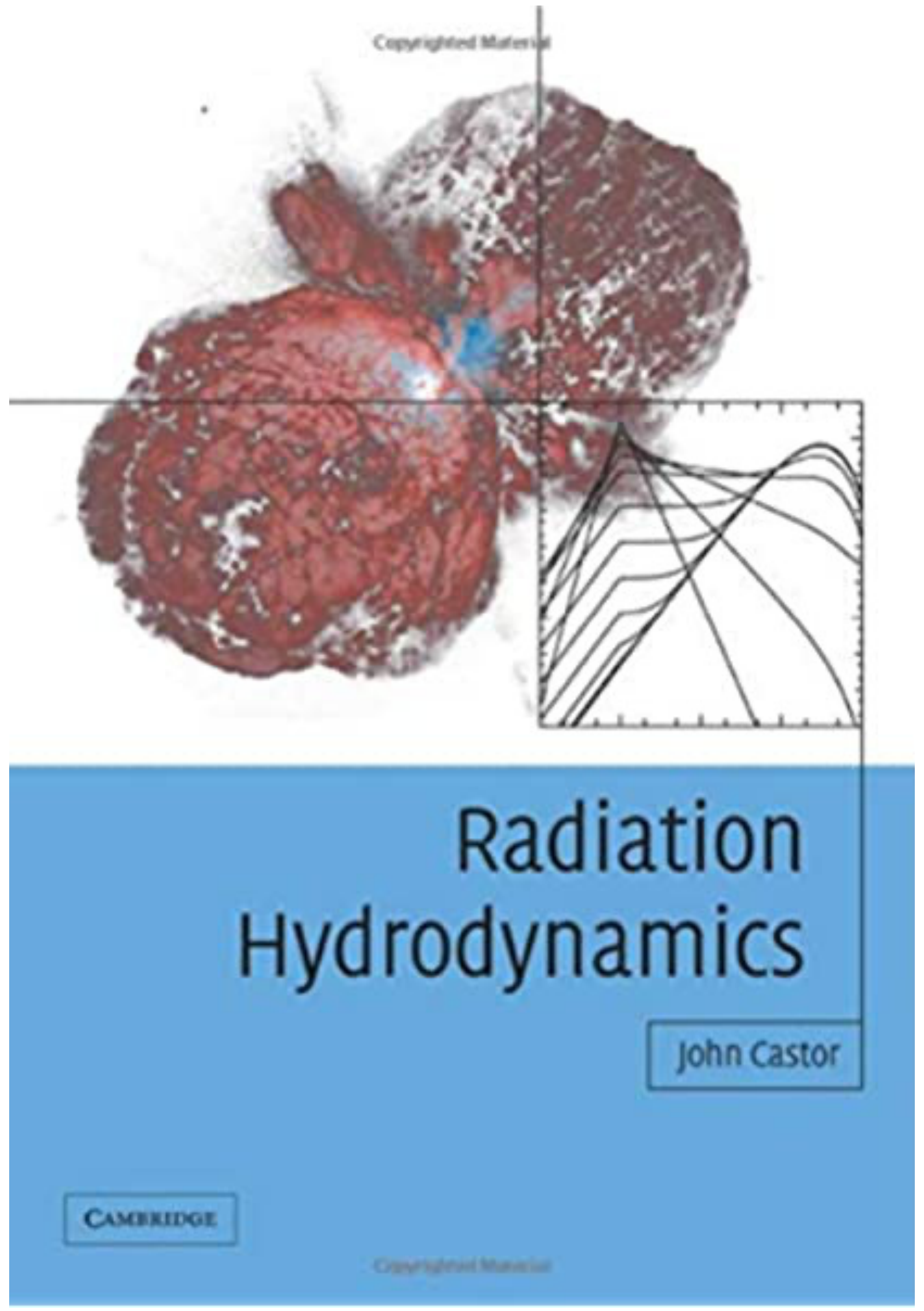
References for Further Exploration



Fundamentals of Radiation.



Bible for Radiation Hydrodynamics



“My” bible for Radiation Hydrodynamics

Youtube Lectures:

- Jim Stone — Solving RHD
- Stan Owocki — RHD in Stellar Winds

Recap: Gas Dynamics

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathsf{v}) = \ldots$$

Mass Conservation

$$\frac{\partial (\rho \mathsf{v})}{\partial t} + \nabla \cdot (\rho \mathsf{v}\mathsf{v}) = \ldots$$

Momentum Conservation

$$\frac{\partial (\rho e)}{\partial t} + \nabla \cdot [(\rho e + P)\mathsf{v}] = \ldots$$

Energy Conservation

Source/sinks of momentum and energy can be: Magnetic fields, Gravity, Chemical or Nuclear reactions etc.

What is Radiation Hydrodynamics

- The study of systems where the source/sink of energy and momentum can be due to radiation interacting with matter.

- Energy of a photon, $E = \frac{hc}{\lambda}$, Momentum of photon p such that, $E = pc$ (massless particle).

  Radiation -> population of photons (like gas is a population of particles).

- Physics involved: i) How do photons travel? — Radiative Transfer ii) How does matter capture E and p from the radiation/photons? — Atomic physics, Quantum Mechanics, Solid state Physics etc, iii) What is the effect on gas motions? — Hydrodynamics, Numerical Methods etc.

- Radiation everywhere in astro though……

Cases where RHD can be ignored or is easy to treat

"Most important part of doing physics is the knowledge of approximation"
- Lev Landau

Streaming Limit

Radiation streams through freely; momentum and energy coupling negligible

E.g.: Interplanetary medium

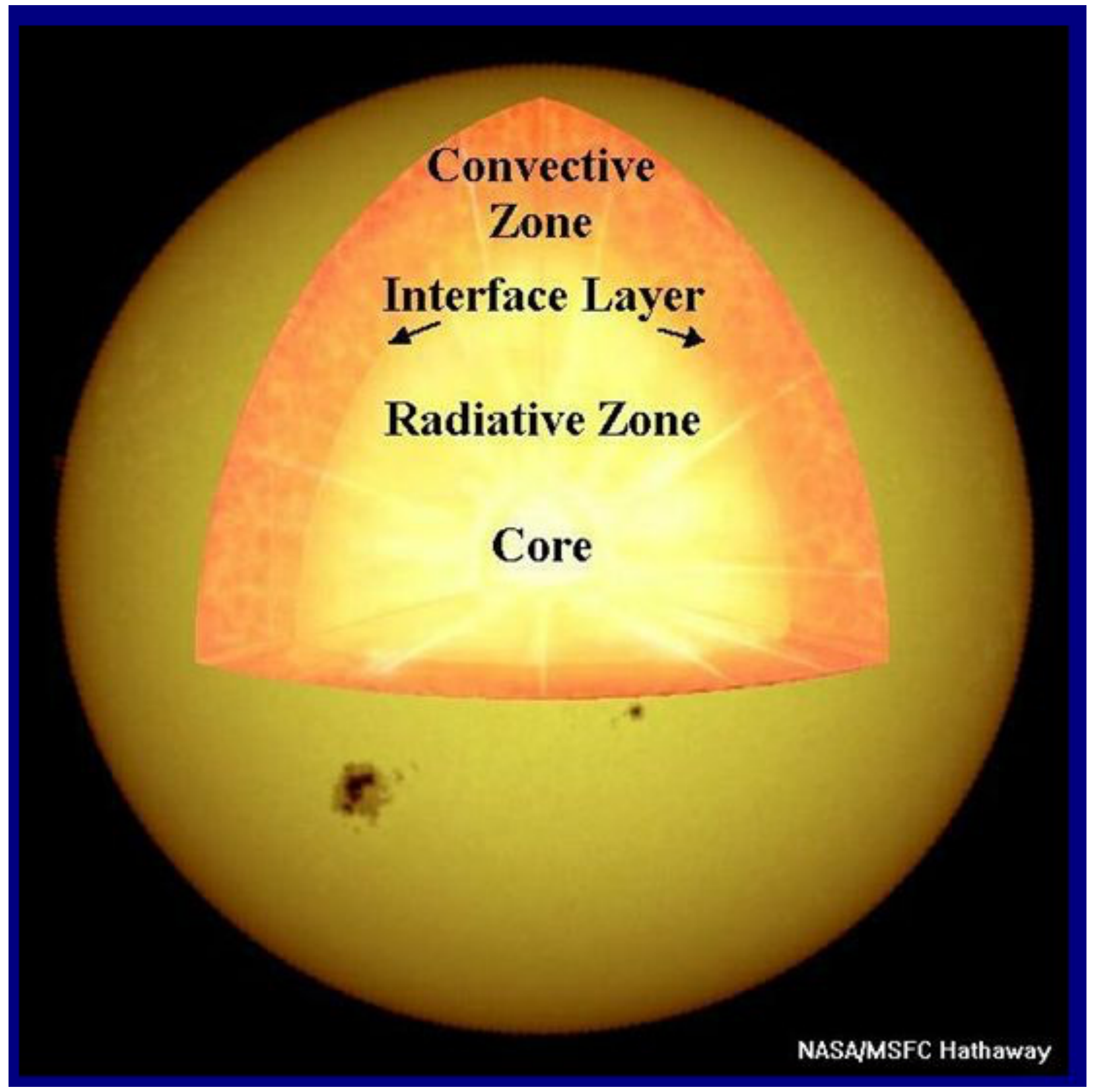
$$n_e \sim 5\,\mathrm{cm}^{-3}$$

$$\lambda_{\rm ph} \sim \frac{1}{\sigma_{\rm Th} n_e}$$

$$\sim 2 \times 10^{23}\,\mathrm{cm} \sim 10^{10}\,\mathrm{AU}$$

Extremely opaque, static limit

Material extremely opaque; radiation essentially static/instantaneous and in equilibrium with matter locally.



E.g.: Stellar ***Interior***

$$\lambda_{\rm ph} \sim 1\,\mathrm{cm}$$

$$t_{\rm diff} \sim 10\,\mathrm{Myr}$$

The Physics of RHD

The concepts and syntax underlying the description of radiating systems

Radiative Transfer I: The Radiation Intensity

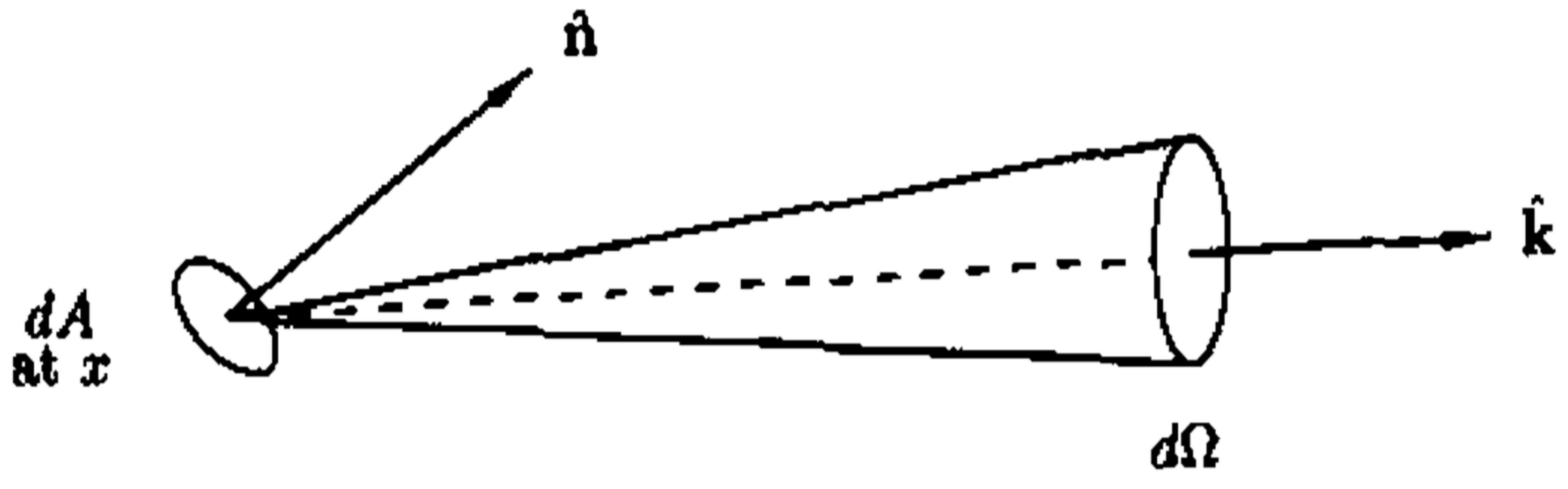
- Radiation Intensity I_ν: Energy per unit area, per unit solid angle, per unit frequency, per unit time carried by a ray moving in a direction $\hat{\mathbf{k}}$.

