Astrophysical Gas Dynamics

TODAY:

- *Rankine-Hugoniot shock jump conditions (→ recap and finish)*
- *Propagation of a 1-dimensional (1D) shock front*

Image credit: M. S. Povich

Astrophysical Gas Dynamics

TODAY:

- *Rankine-Hugoniot shock jump conditions (→ recap and finish)*
- *Propagation of a 1-dimensional (1D) shock front*

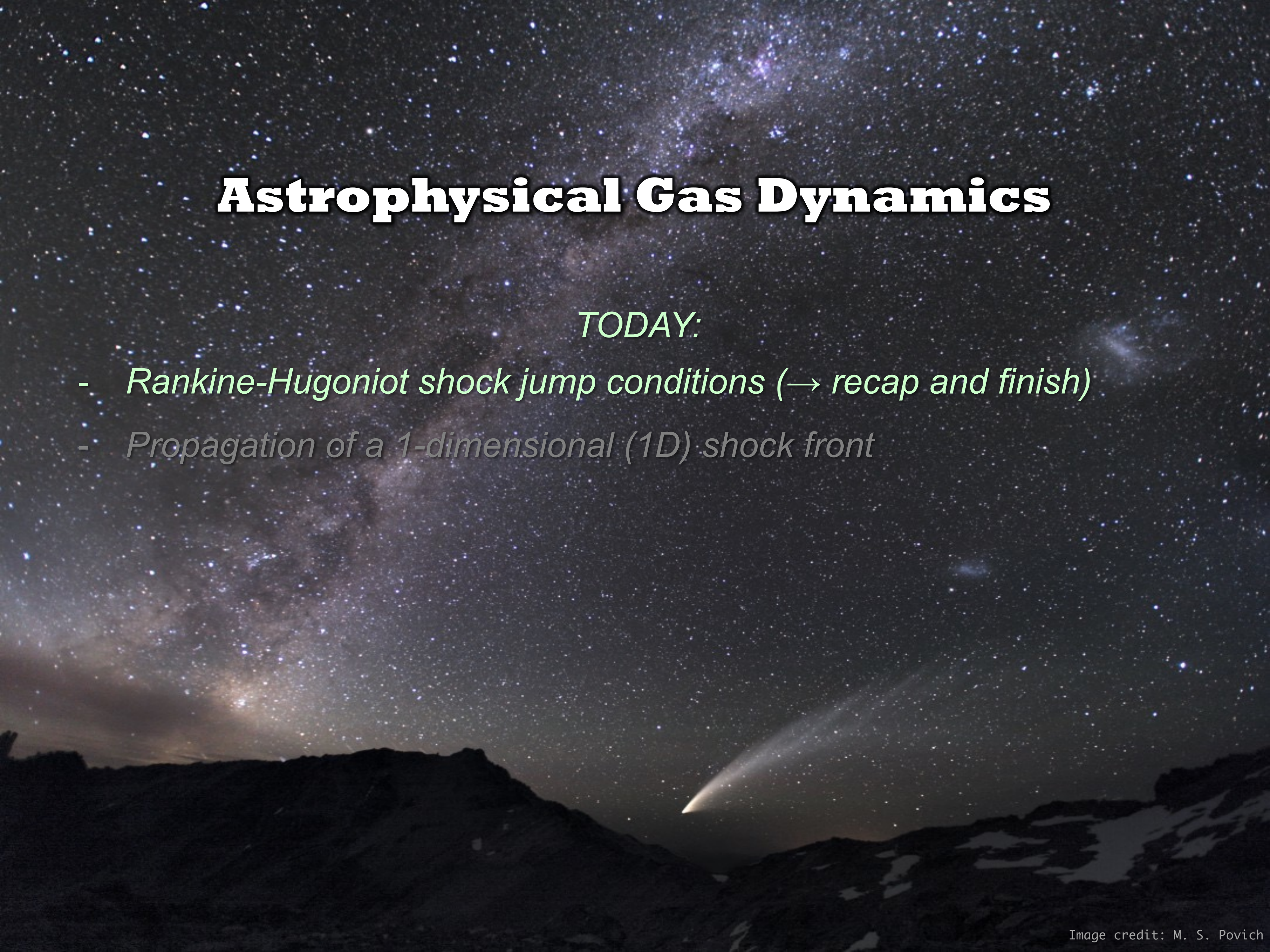
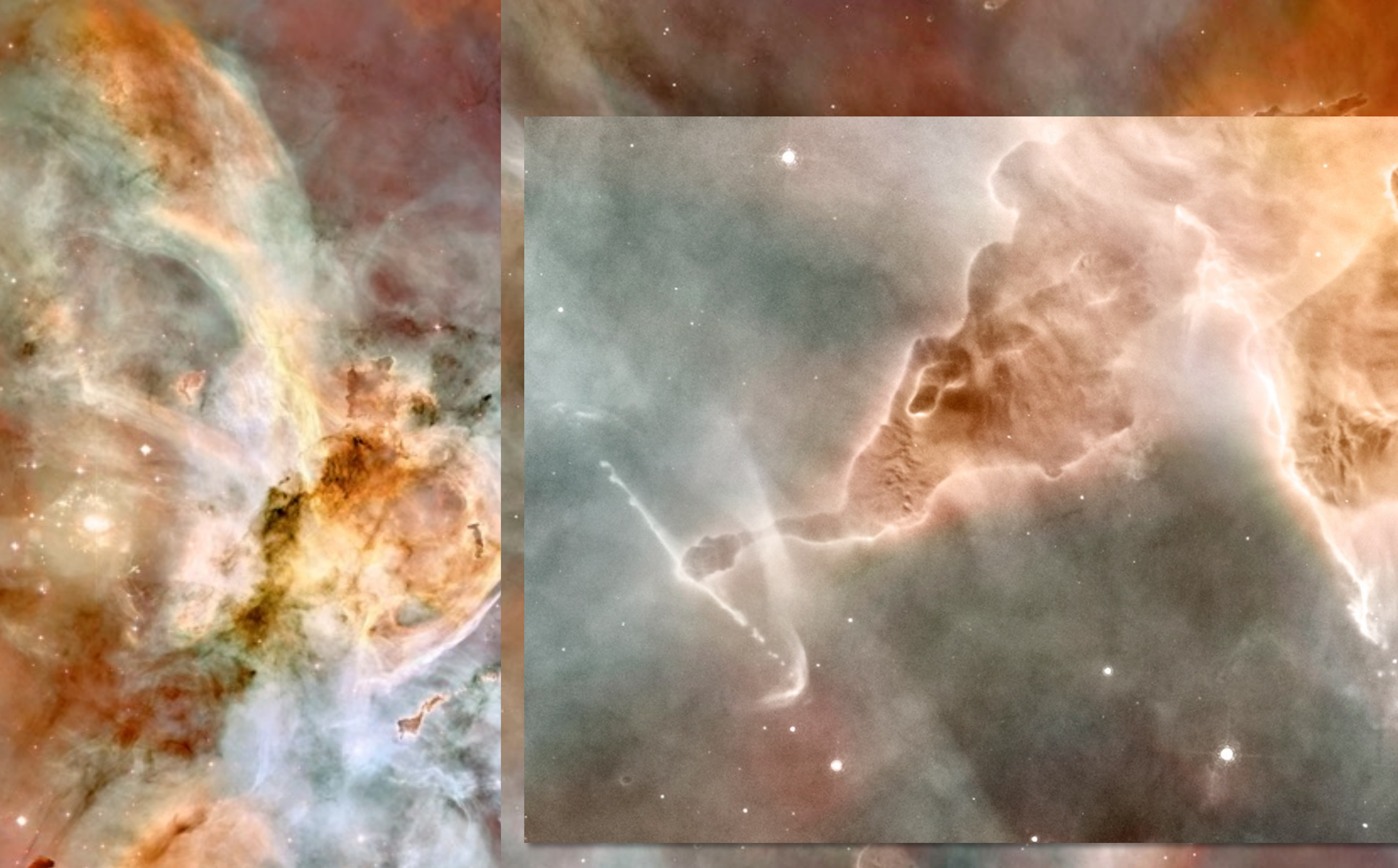


