Introduction to Hybrid-Kinetics

Guest Lecture

ASTR4012/ASTR8002

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Validity of the Fluid/Hydro Approach

Medium	Particle mean free path	Size scale
Water	9×10^{-9} cm	…
Air	5×10^{-6} cm	…
Solar core	2×10^{-8} cm	$\sim R_{sol}/4 \sim 2 \times 10^{10}$ cm
Solar corona	1×10^{8} cm	$\sim R_{sol} \sim 7 \times 10^{10}$ cm
Solar wind	1×10^{13} cm	$\sim$ AU $\sim 1.5 \times 10^{13}$ cm
Interstellar medium	$1 \times 10^{5-15}$ cm	$\sim$ pc $\sim 3 \times 10^{18}$ cm
Galaxy cluster (intracluster medium)	1×10^{23} cm	$\sim$ Mpc $\sim 10^{24}$ cm

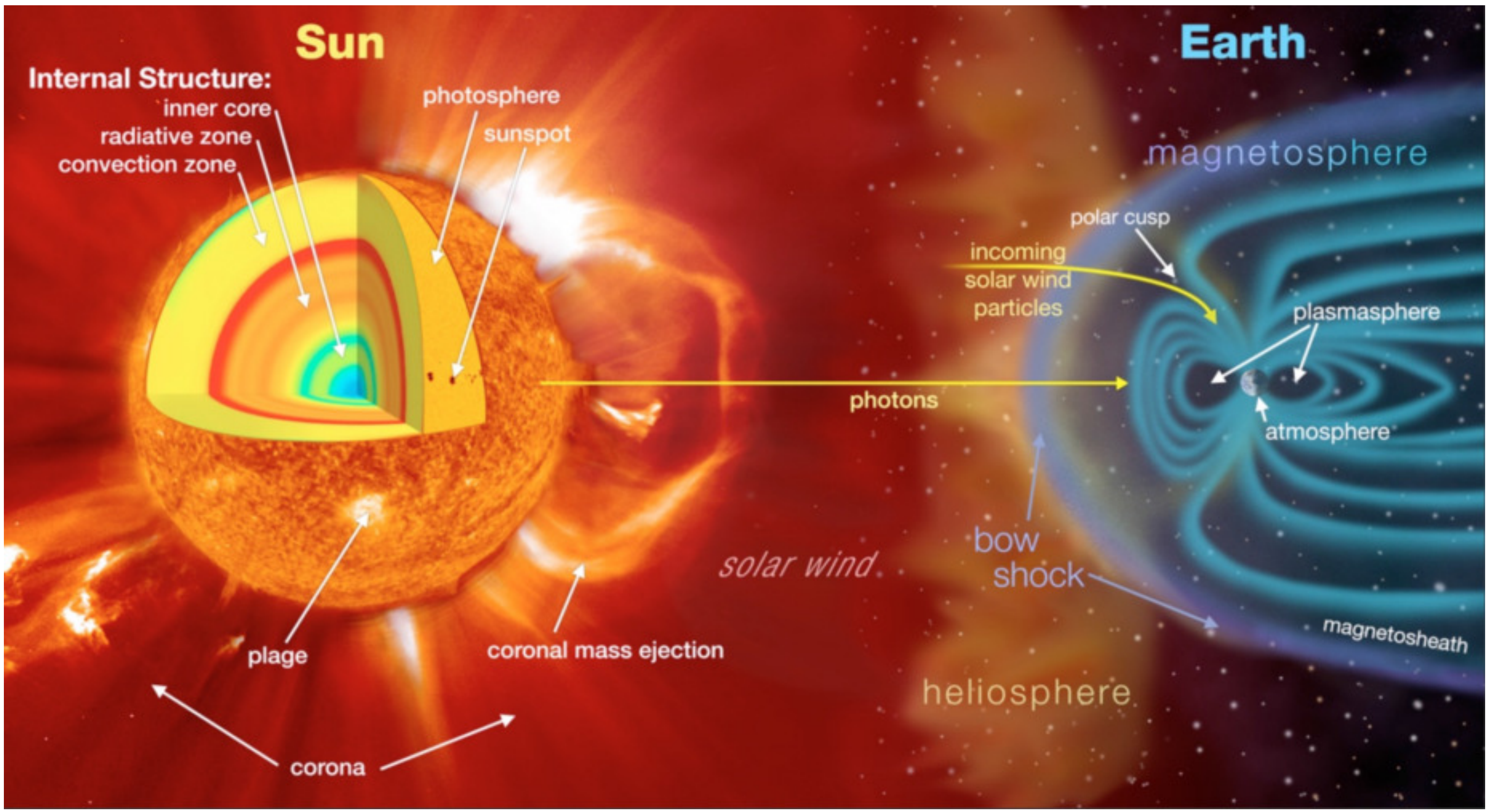
$$\frac{\lambda_{mfp}}{L} << 1$$

frequent collisions between particles -> isotropic temperature, pressure in a fluid region

$$\frac{\lambda_{mfp}}{L} \sim 1$$

Weakly collisional / collision-less gas

Solar Wind



$L \sim 1$ **au** $\lambda_{\text{mfp}} \sim 1$ **au**

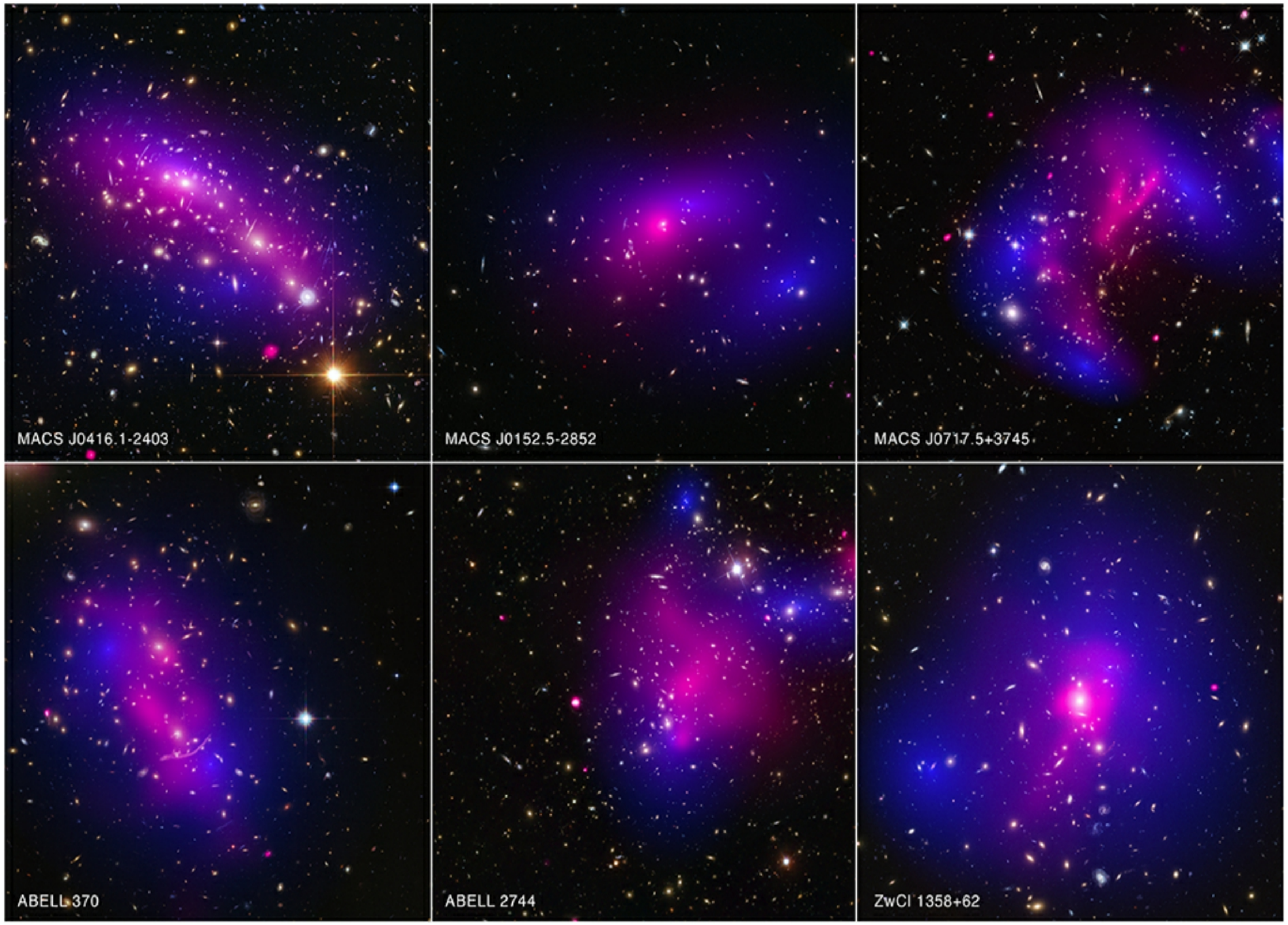
Galactic center

Galactic center (Chandra image)

$L \sim 0.1$ **pc**

$\lambda_{\rm mfp} \sim 0.1$ **pc**

Intracluster Medium



$L \sim 50 - 100$ kpc

$\lambda_{\mathrm{mfp}} \sim 10$ kpc

Hot gas in clusters emitting X-rays detected by Chandra (pink), optical image from Hubble and inferred dark matter distribution (blue)

Beyond the fluid approach

- In weakly-collisional plasma like ICM, $L \sim \lambda_{mfp} \rightarrow$ fluid approximation breaks down → kinetic / particle-in-cell (PIC) methods

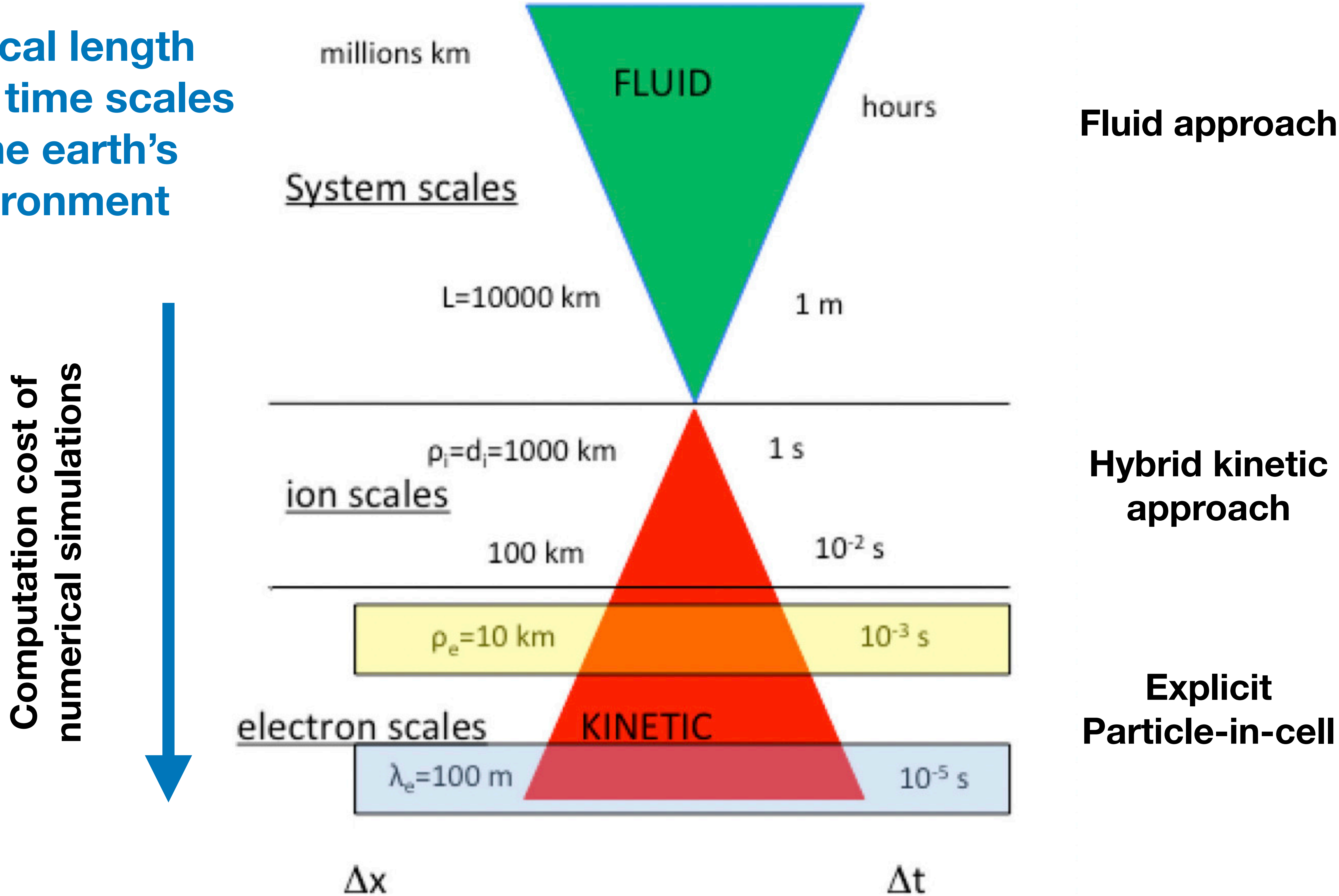
- Model charged particles using distribution functions and study their evolution using the Vlasov equation