- 7-dimensional function: $\hat{\mathbf{k}}, \mathbf{x}, t$

- Represents occupation number/distribution of photons in energy/frequency space.

- Constant along a line-of-sight in a vacuum: Thus purely property of radiating source.

Definition of Radiation Intensity (Shu 1991)



$$\frac{1}{c}\frac{\partial I_\nu}{\partial t} + \hat{\mathbf{k}} \cdot \nabla I_\nu = 0$$

Radiative Transfer II: Radiation Energy, Flux, Pressure

Radiation Energy Density

$$E_\nu = \frac{1}{c}\int_{4\pi} I_\nu \, d\Omega$$

Radiation Flux

$$\mathbf{F}_\nu = \int_{4\pi} \hat{\mathbf{k}} \, I_\nu \, d\Omega$$

Radiation Pressure Tensor

$$\mathbf{P}_\nu^{ij} = \frac{1}{c}\int_{4\pi} \hat{\mathbf{k}}_\mathbf{i}\hat{\mathbf{k}}_\mathbf{j} \, I_\nu \, d\Omega$$

$$\frac{1}{c}\frac{\partial I_\nu}{\partial t} + \hat{\mathbf{k}} \cdot \nabla I_\nu = 0$$

Integrate over all angles →

$$\frac{\partial E_\nu}{\partial t} + \nabla \cdot \mathbf{F}_\nu = 0$$

→ Zeroth Moment Equation (Radiation Energy Conservation)

$$\frac{1}{c}\frac{\partial \mathbf{F}_\nu}{\partial t} + c\nabla \cdot \mathbf{P}_\nu^{ij} = 0$$

→ First Moment Equation (Radiation Momentum Conservation)

Radiative Transfer II: Interactions with Matter—Absorption, Emission and Scattering

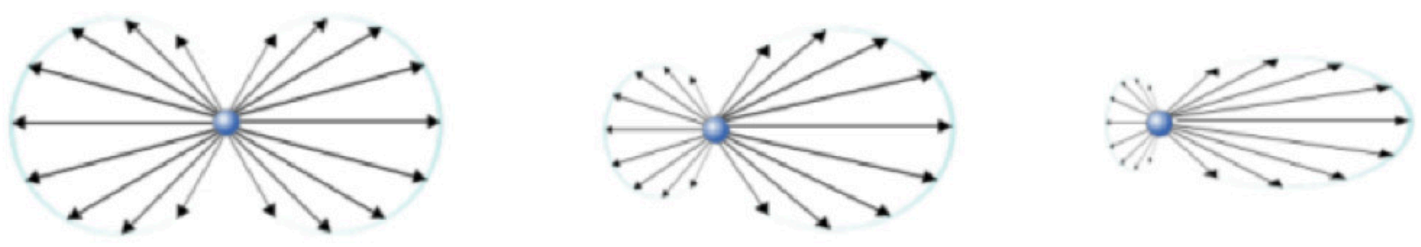
1. **Absorption** : $$\frac{dI_\nu(\hat{\mathbf{k}},x)}{dx} = -\rho\kappa_\nu(x) I_\nu(\hat{\mathbf{k}},x)$$

2. **Emission:** $$\frac{dI_\nu(\hat{\mathbf{k}},x)}{dx} = \rho j_\nu$$

3. **Scattering:**

$$\frac{dI_\nu(\hat{\mathbf{k}},x)}{dx} = -\rho\kappa_\nu^{\rm sca} I_\nu + \rho\kappa_\nu^{\rm sca}\oint \phi_\nu(\hat{\mathbf{k}},\hat{\mathbf{k}}')\, I_\nu(\hat{\mathbf{k}}')d\Omega'$$

Rayleigh and Mie scattering phase function



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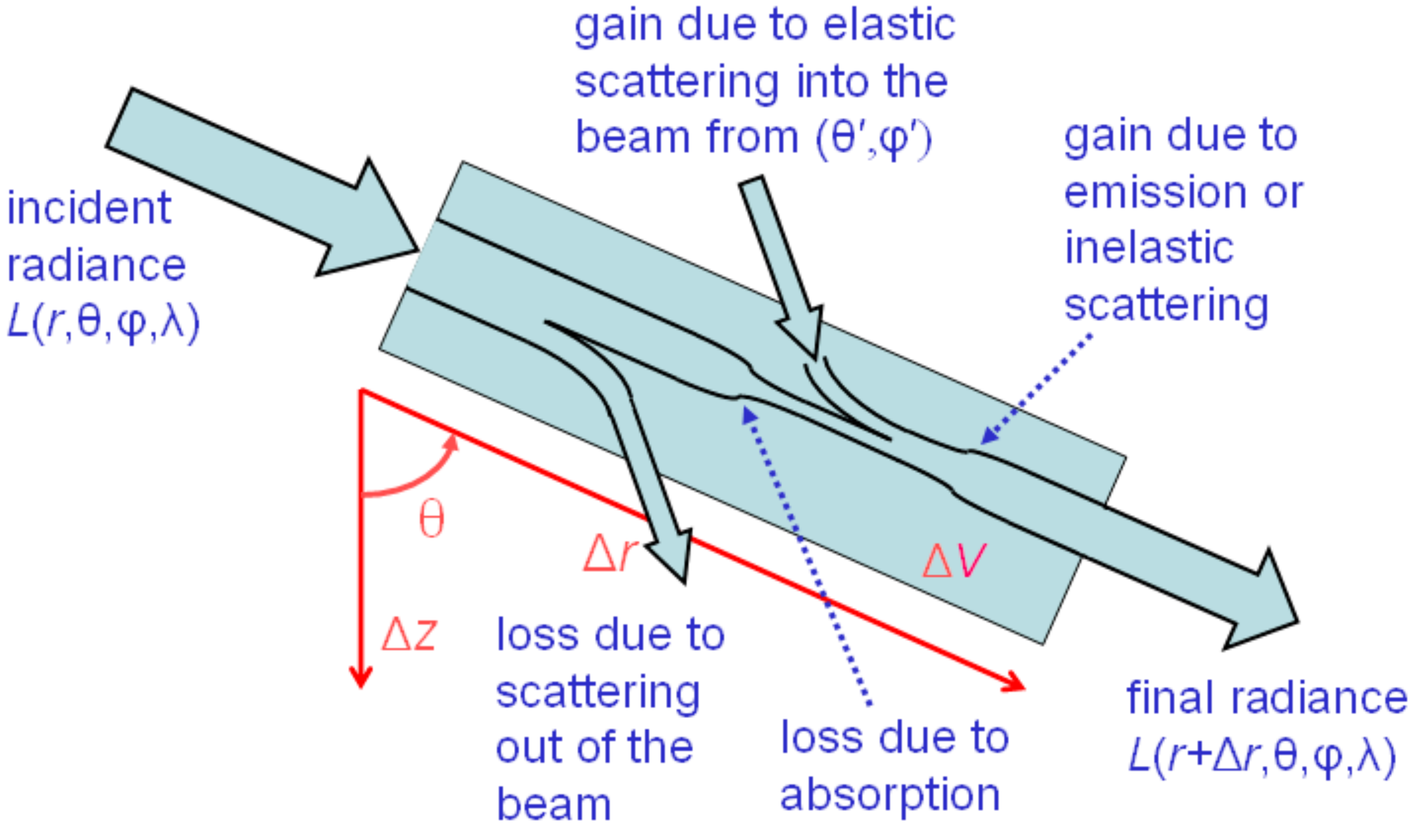


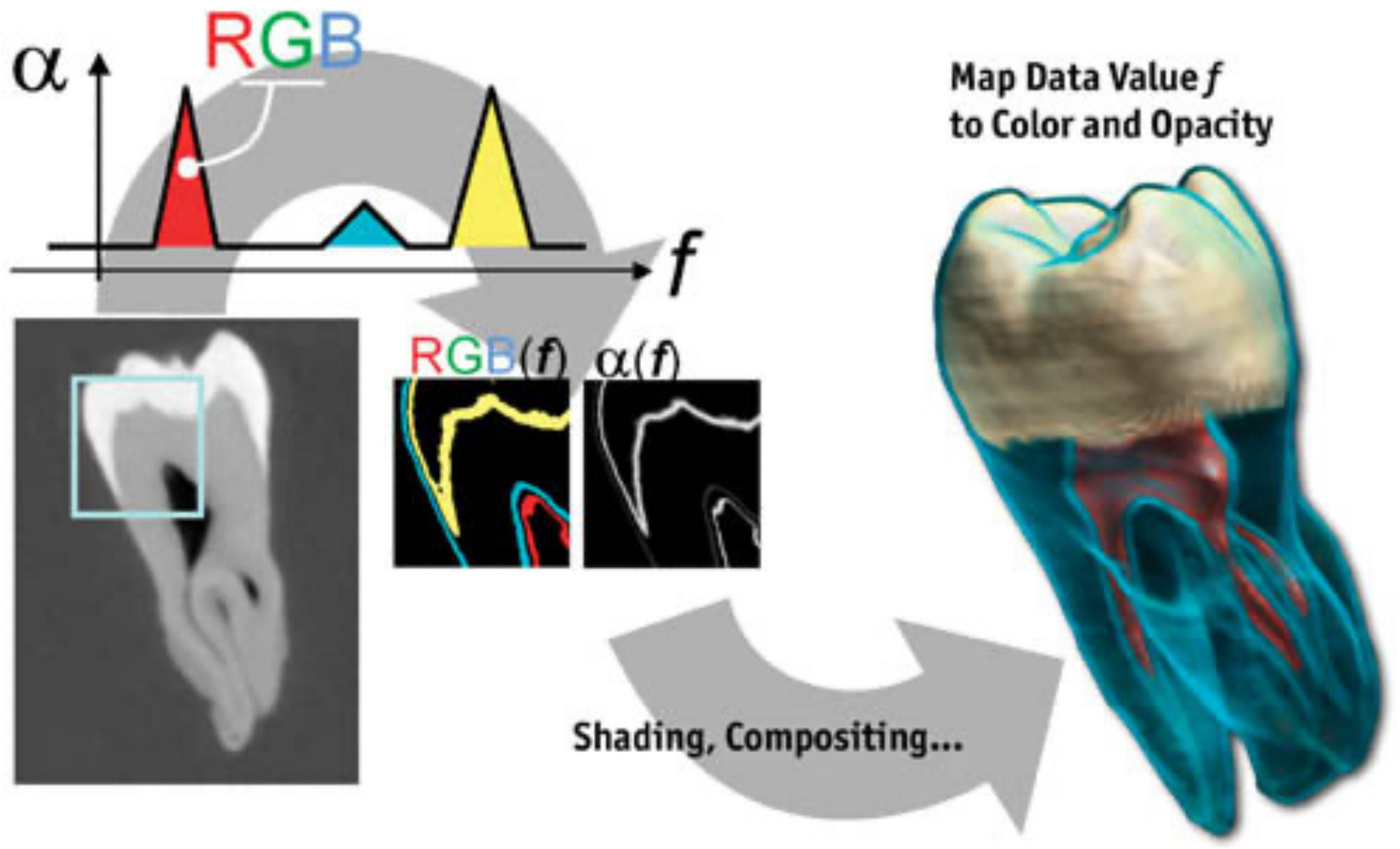
Heuristic picture of radiation-matter interaction