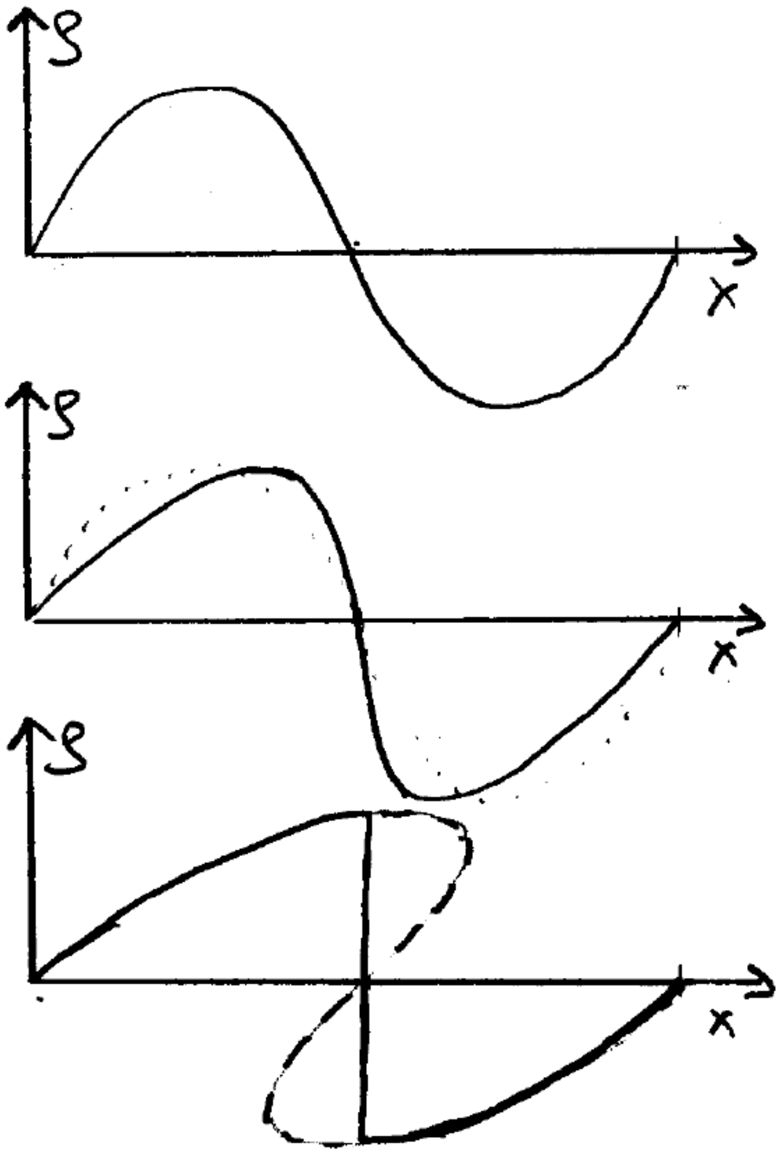
Image credit: M. S. Povich



Shocks occur when gas moves faster than the speed of sound

Carina Nebula, NASA, ESA, N. Smith (University of California, Berkeley), and The Hubble Heritage Team (STScI/AURA), and NOAO/AURA/NSF

Sound waves → Shock waves



Polytropic gas EOS:

$$P \propto \rho^{\Gamma}$$

Sound speed:

$$c_{\mathrm{s}} \propto \rho^{(\Gamma-1)/2}$$

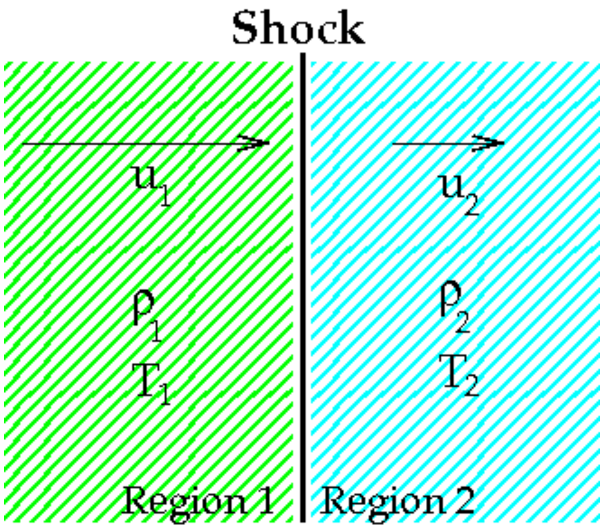
Sound propagates faster in denser regions...