$$\frac{\partial f_i}{\partial t} + \mathbf{v} \cdot \frac{\partial f_i}{\partial \mathbf{r}} + \mathbf{a} \cdot \frac{\partial f_i}{\partial \mathbf{v}} = \left(\frac{\partial f_i}{\partial t} \right)_c$$; $f_i \rightarrow f_i(\mathbf{r}, \mathbf{v}, t)$ and $(\partial f_i / \partial t)_c = 0$ for collision-less plasma

- Charged particles are accelerated by the Lorenz force

$$\mathbf{a} = \frac{q_i}{m_i} \left(\mathbf{E} + \mathbf{v} \times \mathbf{B} \right)$$; q_i, m_i are charge and mass of ion-species i , E and B electric and magnetic field

Typical length and time scales in the earth's environment



Innocenti *et al.* 2017

Hybrid kinetic approach

- Ions (protons) are treated as particles and electrons are treated as a fluid $\rightarrow$ Hybrid kinetics
- Protons (f_p) $\dfrac{\partial f_p}{\partial t} + \mathbf{v} \cdot \dfrac{\partial f_p}{\partial \mathbf{r}} + \mathbf{a} \cdot \dfrac{\partial f_p}{\partial \mathbf{v}} = 0$
- Electrons $\rightarrow$ hydro approach $\rightarrow$ continuity, momentum, energy equations

$$m_e n_e \frac{\mathrm{D} u_e}{\mathrm{D} t} = -\nabla \mathrm{P}_e - e n_e (\mathbf{E} + u_e \times \mathbf{B})$$

where $\rho_e = -e n_e \approx -\rho_p \approx \rho$ (quasi-neutrality) and $\nabla \cdot \mathbf{E} \approx 0$

Ohm's Law

We can re-write the electron momentum equation by using

$$\mathrm{J} = \mathrm{J_I} + \mathrm{J_e} = \rho(u_\mathrm{I} + u_e) \text{ or } u_e = \frac{\mathrm{J}}{\rho} - u_\mathrm{I}$$

$$\mathbf{E} = -\mathbf{u}_\mathrm{I} \times \mathbf{B} + \frac{\mathbf{J} \times \mathbf{B}}{\rho} - \frac{\nabla p_\mathrm{electron}}{\rho} + \frac{m_e}{e}\frac{\mathrm{D}u_e}{\mathrm{D}t}$$

$$\boxed{\frac{m_e}{e}\frac{\mathrm{D}u_e}{\mathrm{D}t}} \to 0$$ (electron kinematic physics excluded)

electron inertial term

Ohm's Law

$$\mathbf{E} = -\mathbf{u}_{\mathrm{I}} \times \mathbf{B} + \frac{\mathbf{J} \times \mathbf{B}}{\rho} - \frac{\nabla p_{\mathrm{electron}}}{\rho} + \boxed{\eta \mathbf{J} - \eta_{\mathrm{hyper}} \nabla^2 \mathbf{J}}$$

added resistive terms

- Again, closure!

$p_{\mathrm{electron}} \propto T_{e0} n_e$ (isothermal) or $p_{\mathrm{electron}} \propto T_{e0} n_e^{\gamma}$ (adiabatic)

Maxwell's equations

Evolution of electromagnetic fields is governed by Maxwell's equations

- $\nabla \cdot \mathbf{B} = 0$
- $\nabla \cdot \mathbf{E} = \dfrac{\rho_{\text{total}}}{\epsilon_0}$ ($\rho_{\text{total}} = \rho_{\text{I}} + \rho_e \approx 0$)
- $\nabla \times \mathbf{B} = \mu_0 \mathbf{J} \; + \; \dfrac{1}{c^2}\dfrac{\partial \mathbf{E}}{\partial t}$

  $\dfrac{1}{c^2}\dfrac{\partial \mathbf{E}}{\partial t} \to 0$ (Ampere's Law) and $c = \dfrac{1}{\sqrt{\mu_0 \epsilon_0}}$
- $\nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t}$

Ohm's Law

$$\mathbf{E} = -\mathbf{u}_{\mathrm{I}} \times \mathbf{B} + \frac{\mathbf{J} \times \mathbf{B}}{\rho} - \frac{\nabla p_{\mathrm{electron}}}{\rho} + \eta \mathbf{J} - \eta_{\mathrm{hyper}} \nabla^2 \mathbf{J}$$

convective term

resistivity

Recap : non-relativistic MHD Ohm's law

$$\mathbf{J} = \sigma(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \rightarrow \mathbf{E} = -\mathbf{v} \times \mathbf{B} + \eta \mathbf{J} \ \left(\eta = \frac{1}{\sigma}\right)$$

$\mathbf{u}_{\mathrm{I}} \sim$ **bulk velocity**

Ohm's Law

$$\mathbf{E} = -\,\mathbf{u}_{\mathrm{I}} \times \mathbf{B} + \frac{\mathbf{J} \times \mathbf{B}}{\rho} - \frac{\nabla p_{\mathrm{electron}}}{\rho} + \eta \mathbf{J} - \eta_{\mathrm{hyper}} \nabla^2 \mathbf{J}$$

thermo-electric term

Recap : Induction equation

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \eta \Delta \mathbf{B}$$

$$\mathbf{B}_{\mathrm{init}} = 0 \rightarrow \mathbf{B} = 0$$

Source/seed magnetic fields ?

$$\frac{\partial \mathbf{B}}{\partial t} = -\,\nabla \times \mathbf{E} = -\,\nabla \times \left(-\frac{\nabla p}{\rho}\right) \approx \frac{\nabla \rho \times \nabla p}{\rho^2}$$

For closures of form $p \propto \rho$**,** $\dfrac{\partial \mathbf{B}}{\partial t} = 0$

If pressure and density gradients are misaligned $\rightarrow \mathbf{B}_{\mathrm{seed}} \neq 0$

Biermann battery!

Ohm's Law

$$\mathbf{E} = -\,\mathbf{u}_{\mathrm{I}} \times \mathbf{B} + \frac{\mathbf{J} \times \mathbf{B}}{\rho} - \frac{\nabla p_{\mathrm{electron}}}{\rho} + \eta \mathbf{J} - \eta_{\mathrm{hyper}} \nabla^2 \mathbf{J}$$

Hall electric field

Ohm's Law

$$\mathbf{E} = -\,\mathbf{u}_{\mathrm{I}} \times \mathbf{B} + \frac{\mathbf{J} \times \mathbf{B}}{\rho} - \frac{\nabla p_{\mathrm{electron}}}{\rho} + \eta \mathbf{J} - \eta_{\mathrm{hyper}} \nabla^2 \mathbf{J}$$

Hyper-resistivity

Whistler oscillations : $\omega \propto k^2 \rightarrow$ **grid-scale oscillations**

Speed of light $c \rightarrow \infty$ **or physical velocities are unbounded**

To remove energy at high-k and for stability of numerical simulations

Hybrid kinetic approach

Source terms in Ohm's law (ρ, u_{I}) → from collision-less particles

Moments of density and bulk velocity

$$n_p = \int f_p \, d^3 v \qquad u_{\mathrm{I}} = \frac{1}{n} \int \mathbf{v} \, f_p \, d^3 v$$

How to solve these equations numerically?

1. Evolution of protons (f_p)

$$\frac{\partial f_p}{\partial t} + \mathbf{v} \cdot \frac{\partial f_p}{\partial \mathbf{r}} + \mathbf{a} \cdot \frac{\partial f_p}{\partial \mathbf{v}} = 0$$

6-D in space and momentum + time evolution

2. Sample the distribution function using (meta) particles and calculate density, velocity … directly from particles

Hybrid Particle-In-Cell Algorithm

Lorentz force

turbulent driving

$$\frac{d\mathbf{r}}{dt} = \mathbf{v} \qquad \frac{d\mathbf{v}}{dt} = \frac{q}{m}(\mathbf{E}^* + \mathbf{v} \times \mathbf{B}) + \mathbf{f}$$

Particle evolution

$$\mathbf{E}^* = \frac{1}{\rho}\left[(\mathbf{J} - \mathbf{J}_\mathrm{I}) \times \mathbf{B}\right] - \frac{\nabla p_{\mathrm{electron}}}{\rho}$$