Formal Equation of Radiative Transfer

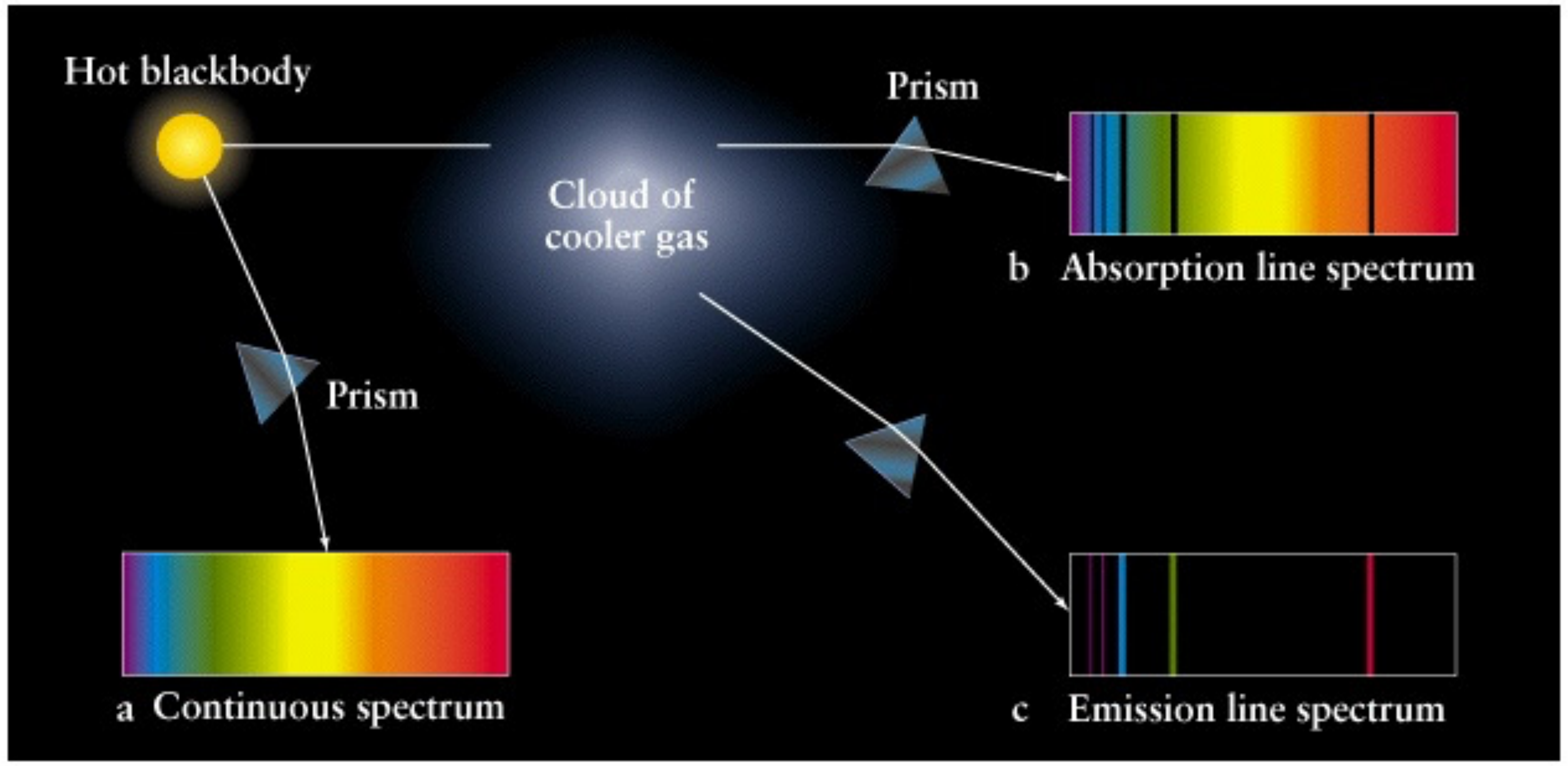
$$\frac{1}{c}\frac{\partial I_\nu}{\partial t} + \hat{\mathbf{k}} \cdot \nabla I_\nu = \frac{1}{4\pi}\rho j_\nu - \rho(\kappa_\nu^{abs} + \kappa_\nu^{sca}) I_\nu + \rho\kappa_\nu^{sca} \oint \phi_\nu(\hat{\mathbf{k}}, \hat{\mathbf{k}}') \, I_\nu(\hat{\mathbf{k}}') d\Omega'$$

If timescale over which system studied/dynamical timescales >> radiation transport timescale

— time independent version can be used, i.e. $\frac{\partial I_\nu}{\partial t} = 0$



Reconstructing the nature of intervening matter with radiation



Similar idea in Astrophysics

Moving to the Radiation Hydrodynamic Equations

Combining radiative transfer with the fundamental axiomatic laws of energy and momentum conservation

Mass Conservation $$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = \ldots$$

Momentum Conservation $$\frac{\partial (\rho \mathbf{v})}{\partial t} + \nabla \cdot (\rho \mathbf{v}\mathbf{v}) = \ldots$$

Energy Conservation $$\frac{\partial (\rho e)}{\partial t} + \nabla \cdot [(\rho e + P)\mathbf{v}] = \ldots$$

Radiative Transfer $$\frac{1}{c}\frac{\partial I_\nu}{\partial t} + \hat{\mathbf{k}} \cdot \nabla I_\nu = \underbrace{\frac{1}{4\pi}\rho j_\nu - \rho(\kappa_\nu^{abs} + \kappa_\nu^{sca}) I_\nu + \rho \kappa_\nu^{sca} \oint}_{I_{\nu,\mathrm{coupling}}} \phi_\nu(\hat{\mathbf{k}}, \hat{\mathbf{k}}')\, I_\nu(\hat{\mathbf{k}}') d\Omega'$$

Net Energy gained from matter per volume, per time, per frequency, per solid angle

$$\frac{1}{c}\frac{\partial I_\nu}{\partial t} + \hat{\mathbf{k}} \cdot \nabla I_\nu = I_{\nu,\mathrm{coupling}}$$

Energy lost by matter: $g_0 = \int_\nu d\nu \int_{4\pi} d\Omega\, I_{\nu,\mathrm{coupling}}$

Momentum lost by matter: $\mathbf{g} = \frac{1}{c}\int_\nu d\nu \int_{4\pi} d\Omega\, \hat{\mathbf{k}} I_{\nu,\mathrm{coupling}}$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = \ldots$$

$$\frac{\partial (\rho \mathbf{v})}{\partial t} + \nabla \cdot (\rho \mathbf{v}\mathbf{v}) = -\,\mathbf{g}$$

$$\frac{\partial (\rho e)}{\partial t} + \nabla \cdot [(\rho e + P)\mathbf{v}] = -\,g_0$$

$\mathbf{g}$ and g_0 can be positive (matter -> radiation) or negative (radiation->matter)

Radiation Moment Formulation of Radiation Hydrodynamics

$$E_\nu = \frac{1}{c}\int_{4\pi} I_\nu \, d\Omega$$

$$\mathbf{F}_\nu = \int_{4\pi} \hat{\mathbf{k}} \, I_\nu \, d\Omega$$

$$\mathbf{P}_\nu^{ij} = \frac{1}{c}\int_{4\pi} \hat{\mathbf{k}}_\mathbf{i}\hat{\mathbf{k}}_\mathbf{j} \, I_\nu \, d\Omega$$

Without matter interactions

$$\frac{\partial E_\nu}{\partial t} + \nabla \cdot \mathbf{F}_\nu = 0$$

$$\frac{1}{c}\frac{\partial \mathbf{F}_\nu}{\partial t} + c\nabla \cdot \mathbf{P}_\nu^{ij} = 0$$

With matter interactions

$$\frac{\partial E_\nu}{\partial t} + \nabla \cdot \mathbf{F}_\nu = g_0$$

$$\frac{1}{c}\frac{\partial \mathbf{F}_\nu}{\partial t} + c\nabla \cdot \mathbf{P}_\nu^{ij} = \mathbf{g}$$

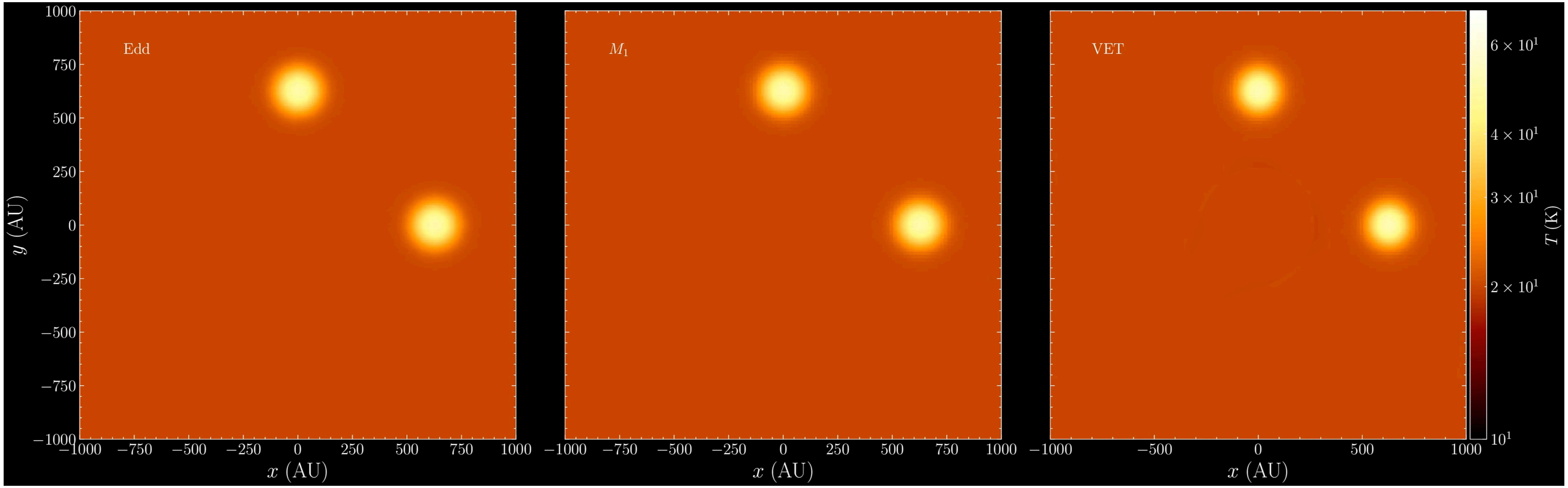
This is the most common approach in RHD: alleviates having to solve the RT equation along rays.