Density (ρ) and ion currents $(\mathbf{J}_\mathrm{I})$

Calculated on particles interpolated to grid

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$$

Compute magnetic field

Thermoelectric effect

Resistivity

Ampere's law

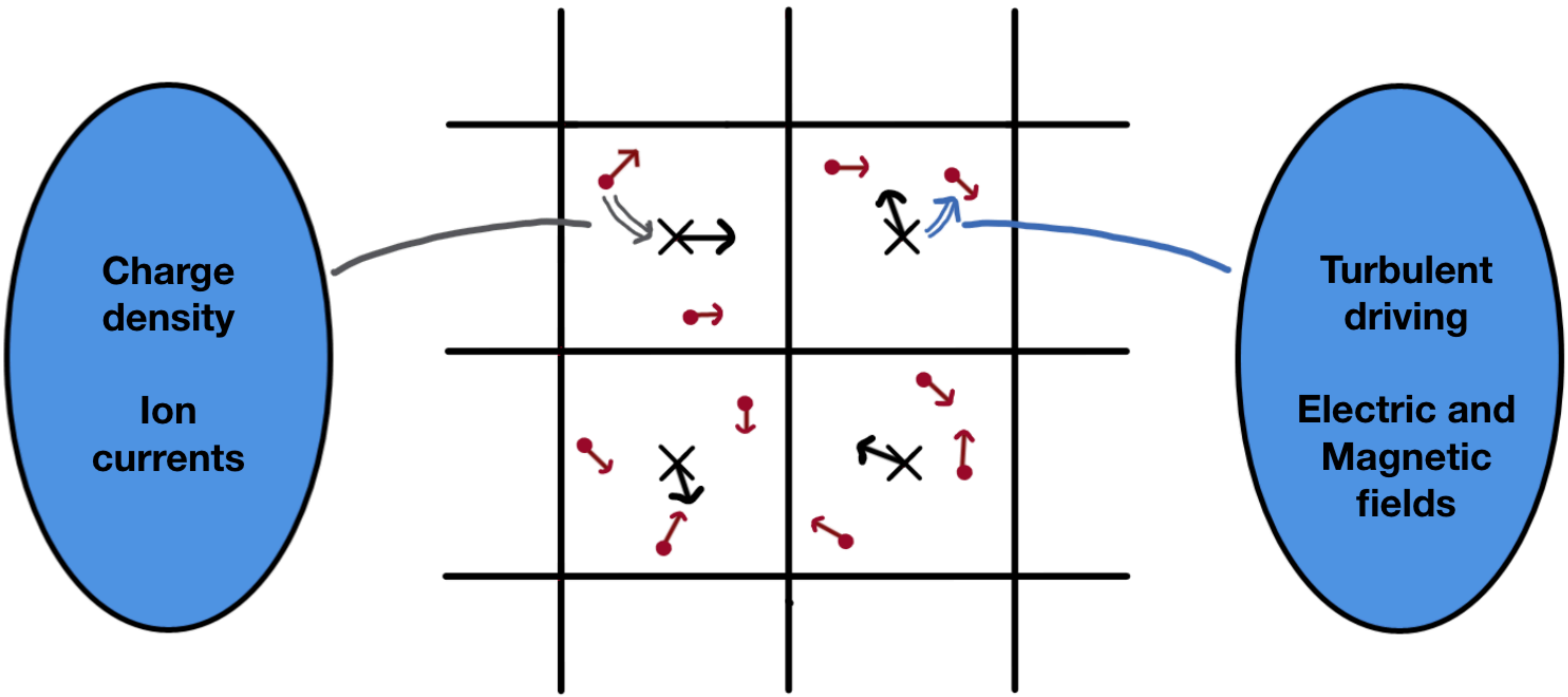
$$\mathbf{E} = \frac{1}{\rho}\left[(\mathbf{J} - \mathbf{J}_\mathrm{I}) \times \mathbf{B}\right] - \frac{\nabla p_{\mathrm{electron}}}{\rho} + \eta \mathbf{J} - \eta_{\mathrm{hyper}} \nabla^2 \mathbf{J}$$

$$\mathbf{J} = \frac{\nabla \times \mathbf{B}}{\mu_0}$$

Compute electric field

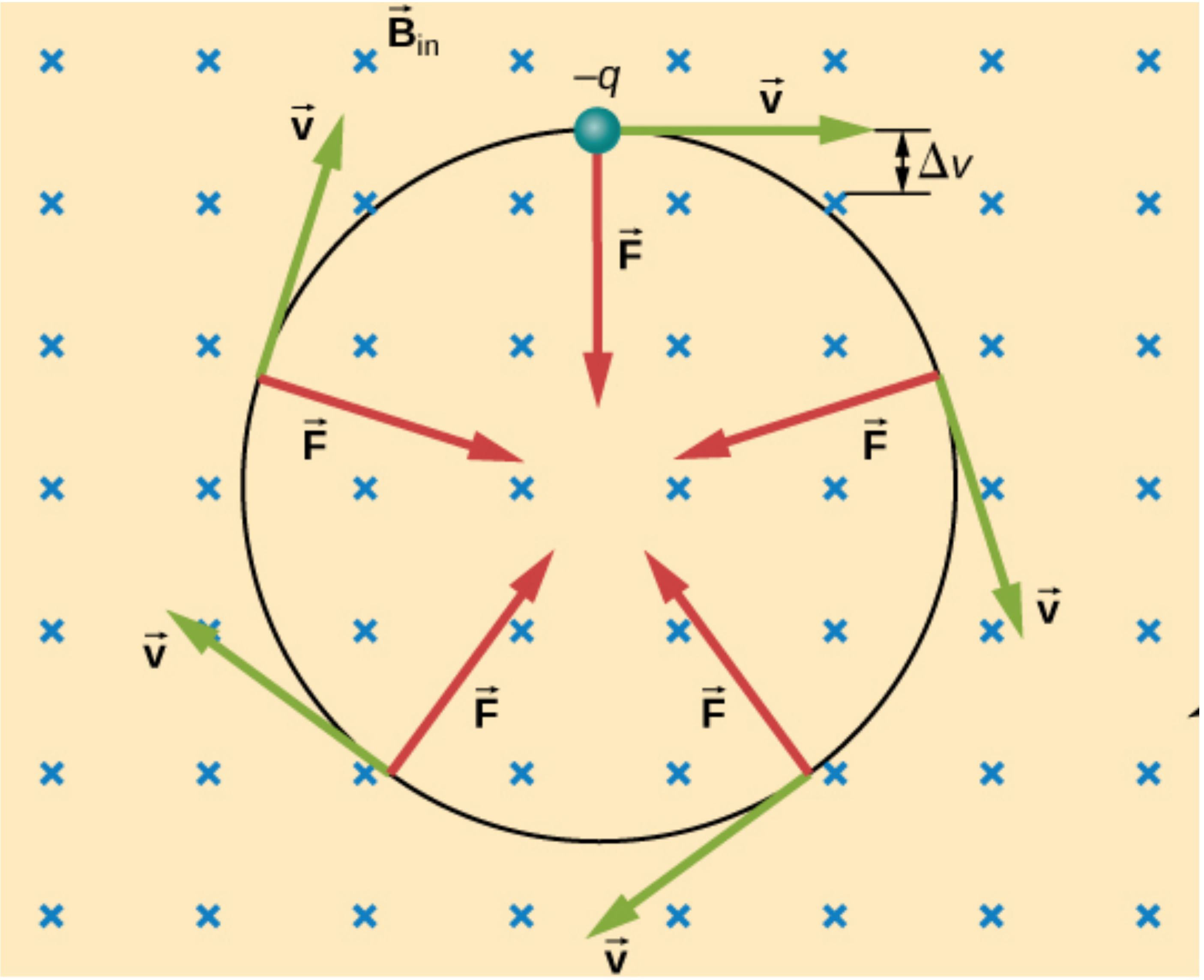
Hyper-resistivity

Particle ↻ Grid Interpolation



Charged Particle in Uniform Magnetic Field

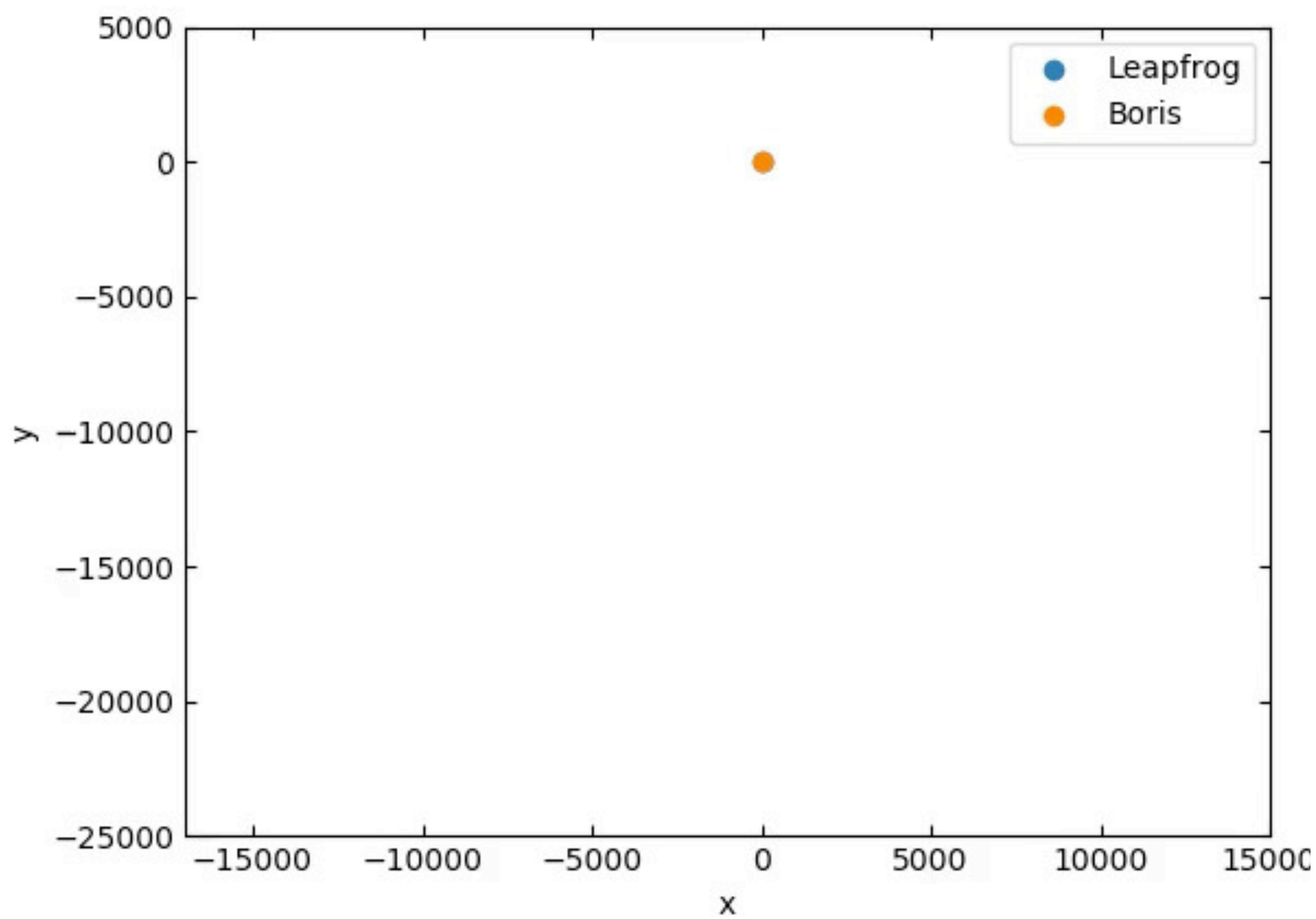
$$\mathbf{F} = mq\mathbf{V} \times \mathbf{B}$$

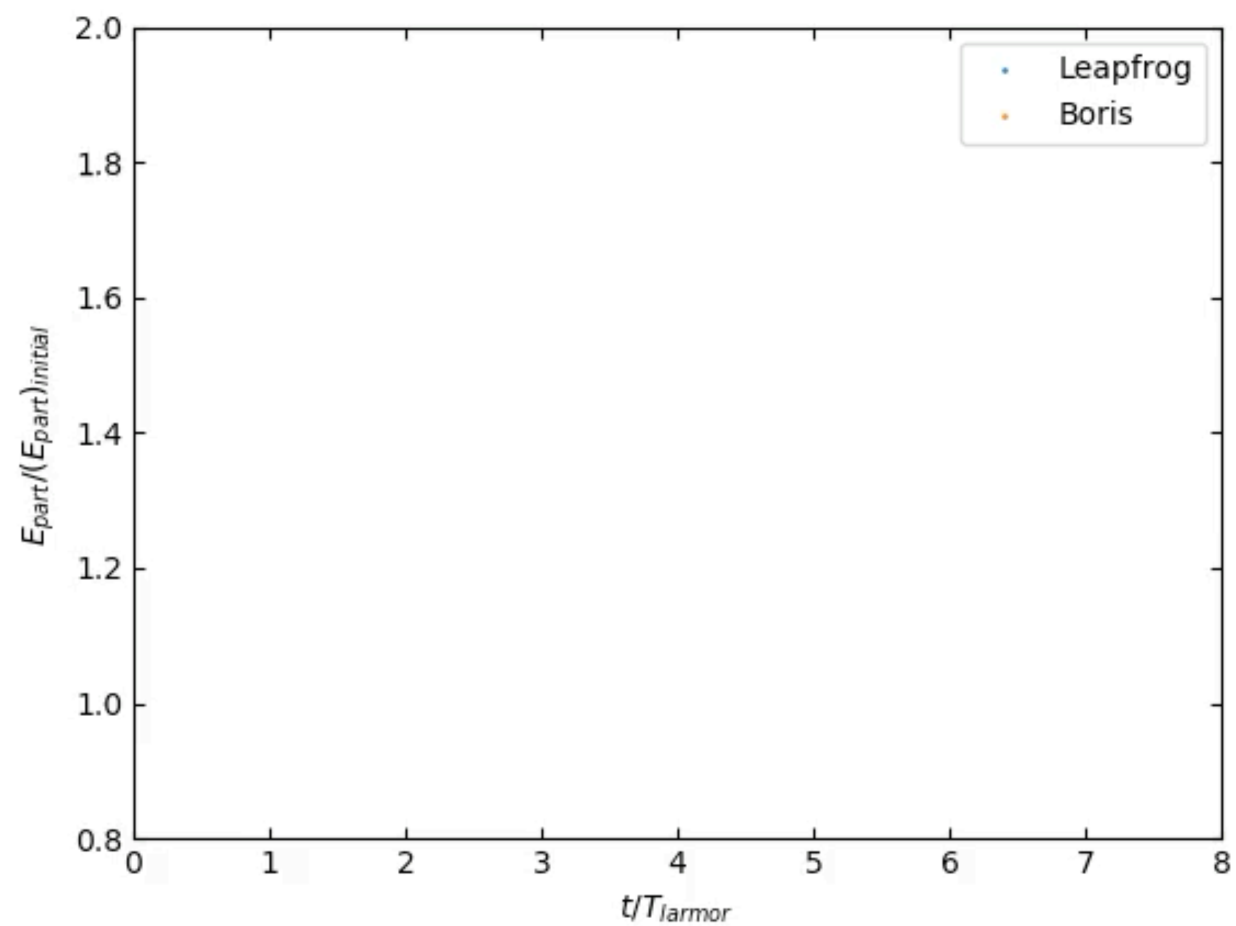


$$r_{\text{Larmor}} = \frac{m\text{V}}{q\text{B}}$$

$$t_{\text{Larmor}} = \frac{2\pi m}{q\text{B}}$$

Charged Particle in Uniform Magnetic Field





Trajectory of charged particle

Energy of charged particle

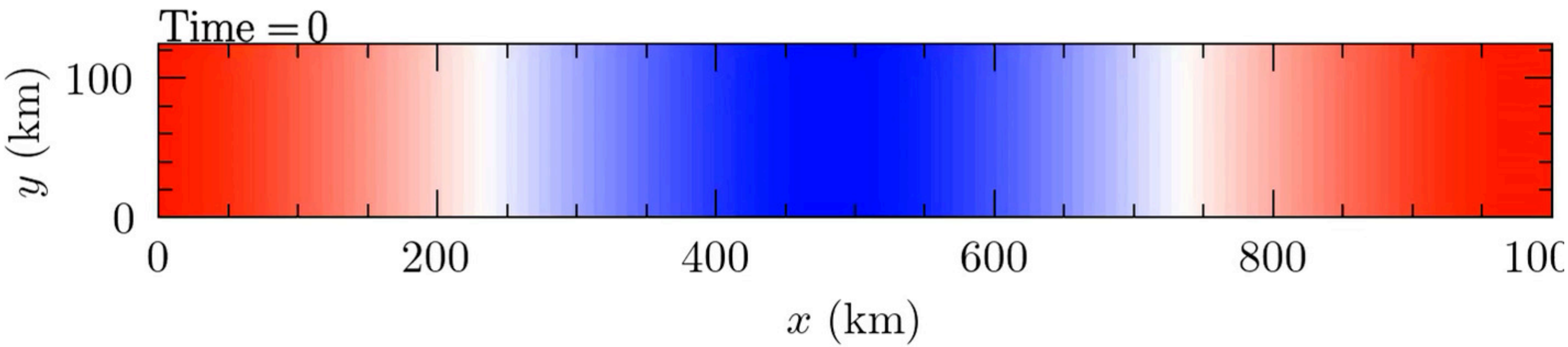
Waves in cold plasma

- Cold plasma $T_p = T_e = 0$ without resistivity ($\eta = 0$)
- Consider a uniform magnetic field ($B_0\hat{x}$) with fluctuations in a perpendicular direction $B_z \sim (B_0/1000)$ cos(kx)
- Linearising the hybrid-kinetic equations (perturbation analysis)
  Recap : Wave solutions to Linearised MHD equations $\rightarrow$ Wave solutions in a collision-less plasma
- Ion-inertial length (d_i) and larmor-gyration time $\left(t_{\text{Larmor}} = \dfrac{2\pi m}{qB_0}\right)$

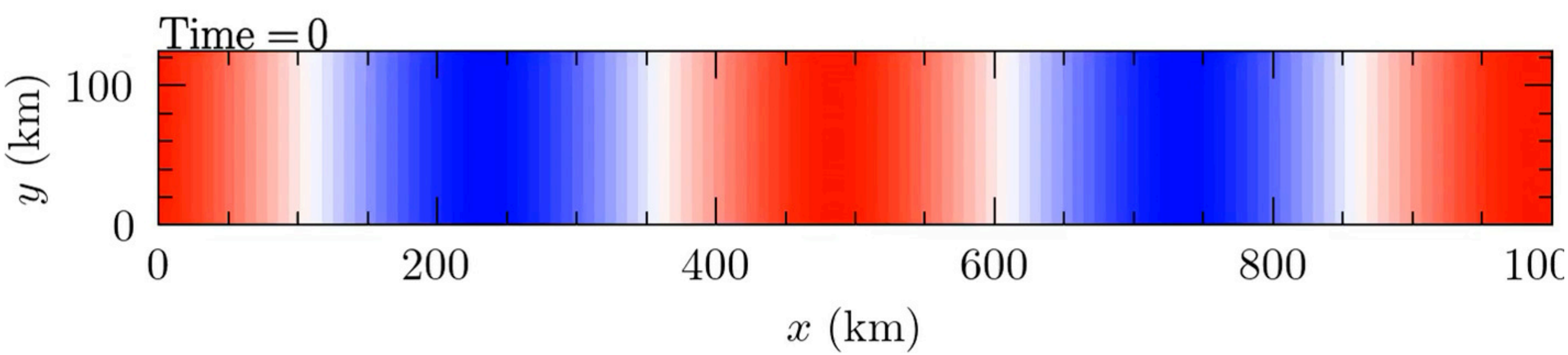
Alfven waves

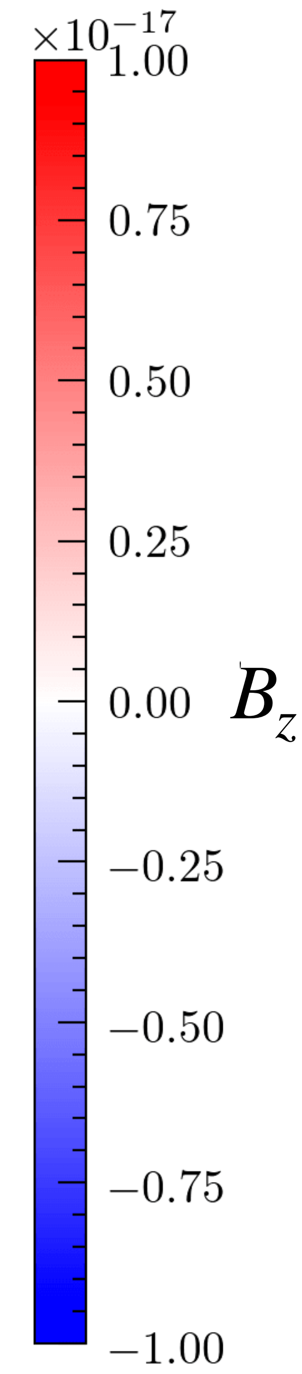
ppc = 100

k = 1 ($kd_i = 0.045$)



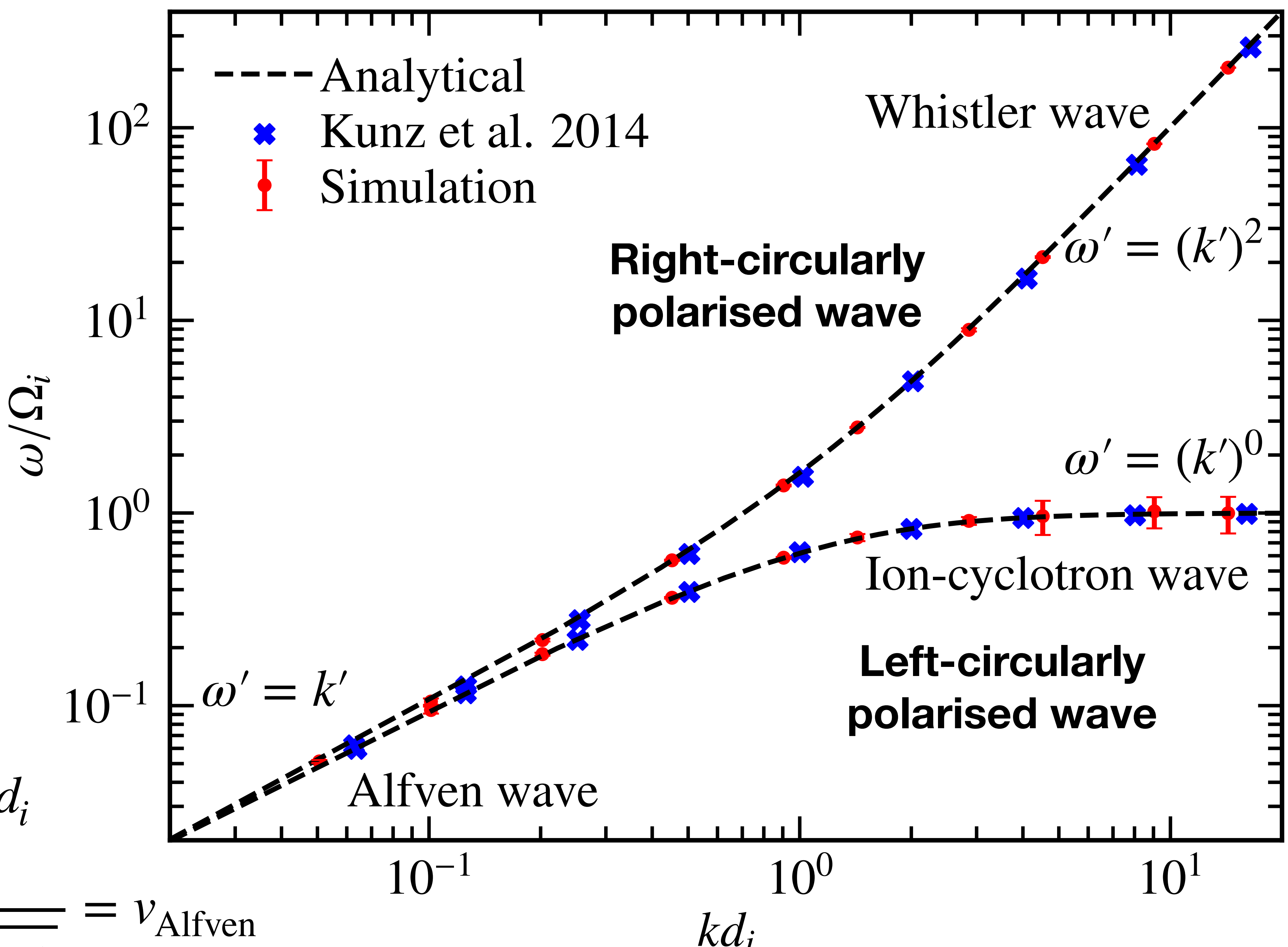
k = 2 ($kd_i = 0.09$)