However, this introduces more unknowns than equations, and an extra "closure" equation is required

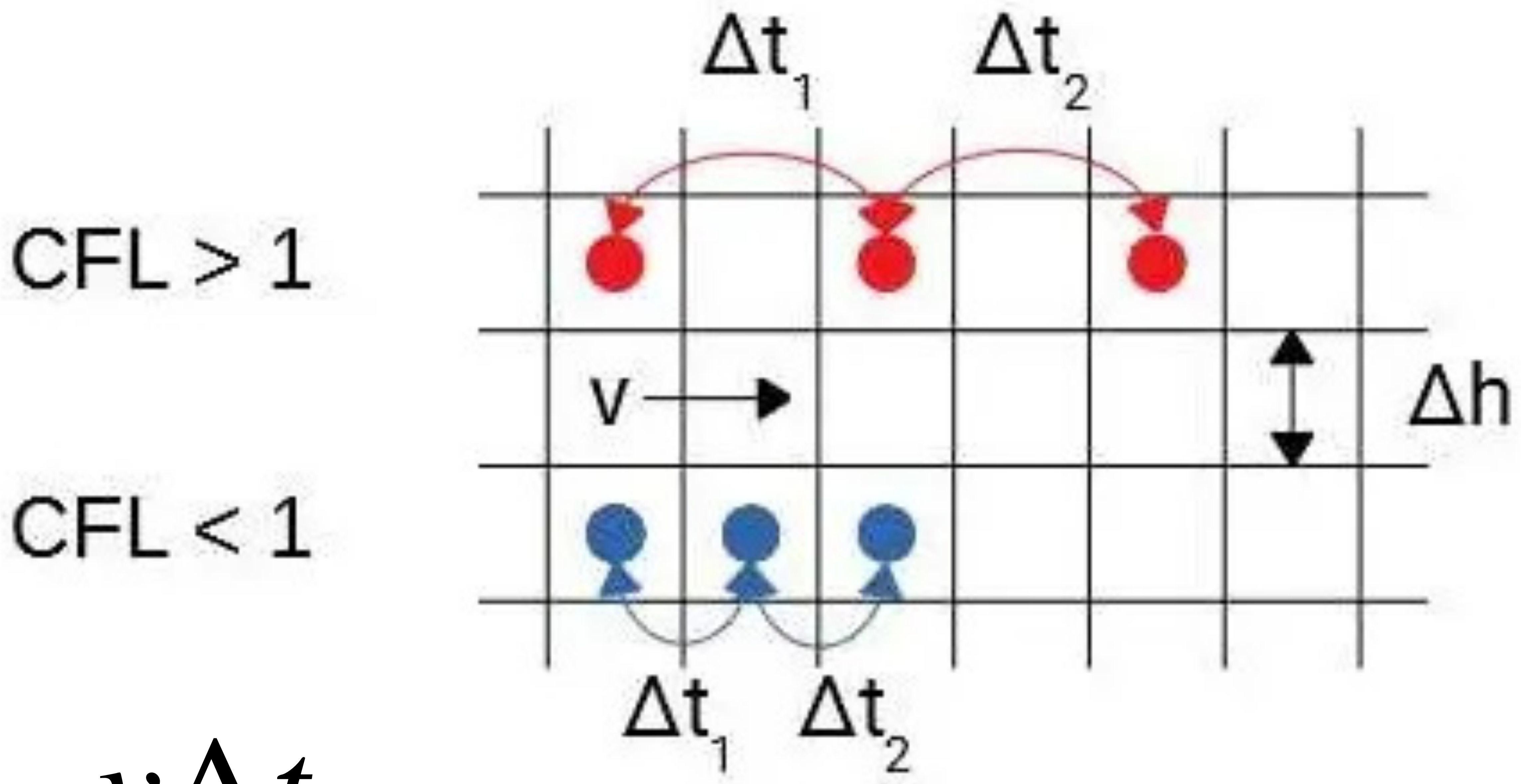
Practical Issue 1: Light travels along rays, and can be anisotropic: Capturing that effect is important

$$\mathbf{P}_\nu^{ij} = \mathbf{T}_\nu^{ij} E_\nu$$ → **Closure relation**

The usual approach is to assume an estimate for the closure making some assumptions regarding the nature of the radiation field, which in many instances is not satisfied.



Practical Issue 2: Light is (usually) *much* Faster than Gas



$$\frac{v\Delta t}{\Delta x} \leq 1$$ For stable hydrodynamics

$$\frac{c\Delta t}{\Delta x} \leq 1$$ For stable RHD

Renders explicit schemes very computationally expensive

$$\frac{\Delta t_{\mathrm{RHD}}}{\Delta t_{\mathrm{HD}}} = v/c$$

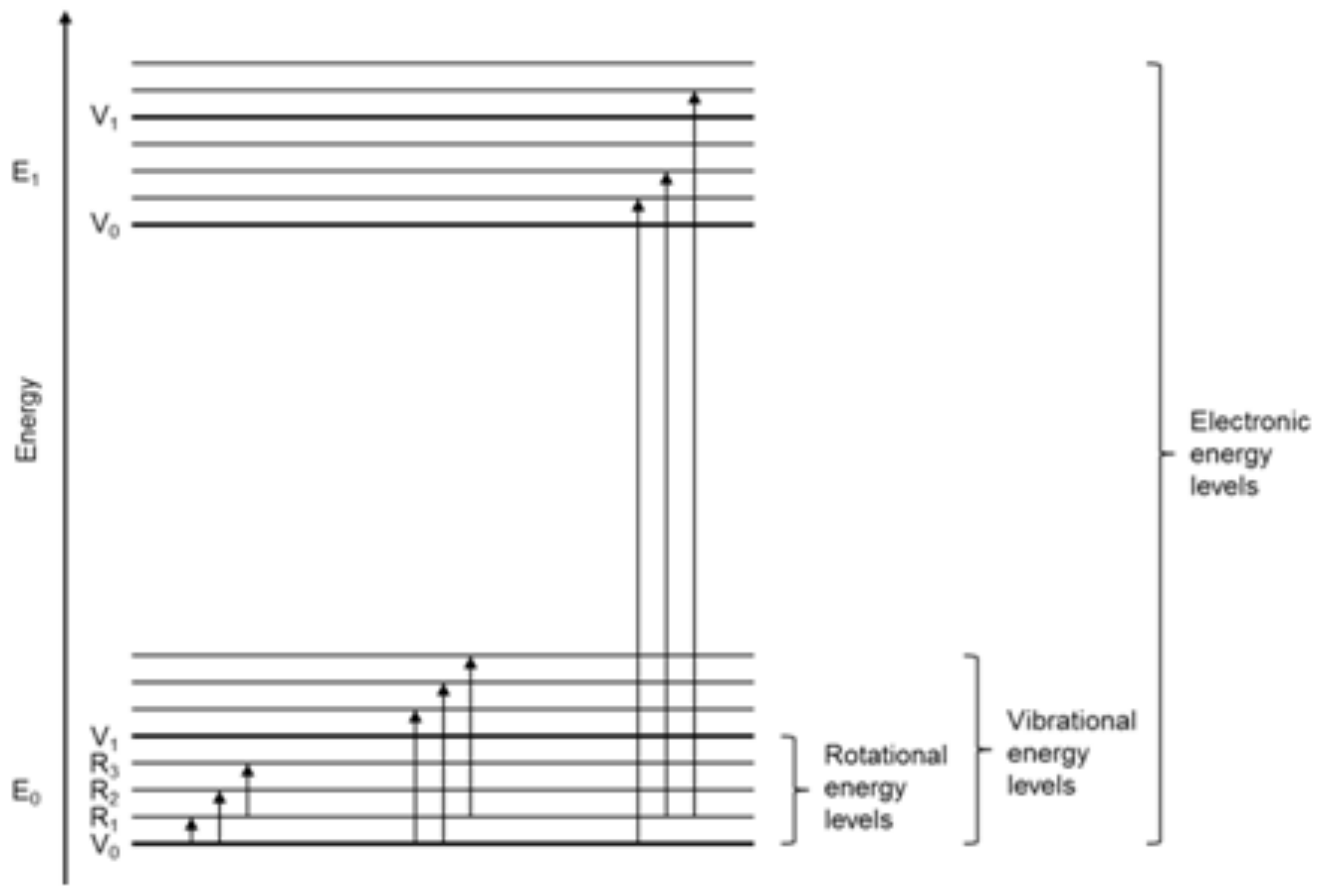
Alternatives:

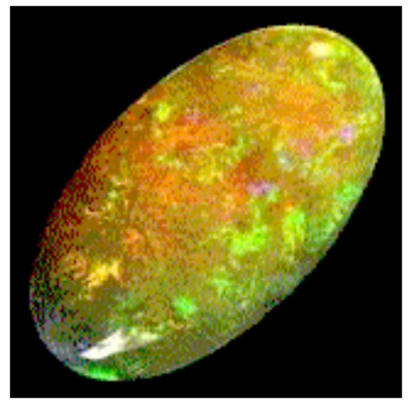
1. Reduced speed of light: Permits explicit updates; Sometimes changes the physics of the problem

2. Implicit Methods: Physics captured correctly; requires matrix inversions and complex numerical methods

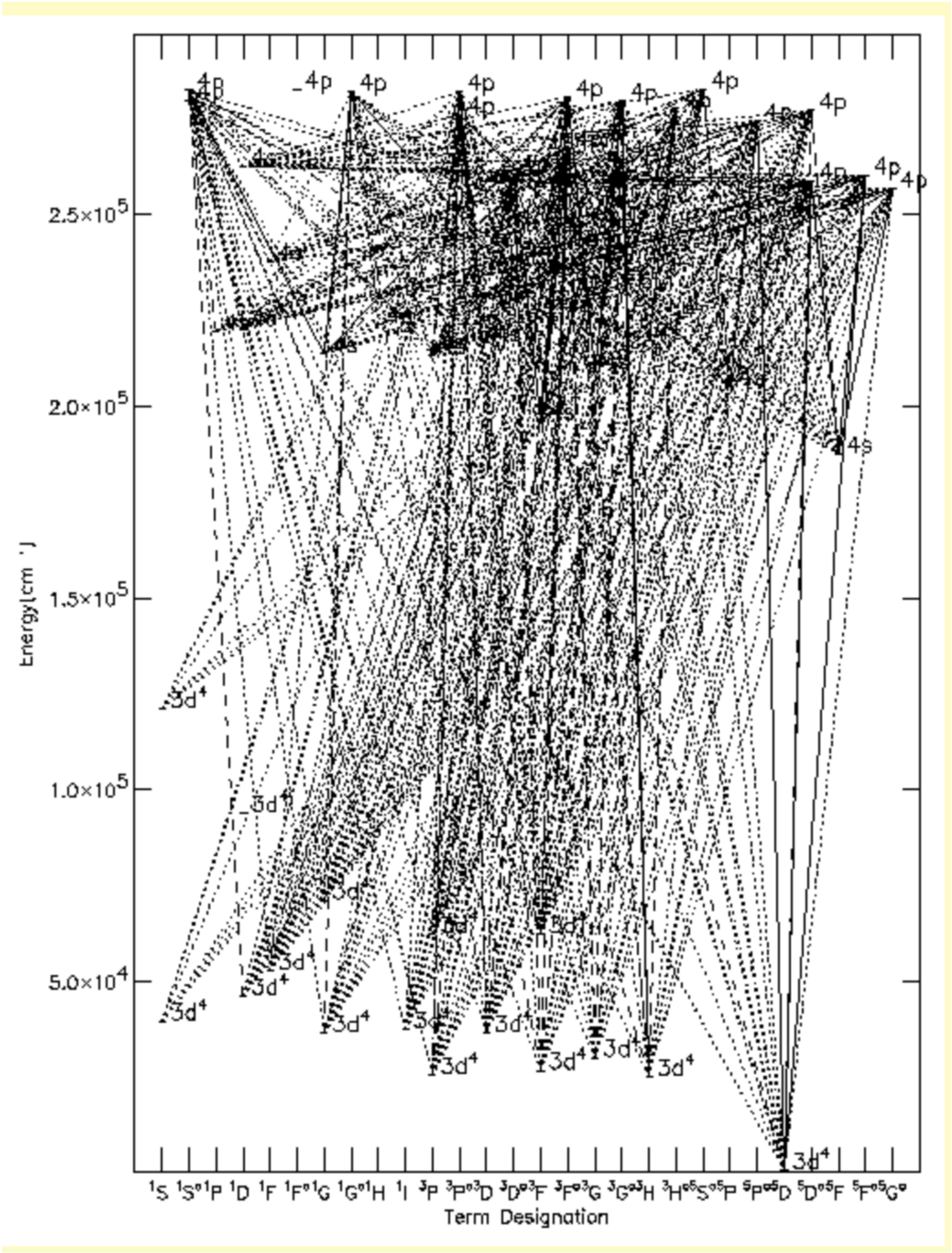
Practical Issue 3: Matter-Radiation Interactions are insanely difficult to model self-consistently

In reality we don't know $j_\nu, \kappa_\nu^{\rm abs}, \kappa_\nu^{\rm sca}$ for a distribution of gas : depends on level populations, which depend on I_ν , ρ and T.