Waves propagating through the computational domain

Dispersion relation



$$\frac{\omega}{k} = \Omega_i d_i$$

$$= \frac{\mathrm{B}}{\sqrt{(\mu_0 n m_p)}} = v_{\mathrm{Alfven}}$$

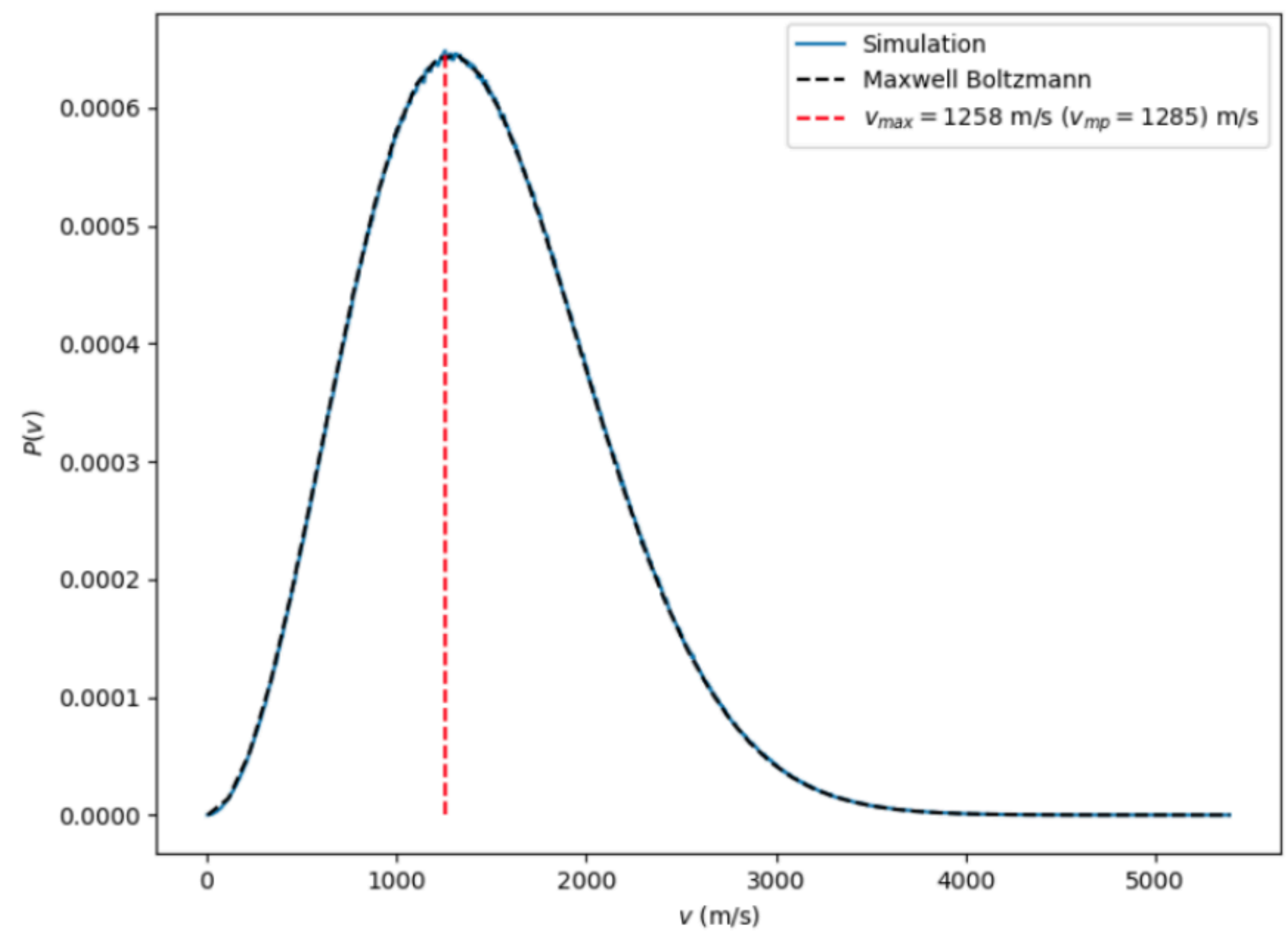
Waves in collision-less plasma !

Thermal distribution of protons

- Collection of protons are initialised with a Maxwellian distribution with temperature T_p

- Thermal velocity of the protons

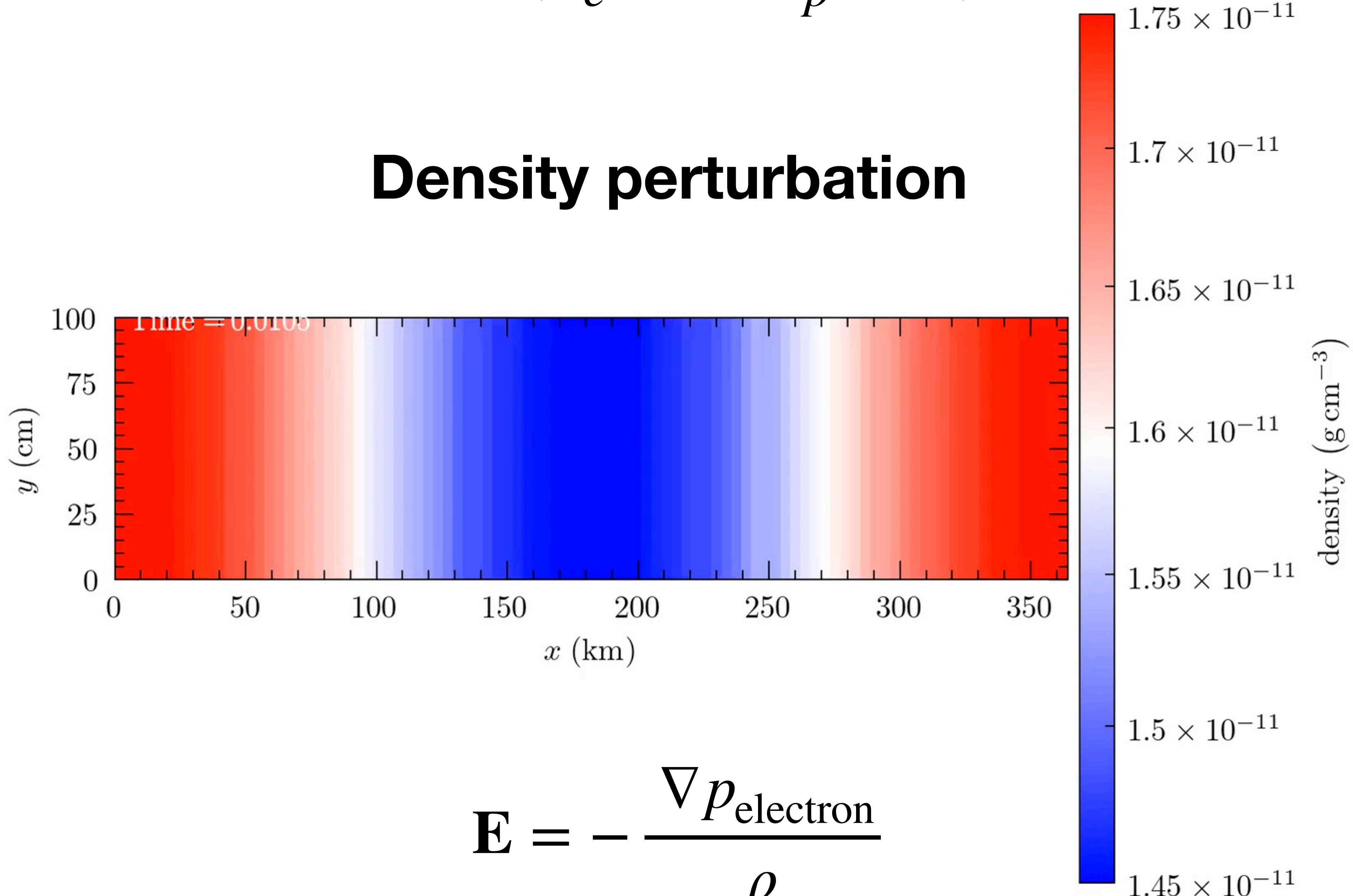
$$v_{\mathrm{rms}} = \sqrt{\frac{3k_b\mathrm{T}_p}{m_p}}$$



Landau Damping

Thermal plasma $(T_e \neq 0, T_p \neq 0)$ and $\mathbf{B} = 0$

Density perturbation



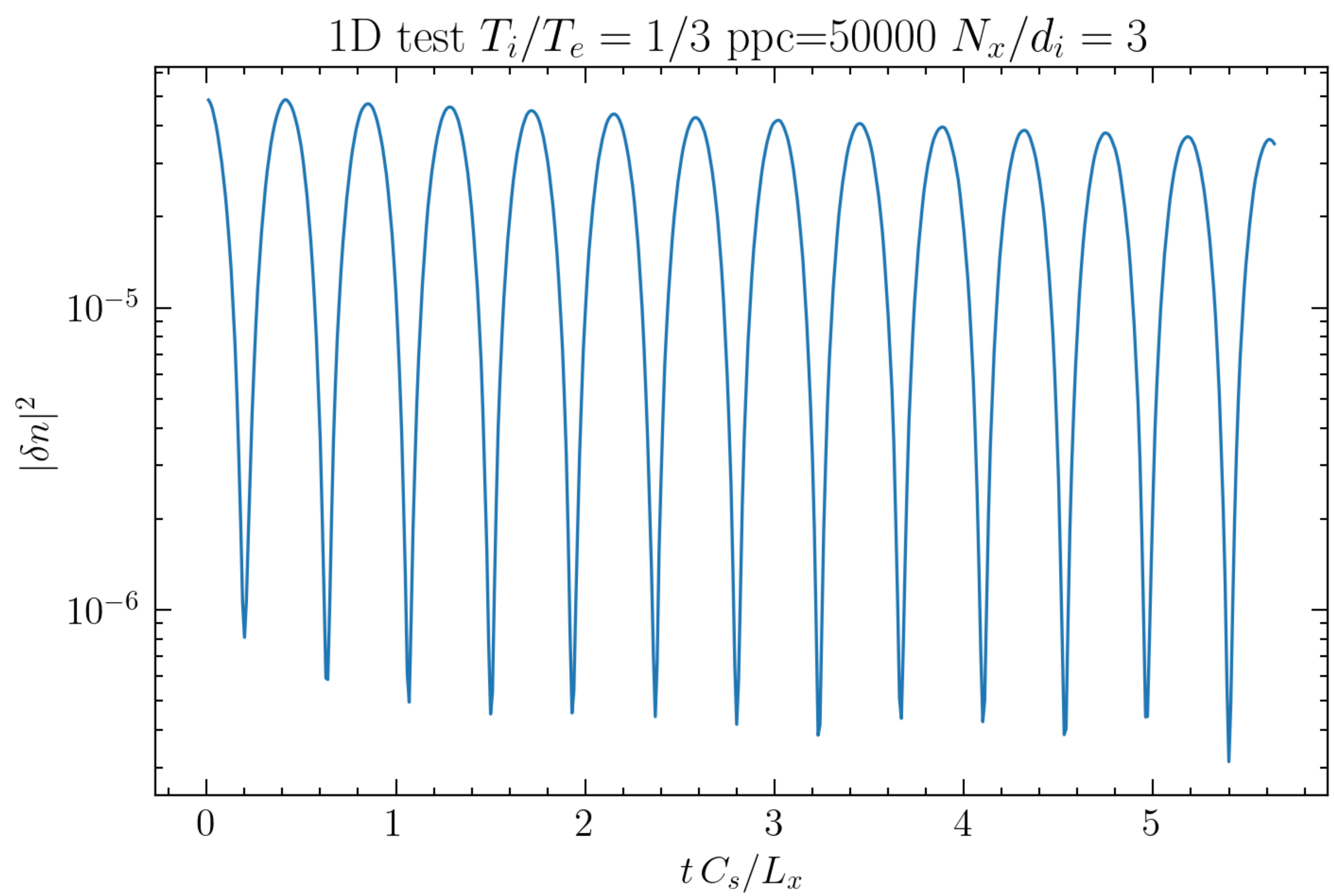
$$\mathbf{E} = -\frac{\nabla p_{\mathrm{electron}}}{\rho}$$