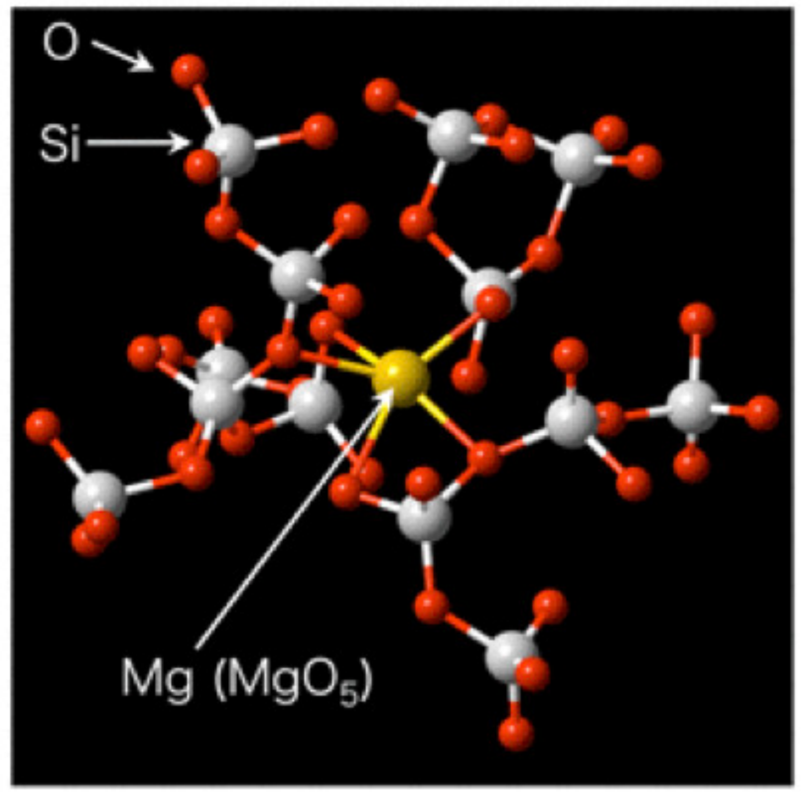
OPAL Opacity Project

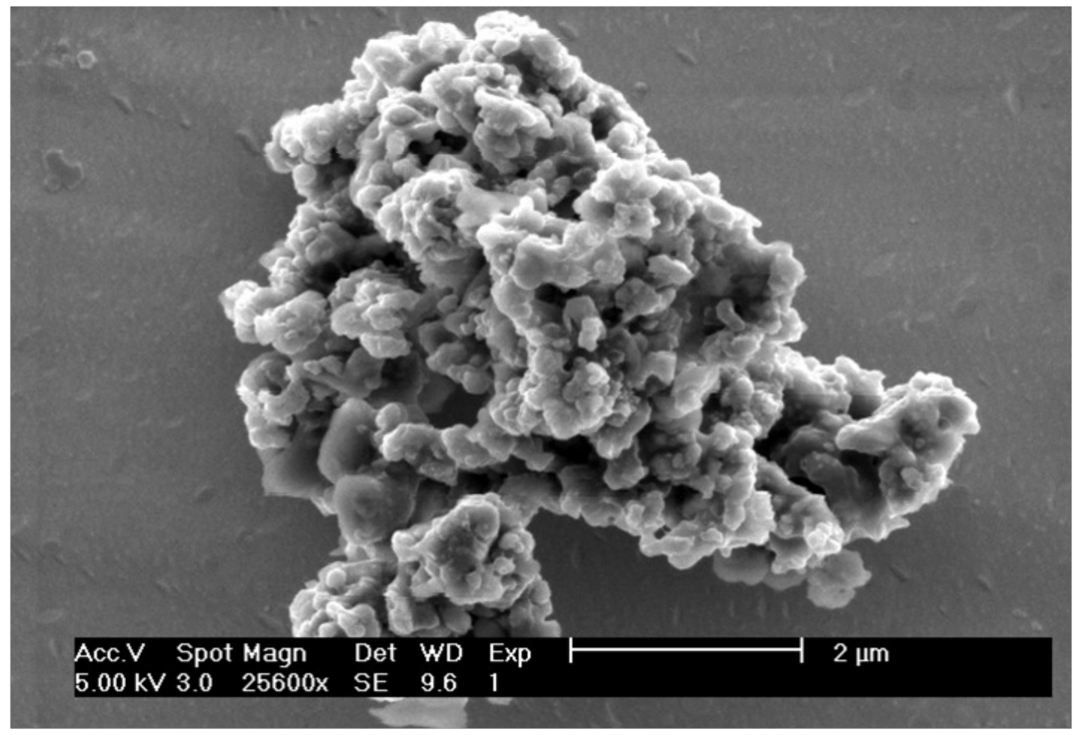


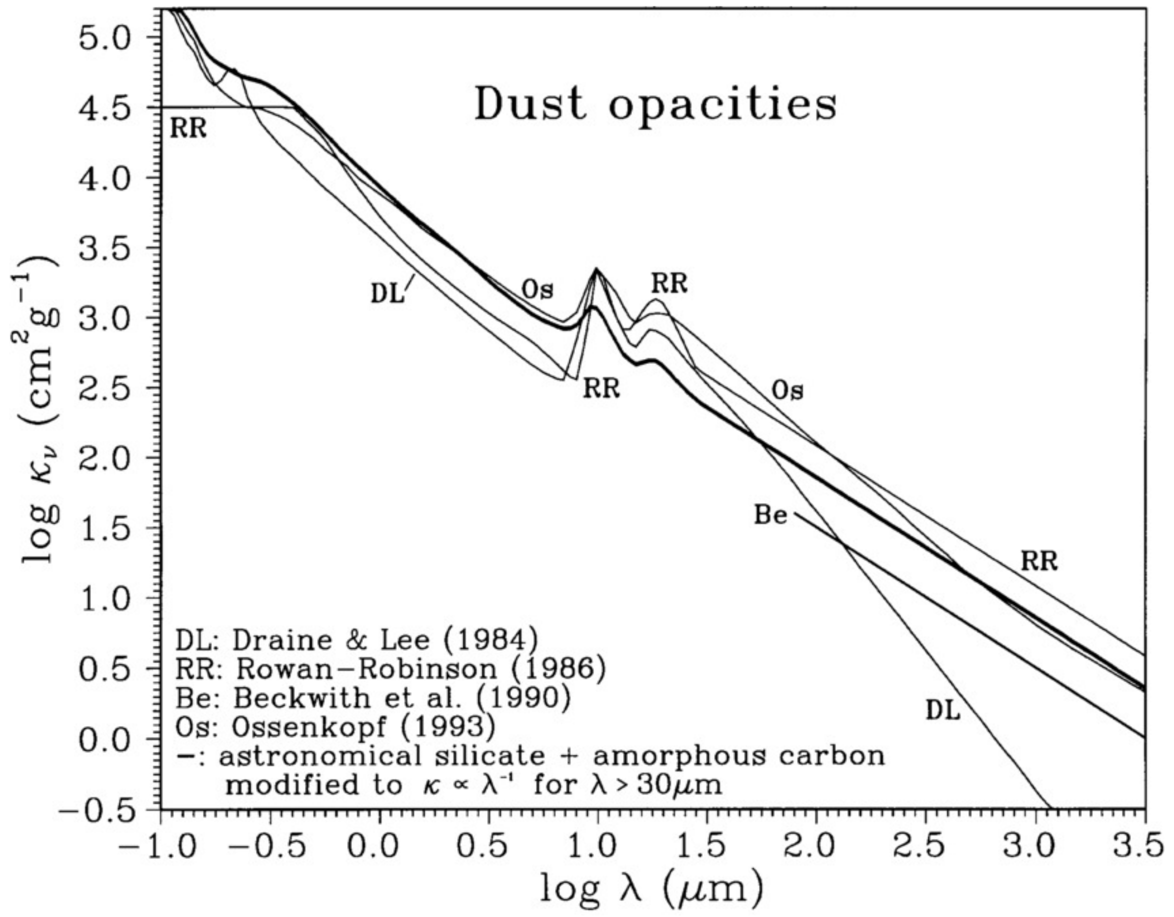
Line-transitions for ionised Iron (FeV)

For Dust: depends on things like dielectric properties of grains, solid-state structure etc.