Landau Damping

- Setup a density perturbation in the plasma (ion-acoustic wave)

- $$\mathbf{E} = -\frac{\nabla p_{\text{electron}}}{\rho}$$

- Wave loses energy to particles



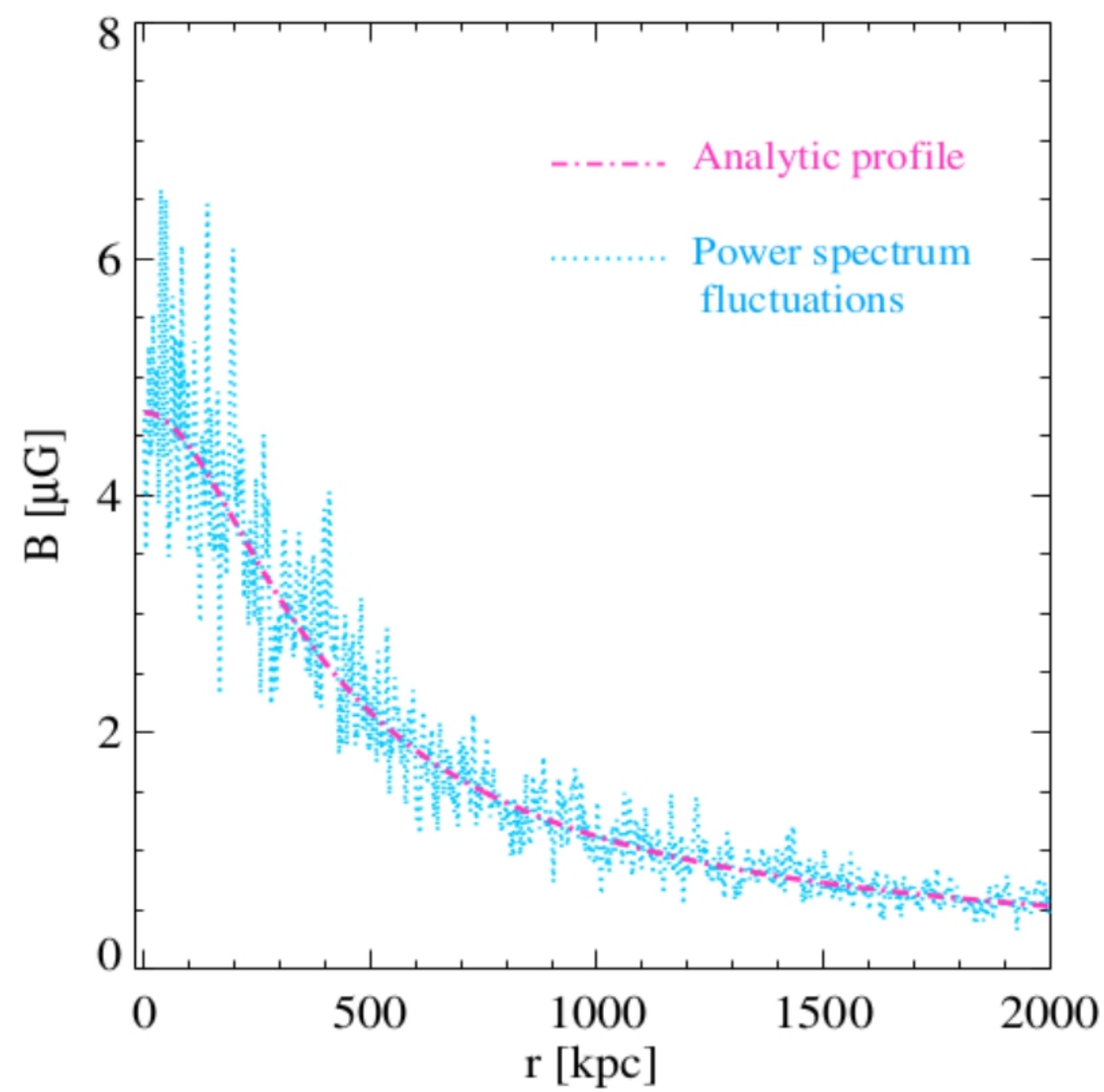
Wave-particle interaction!

Hot Intracluster Medium

- Very hot ($T \sim 10^7 - 10^8 K$) and very diffuse plasma $(n \sim 10^{-3} cm^{-3})$
- $\lambda_{mpc} \sim 10$ kpc and $L \sim 100$ kpc; $\lambda_{mfp}/L \sim 0.1$
- $\mathscr{M}_{avg} \sim 0.1$
- Kinetic Reynolds number $Re \lessapprox 100$
- Magnetic Reynolds number $Rm \gg 1$
- Magnetic Prandtl number $Pm \gg 1$
- Turbulent : Galaxy mergers, wakes of infall events, AGN feedback events, shocks

Magnetic Field

- Magnetic field observations: Faraday rotation, synchrotron radiation
- Strong fields with strength ~ μGauss observed in galaxy clusters, close to the equipartition value of magnetic field for the ICM (Carilli & Taylor 2002, Govoni & Feretti 2004, Bonafede et al. 2010)



Bonafede et. al. 2010

Small-Scale Turbulent Dynamo

- In the ideal MHD limit, the magnetic field lines are frozen into the plasma
- Fluid motions stretch-twist-fold the flux tubes
- Exponential amplification of magnetic energy, $E_m \propto e^{\Gamma t}$
- SSD is a generic mechanism for turbulent magnetised plasma (Vainshtein & Zel'dovich 1972; Schekochihin et al. 2004; Federrath et al. 2011; Seta et al. 2020)

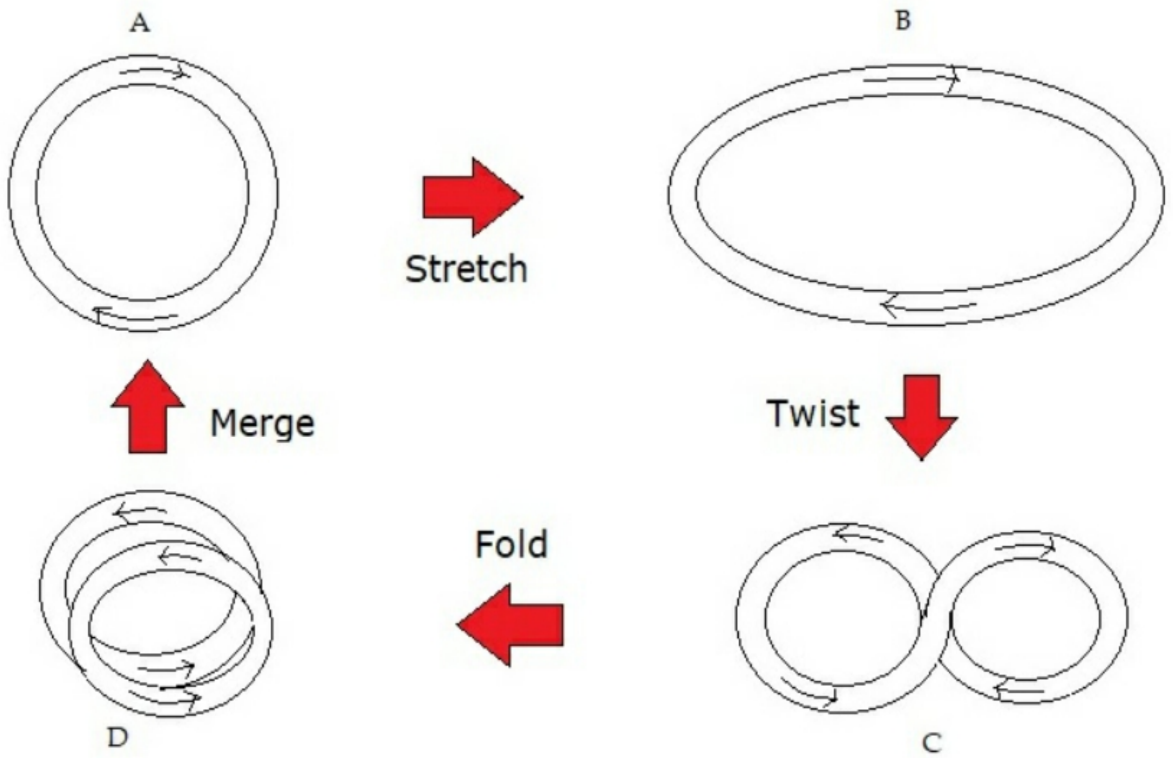


Illustration of the stretch-twist-fold model of the MHD small-scale dynamo

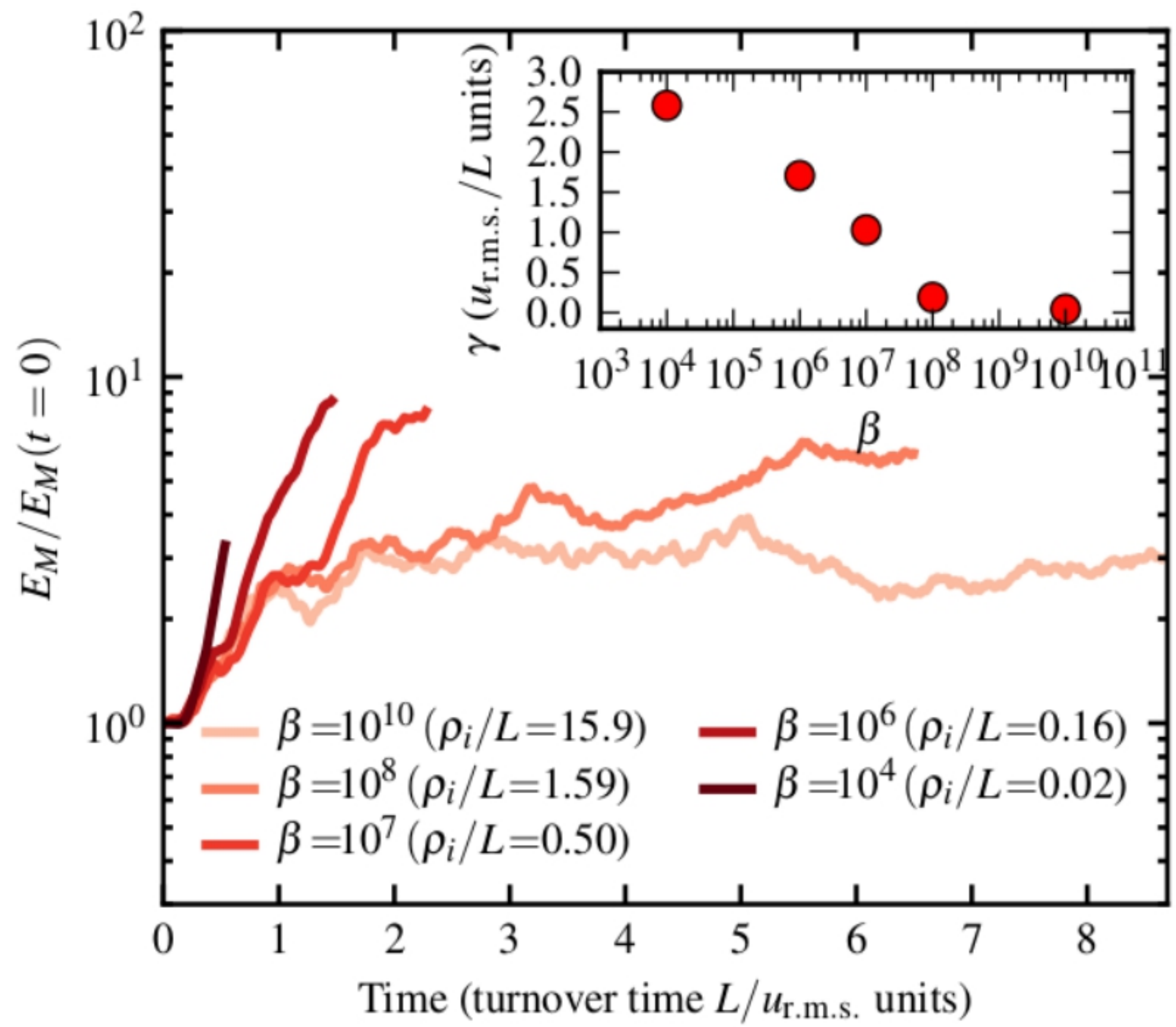
PIC Dynamo Simulations

Possibility of a “Plasma dynamo" has only been explored recently by numerical simulations

(Rincon et al.2016, St-Onge and Kunz 2018)

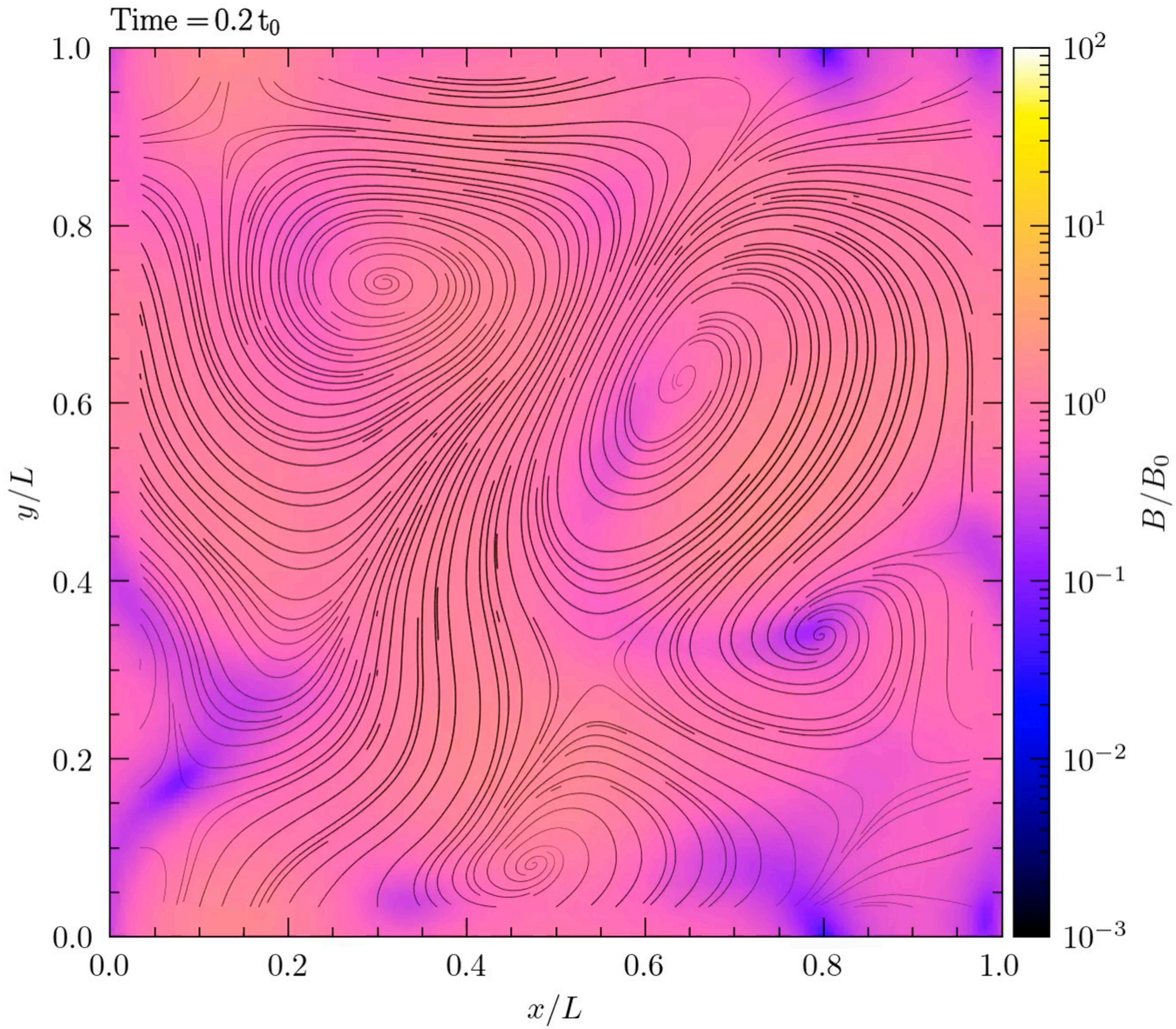
$$Re_{||} \sim \mathscr{M} \frac{L}{\lambda_{mfp}}$$

Schekochihin & Cowley 2005, 2006

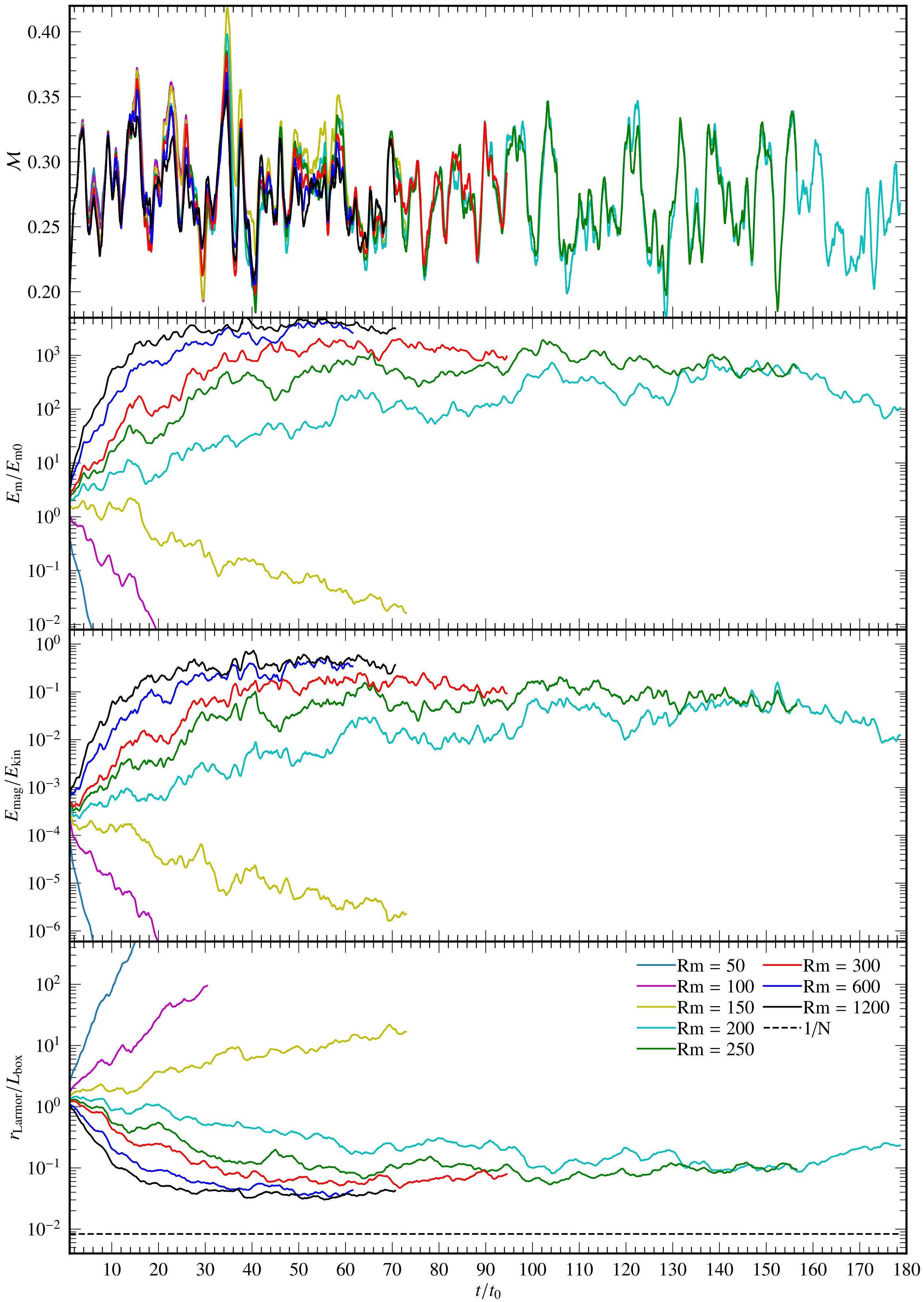


Growth of magnetic energy as a function of time from plasma dynamo simulations adopted from Rincon et al. 2016