$MgSiO_3$ glass







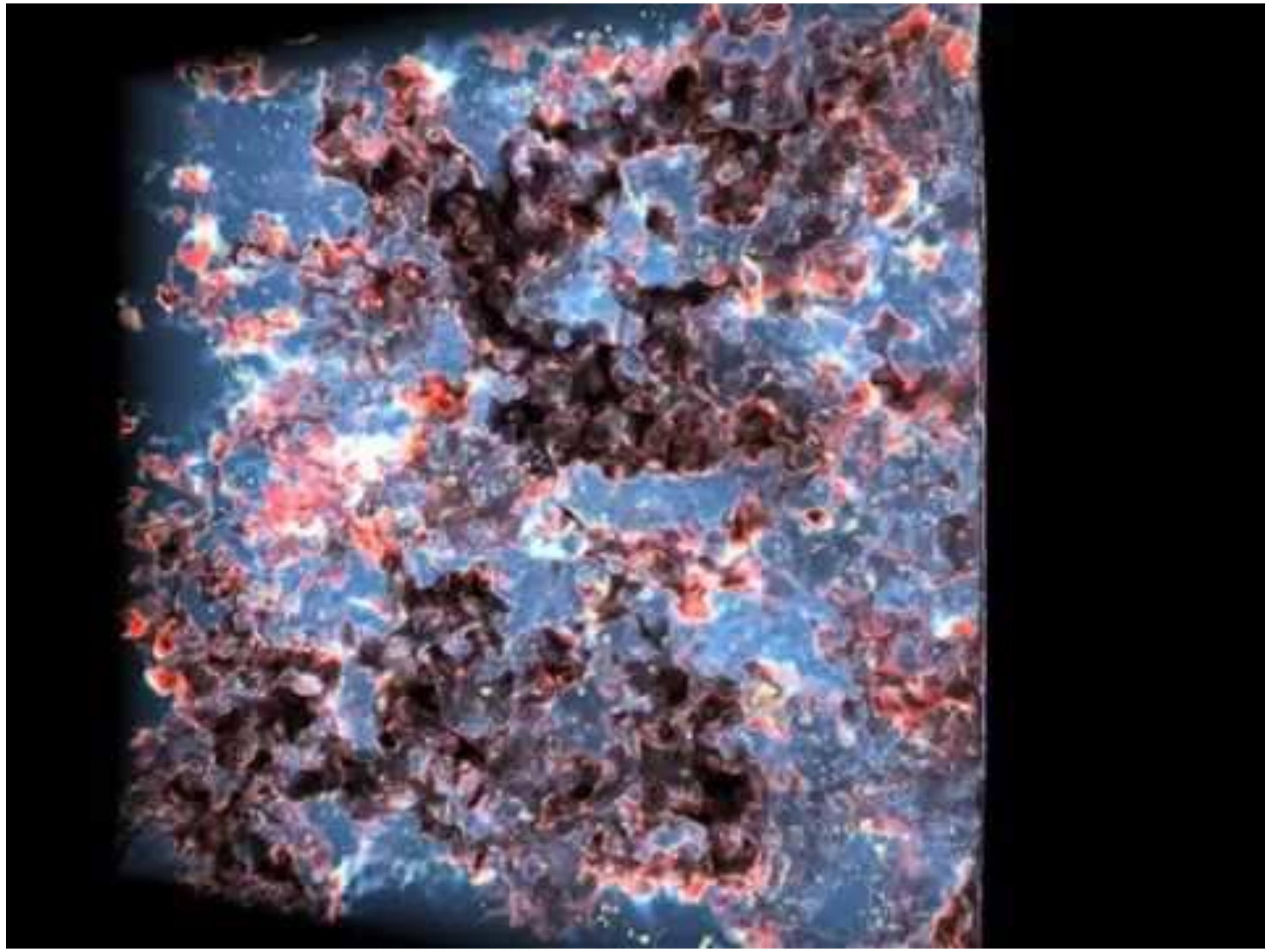
Radiation Hydrodynamics in all its glory*

Astrophysical Phenomena where RHD crucial — at various scales and via different radiation-matter interaction mechanisms

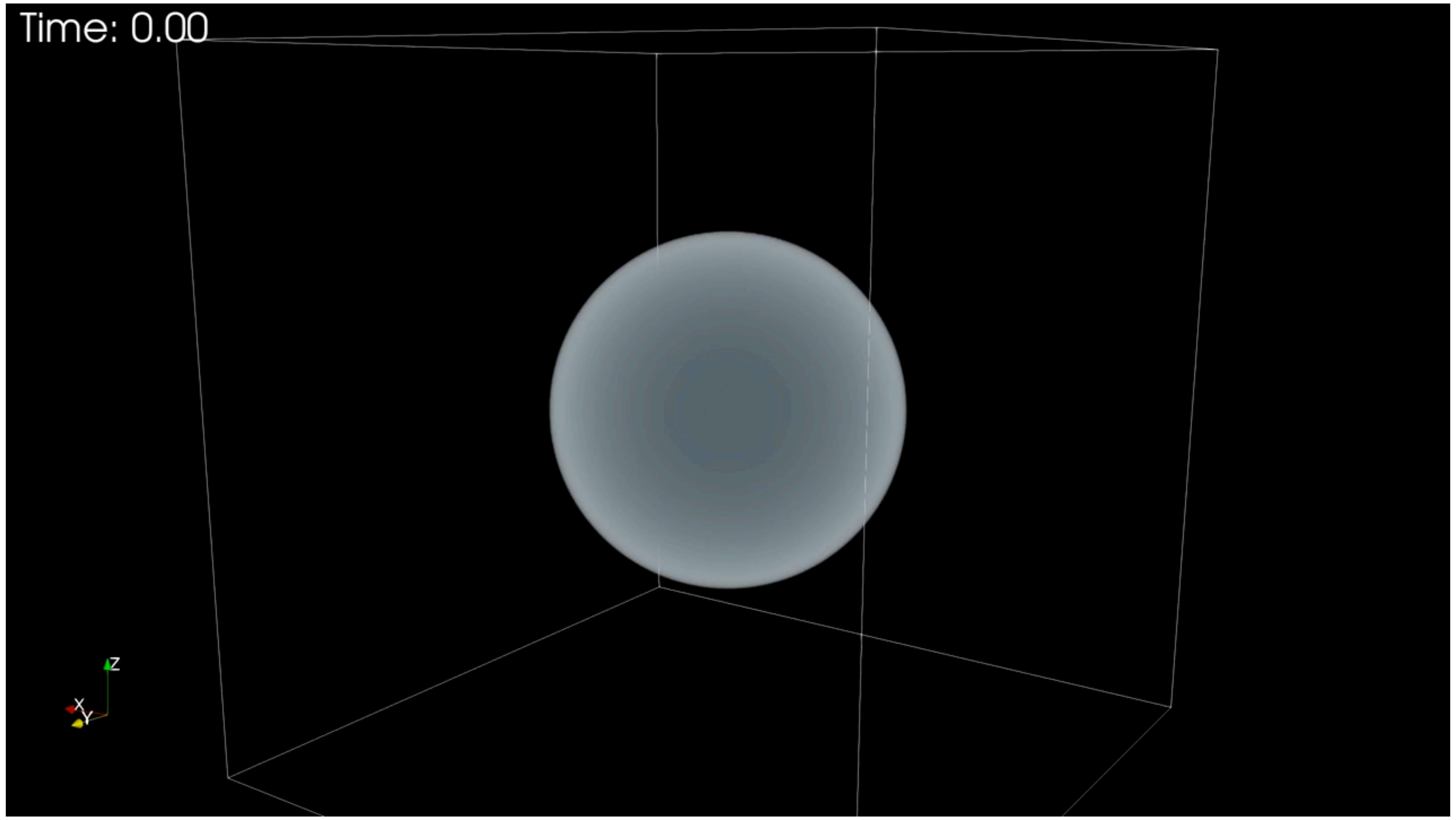
* if you can get around the horrible practical difficulties mentioned earlier

Photoionization of Gas: Cosmological Reionization, HII Regions in Star-Forming Regions

Physics: Ionisation (Bound-free transition) of Hydrogen (or other) atom(s)



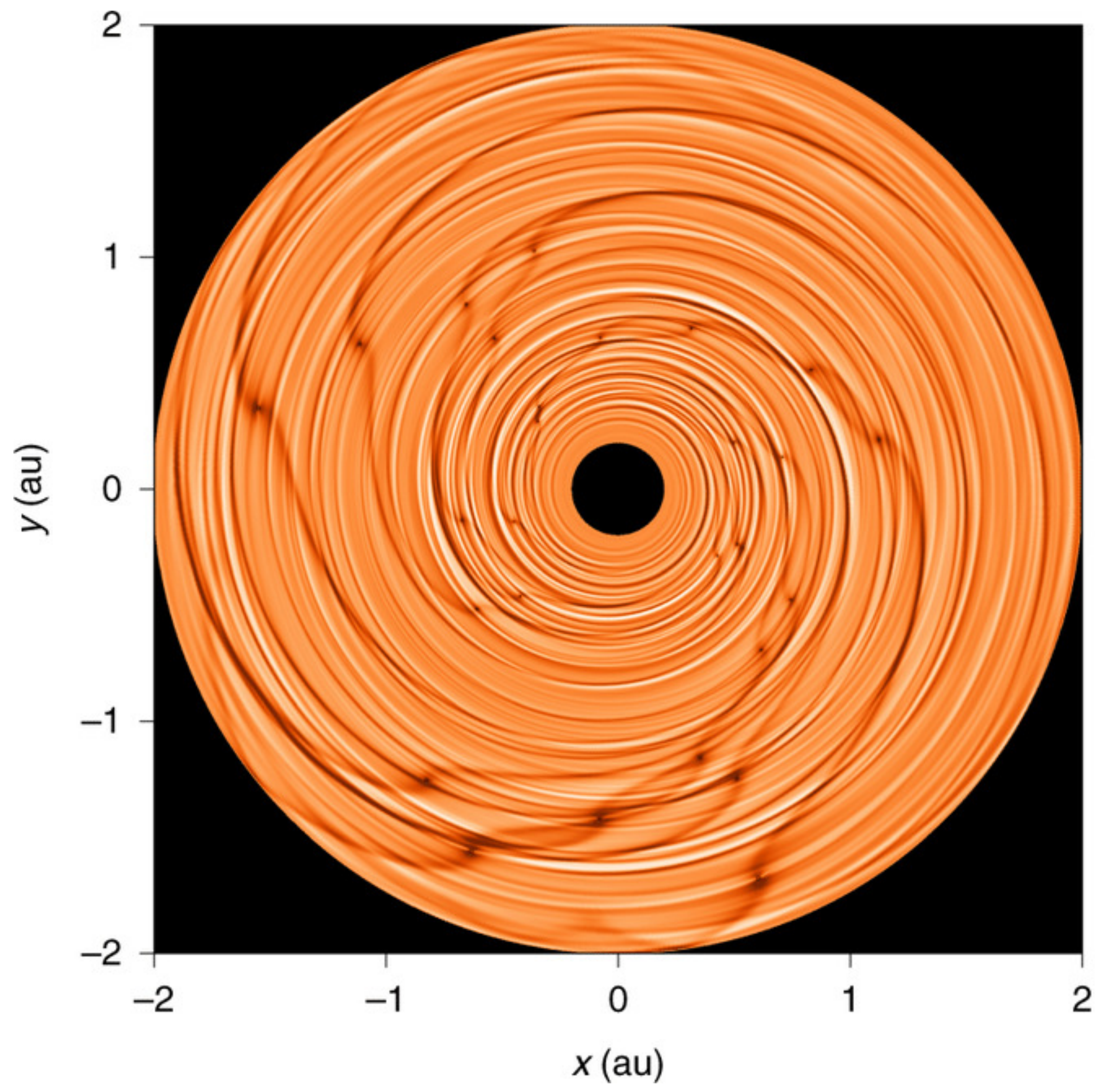
Credit: John Wise



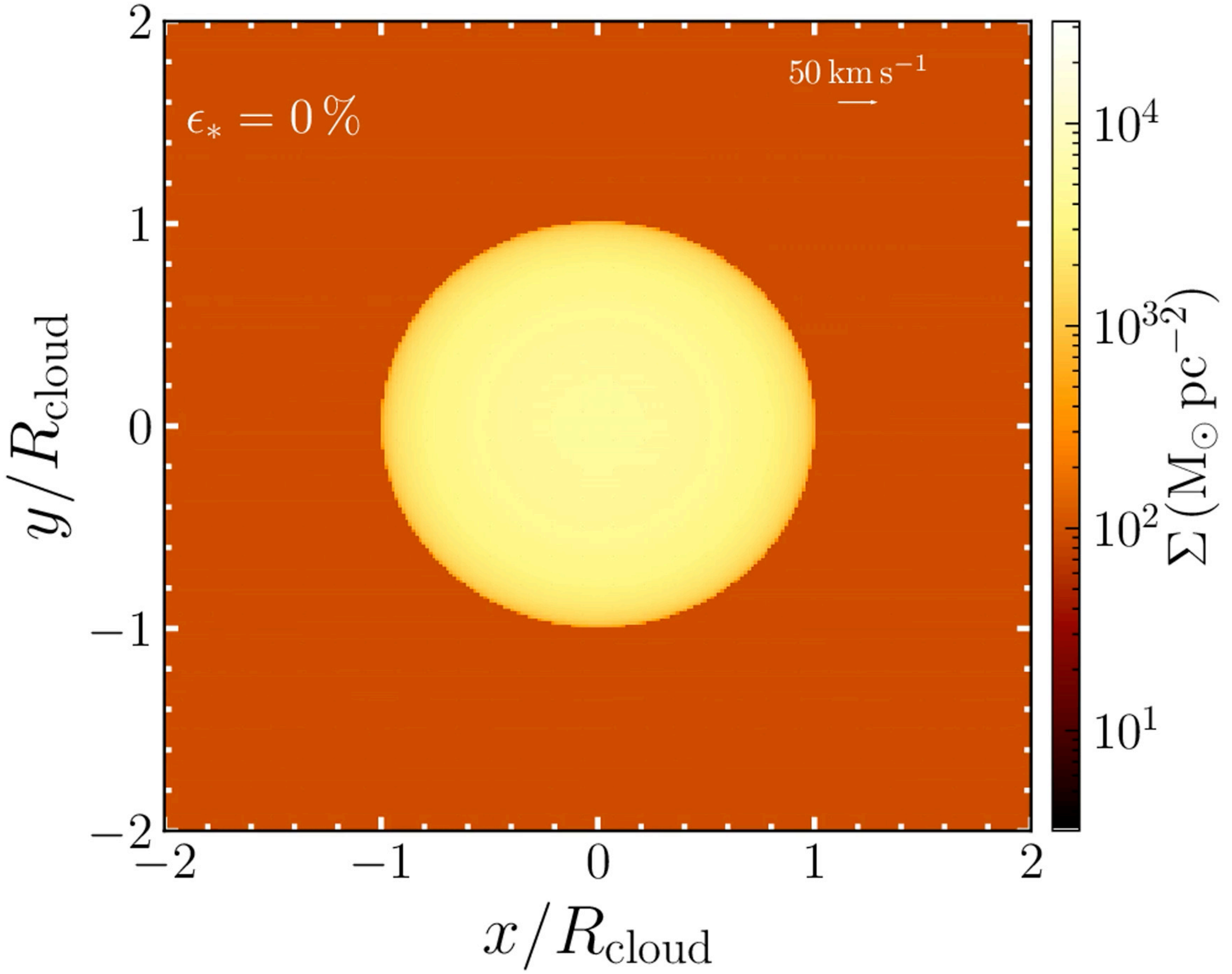
Kim et al. 2018

Dust-mediated RHD: Protoplanetary Disks, Star Clusters, Evolved Stars

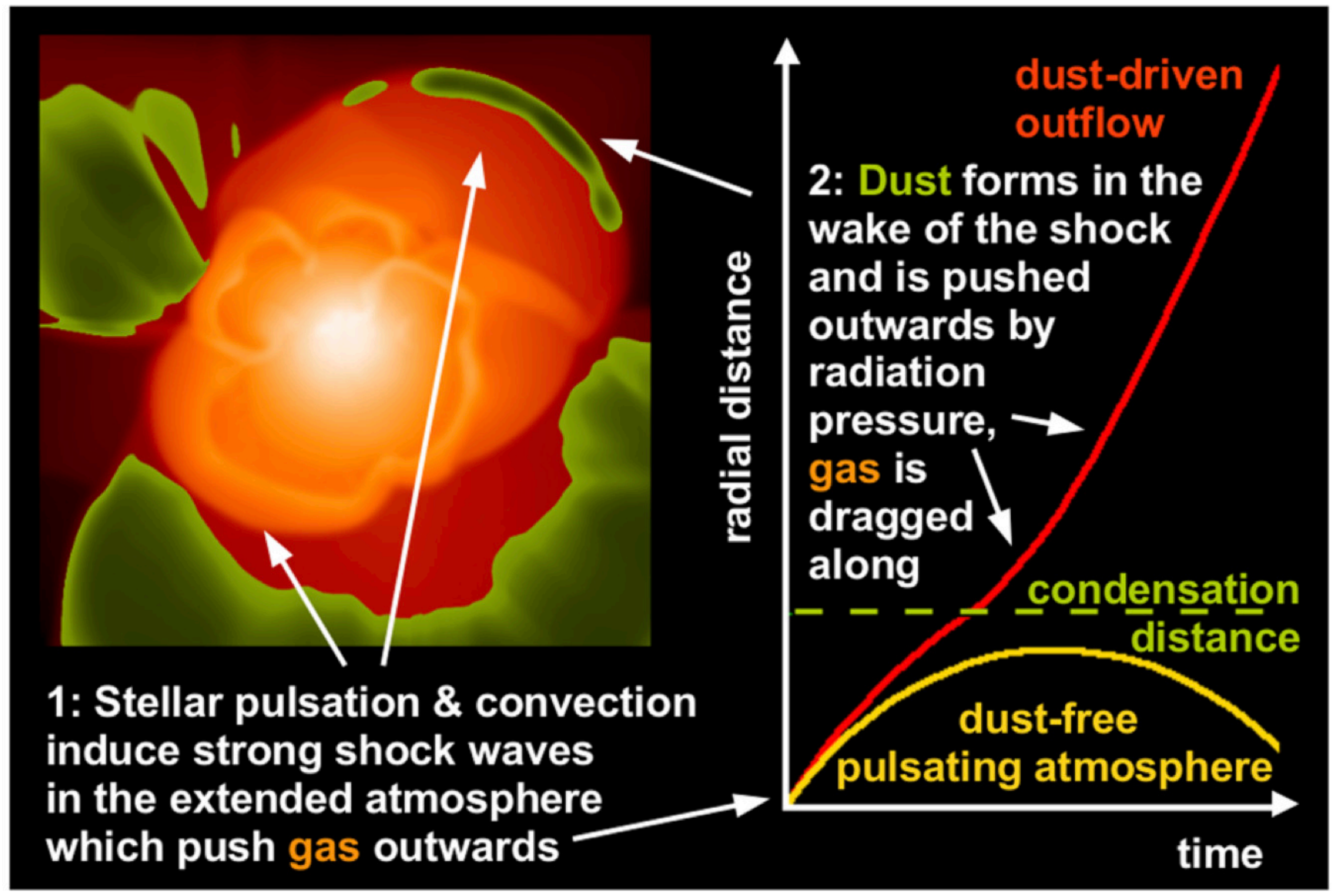
Physics: Dust interacts with radiation, couples to gas via collisions



Brož+ 2021



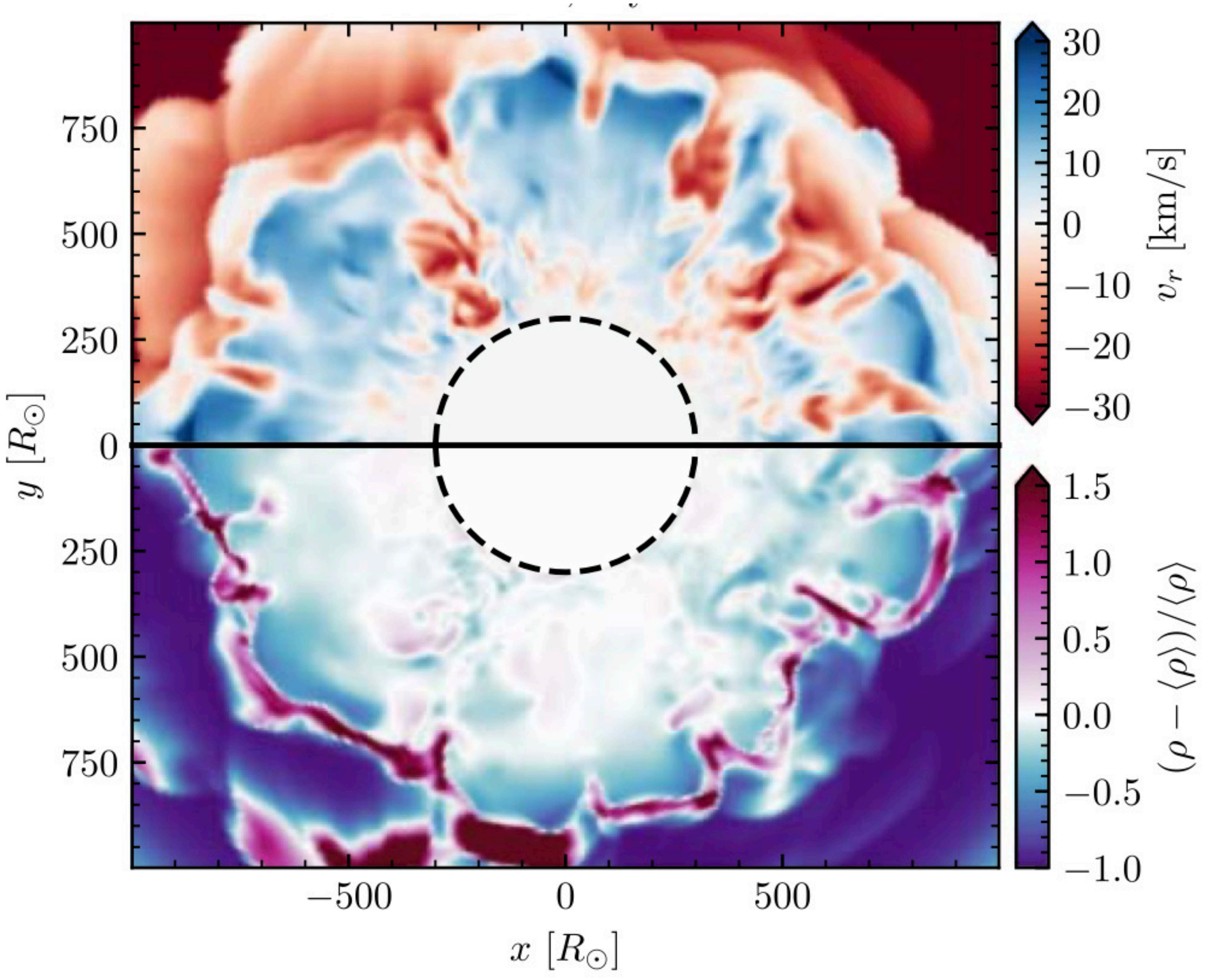
Radiation Pressure-Driven Outflows in Massive Star Clusters: Menon+ 2022c