Magnetic energy



Time Evolution of the Plasma Dynamo



$$\mathscr{M} \sim \frac{V}{c_s} \sim 0.3$$

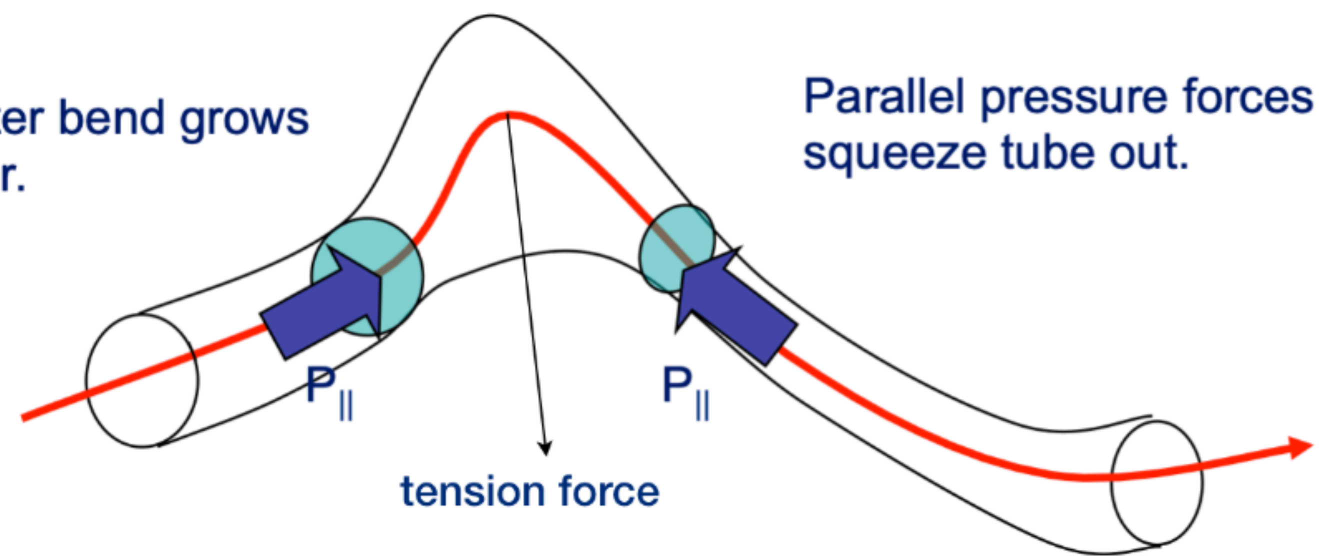
Solenoidal (rotational) driving

Reynolds number ~ 50 - 1200

Growth rate : $E_m \propto e^{\Gamma t}$

Saturation efficiency :
$E_{\rm mag}/E_{\rm kin}$ **at saturation**

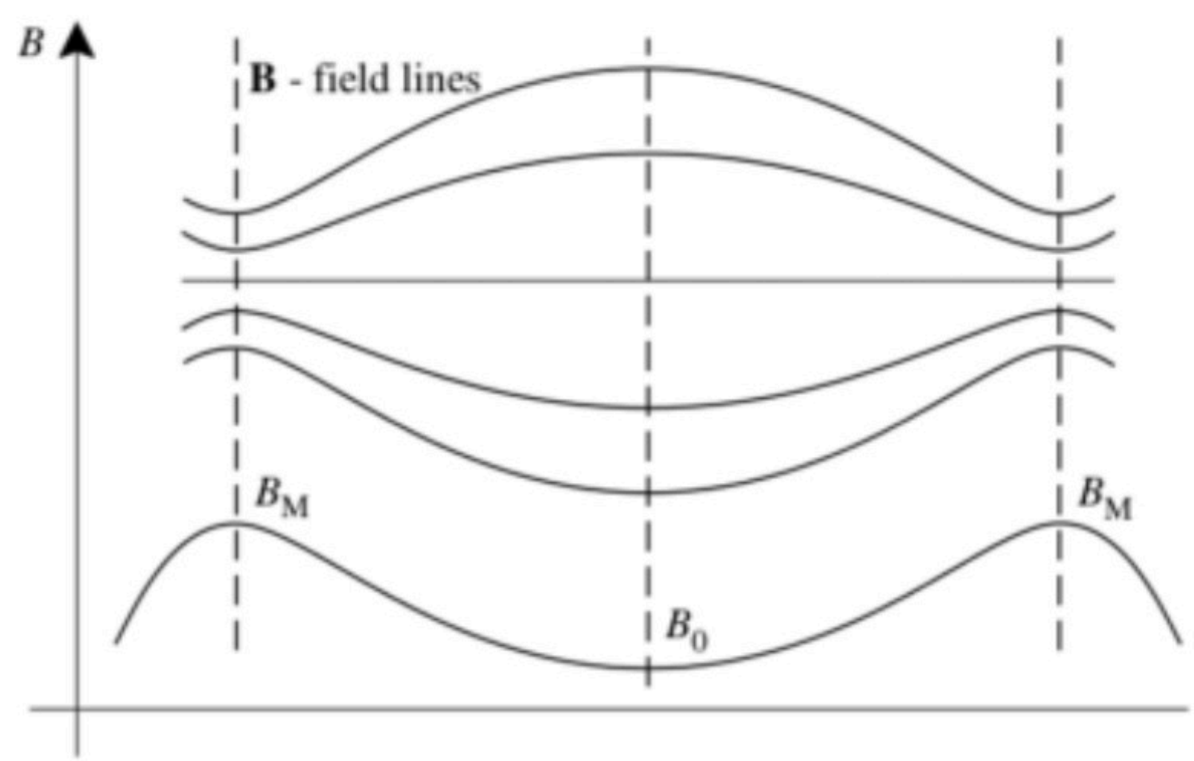
Kinetic Instabilities



Rosenbluth 1956
Parker 1958

St-Onge 2019

$$\left|\frac{p_\perp - p_\parallel}{p_\parallel}\right| \gtrapprox \frac{1}{\beta}$$



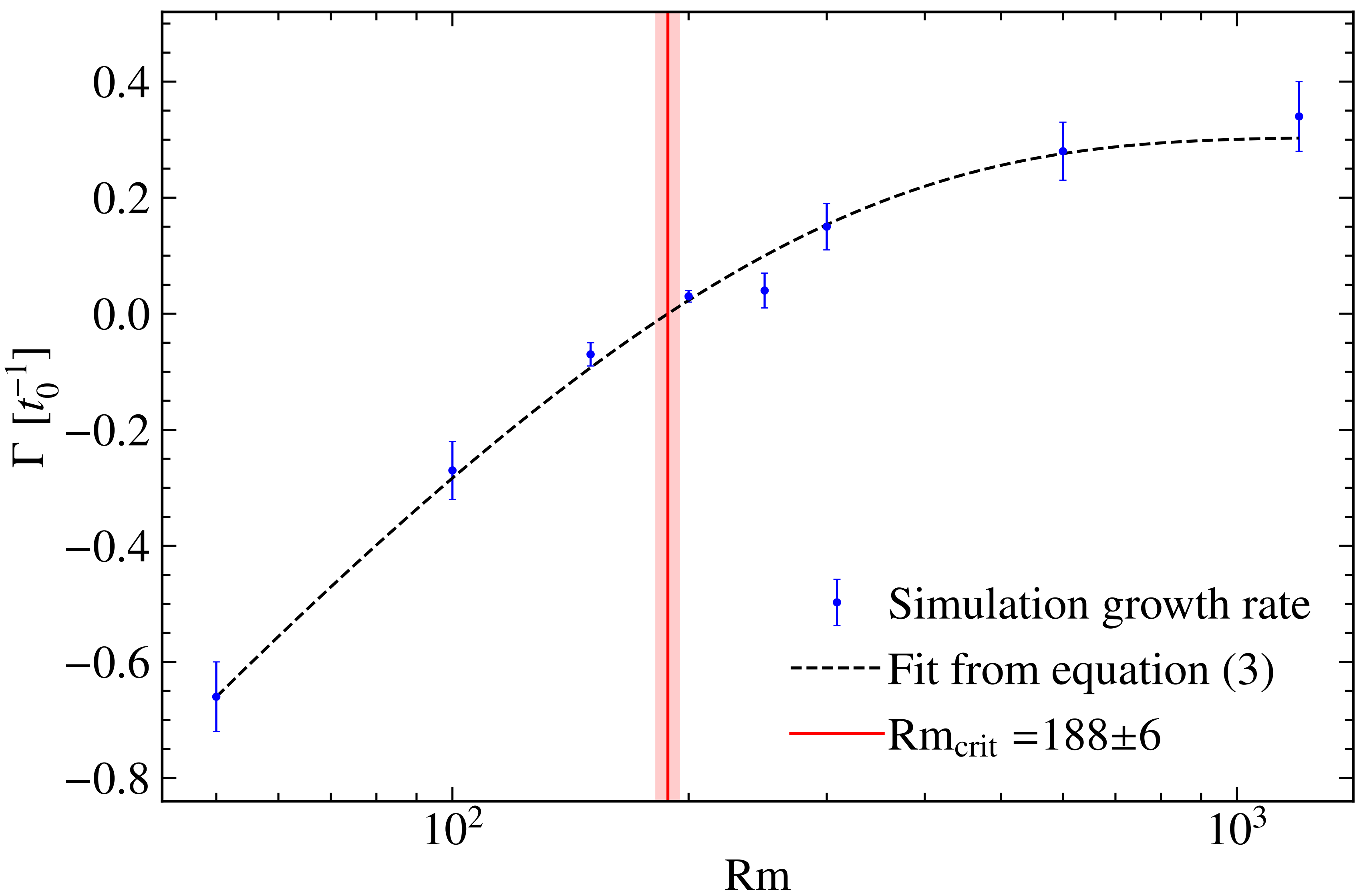
Kunz, Schekochihin, Stone 2014

Melville, Schekochihin, Kunz 2016

Boyd & Sanderson 2004

Growth Rate Vs Rm

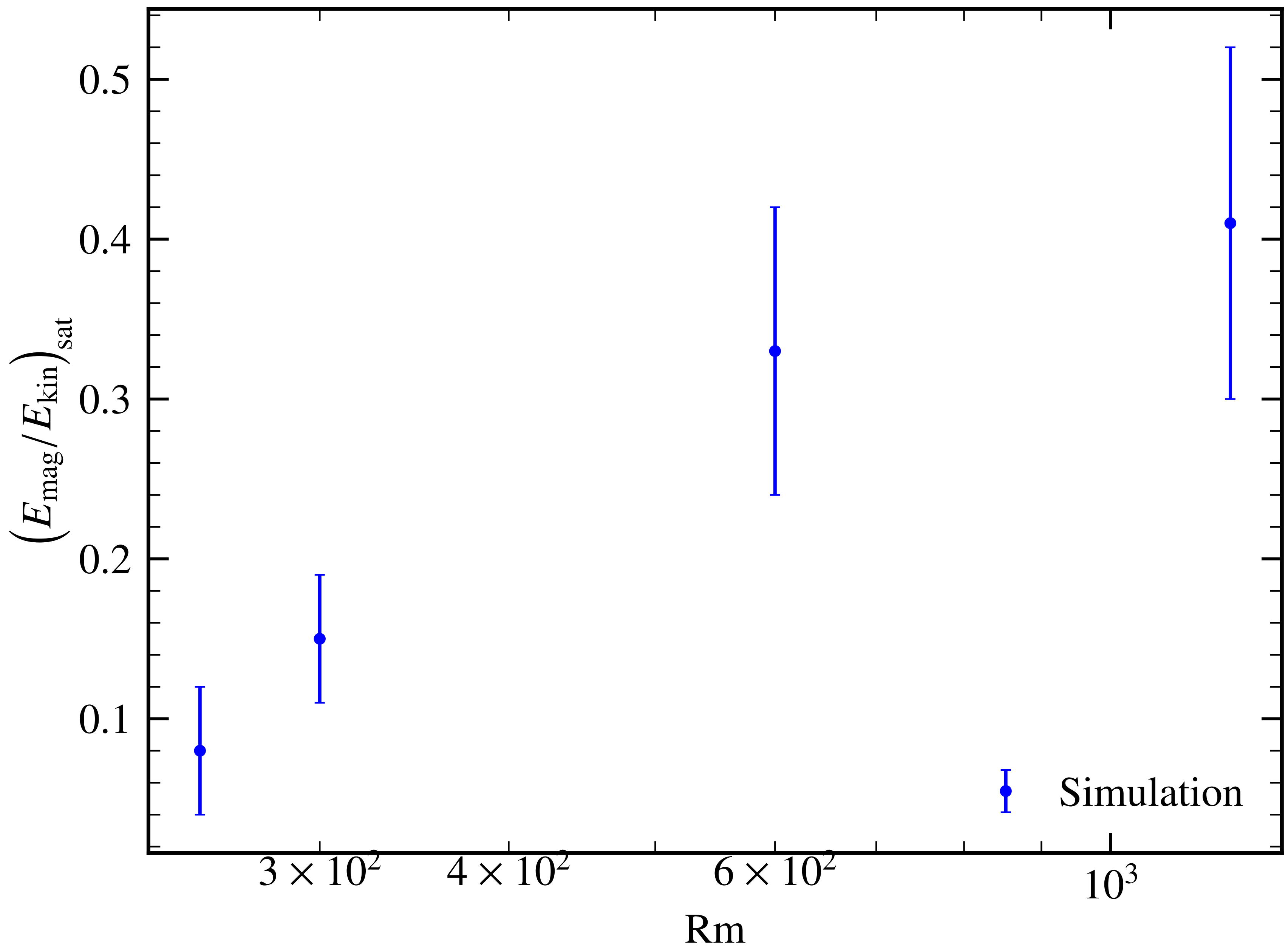
$E_m \propto e^{\Gamma t}$



$\mathrm{Rm_{crit}} \sim 188$

~ MHD turbulent dynamo

Saturation Efficiency Vs Rm



Close to equipartition values for magnetic energy at high Rm

Summary

- Motivation for kinetic approaches
- Hybrid kinetics - derivation and numerical scheme
- Particle motion with a hybrid-PIC code
- Waves in a collision-less plasma
- Wave-particle interaction —> energy transfer in collision-less plasma
- Collision-less turbulent dynamo in the ICM