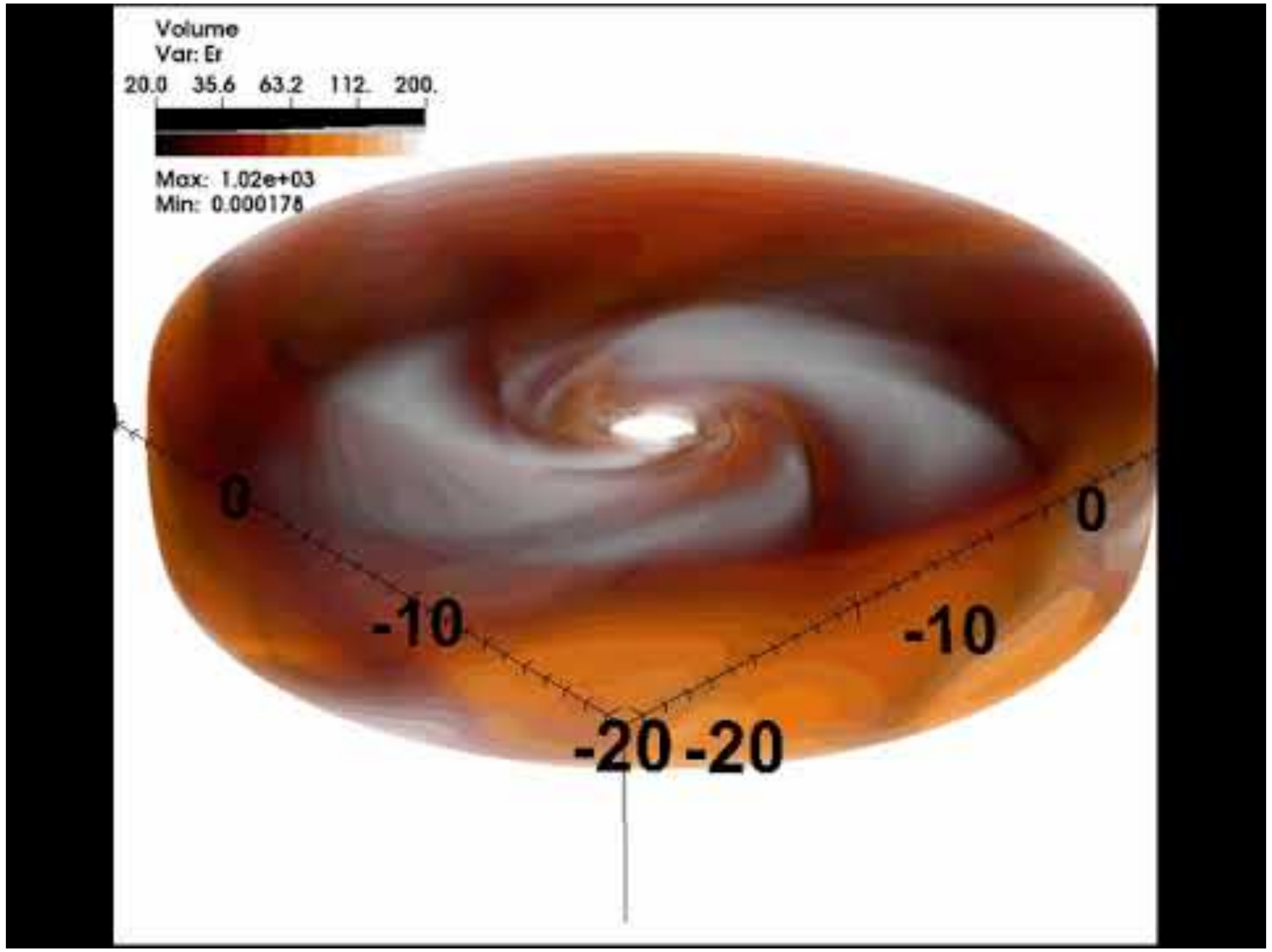
Credits: Höffner & Olofsson

Electrons/Ions: Evolved Star Interiors, BH Accretion Disks

Physics: Free Electrons and ionic transitions absorb and scatter radiation



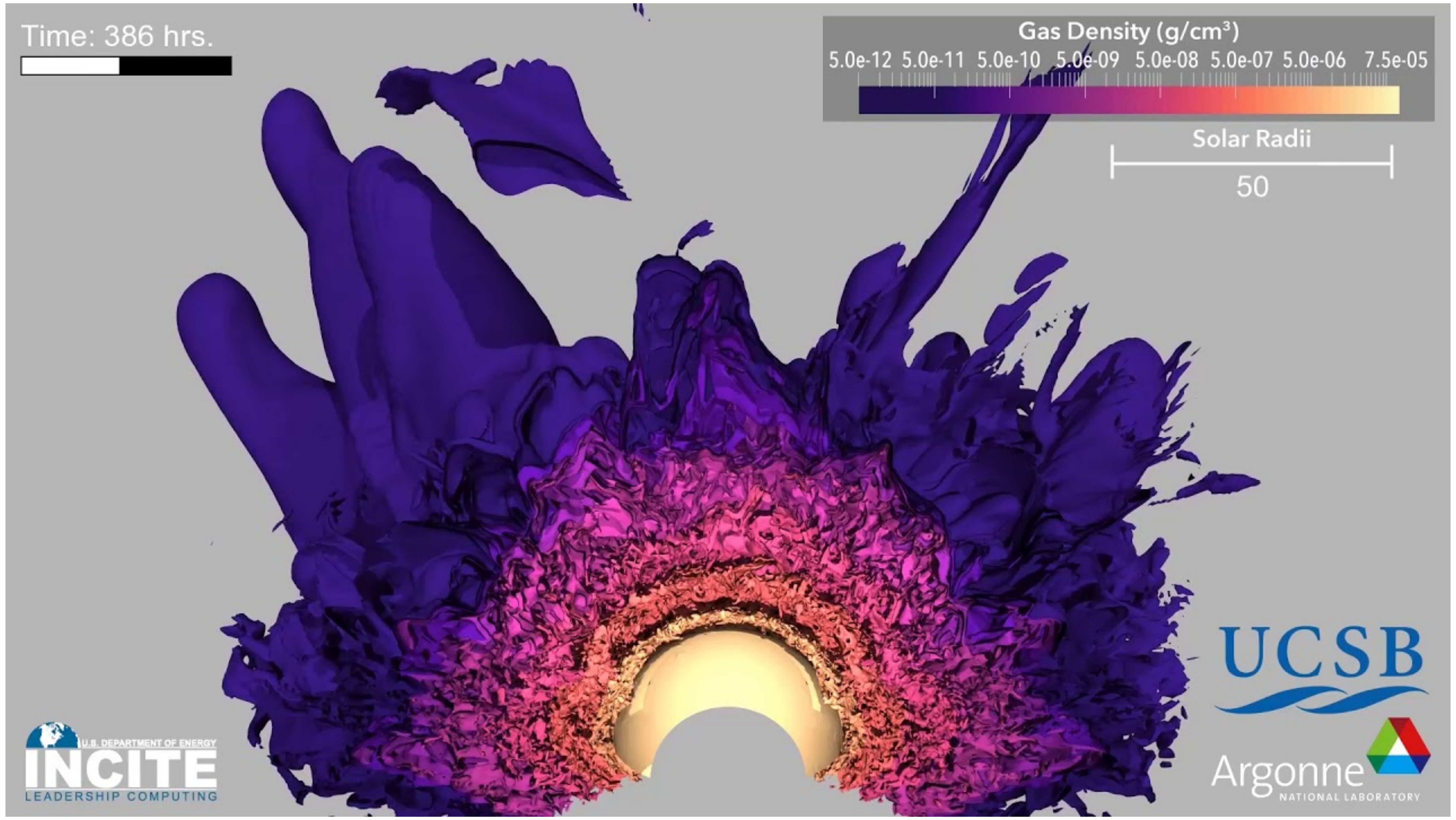
Red-Supergiant Pulsations driven by Radiation Pressure: Goldberg+ 2022



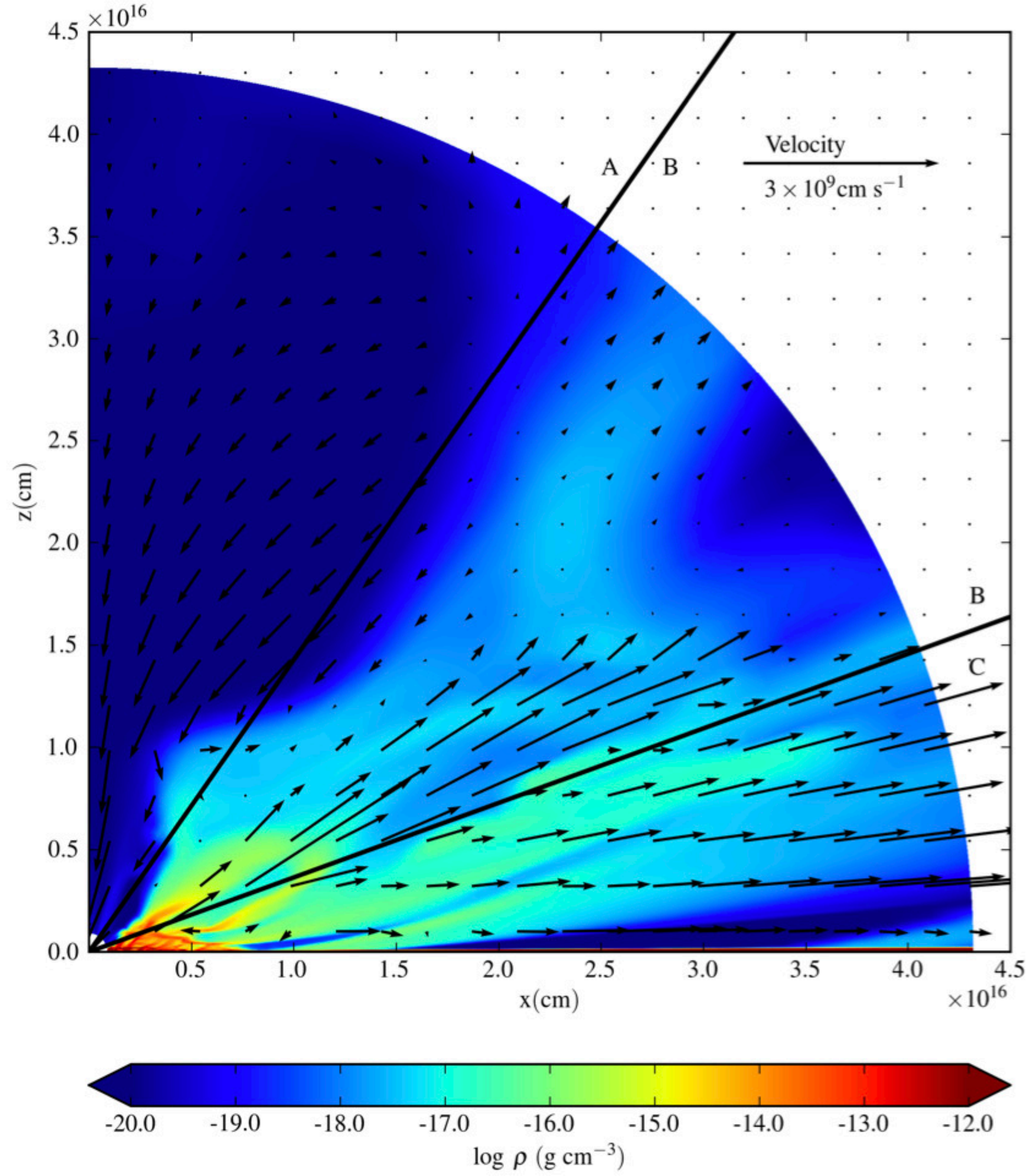
Global BH Accretion Disk Simulations: Jiang+ 2014

Doppler Shifted Atomic/Molecular Line-Driven Winds: Massive Stars and AGN

Physics: Scattering of radiation from the star/BH by UV line-transitions of metal ions



Jiang et al 2018



Wind from AGN disk. Proga & Kallman 2004

One Slide Summary of RHD

- Radiation Hydrodynamics (RHD) is hard, but important and interesting.
- Involves solving the coupled equations of how gas moves (Euler equations) + how radiation is transported (RT equation) + how the two transfer energy and momentum to each other.
- Ideally solve RT equation as-is along rays. Practically very expensive — moment methods preferred in MHD simulations. Appropriate closure important for this though.
- Astrophysical relevant in a broad range of problems: especially when there is hot, opaque gas and/or strongly radiating sources: e.g. AGN, Stellar Interiors, Massive Star Clusters, Inertial Fusion/Laser Experiments, Bombs etc..