→ Steepening → Shock

Steepening of a sound wave (Klessen lecture notes on Theoretical Astrophysics)

Shock waves

- Python program to solve 1D hydro equations: hydro.py
- > ./hydro.py -h
- > ./hydro.py -sim shocktube_test



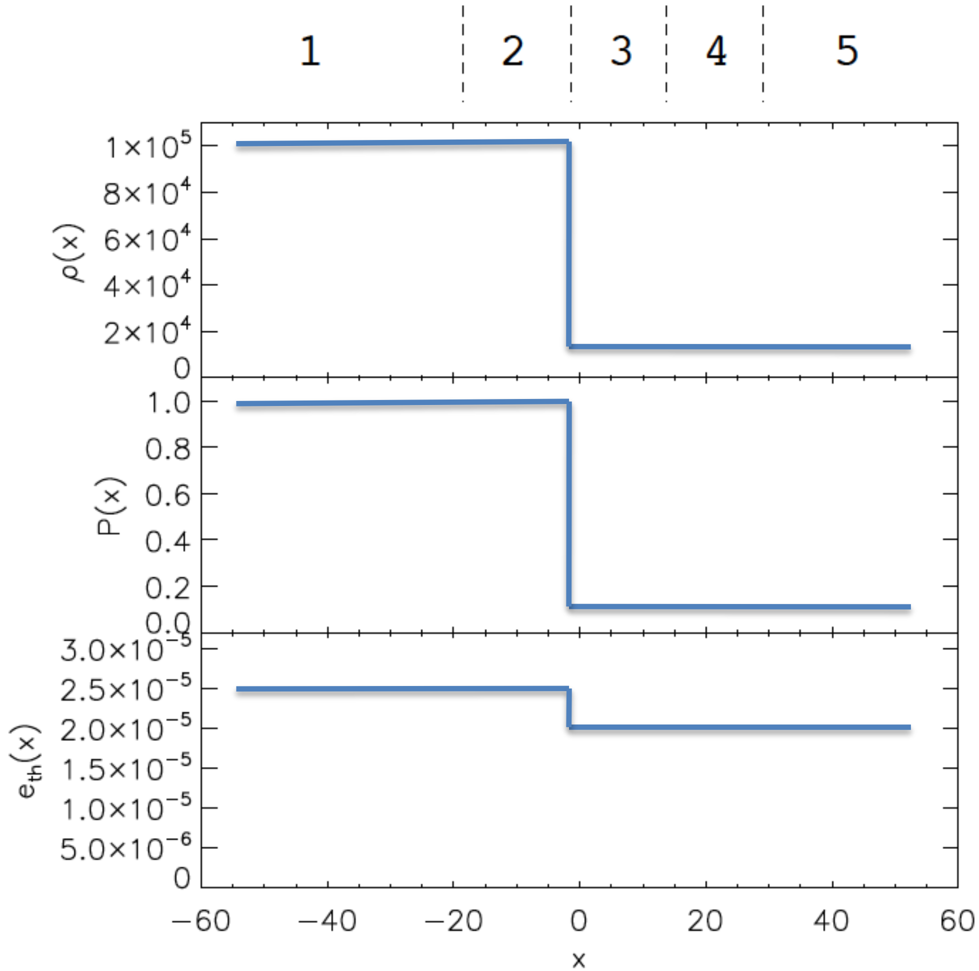
→ Rankine-Hugoniot shock jump conditions

This code uses cftools:
https://www.mso.anu.edu.au/~chfeder/teaching/astr_4012_8002/codes/cftools/

IDL> shocktube_test

Sod shocktube test: $\rho_l = 10^5, P_l = 1$ $\rho_r = 1.25 \times 10^4$ and $P_r = 0.1$

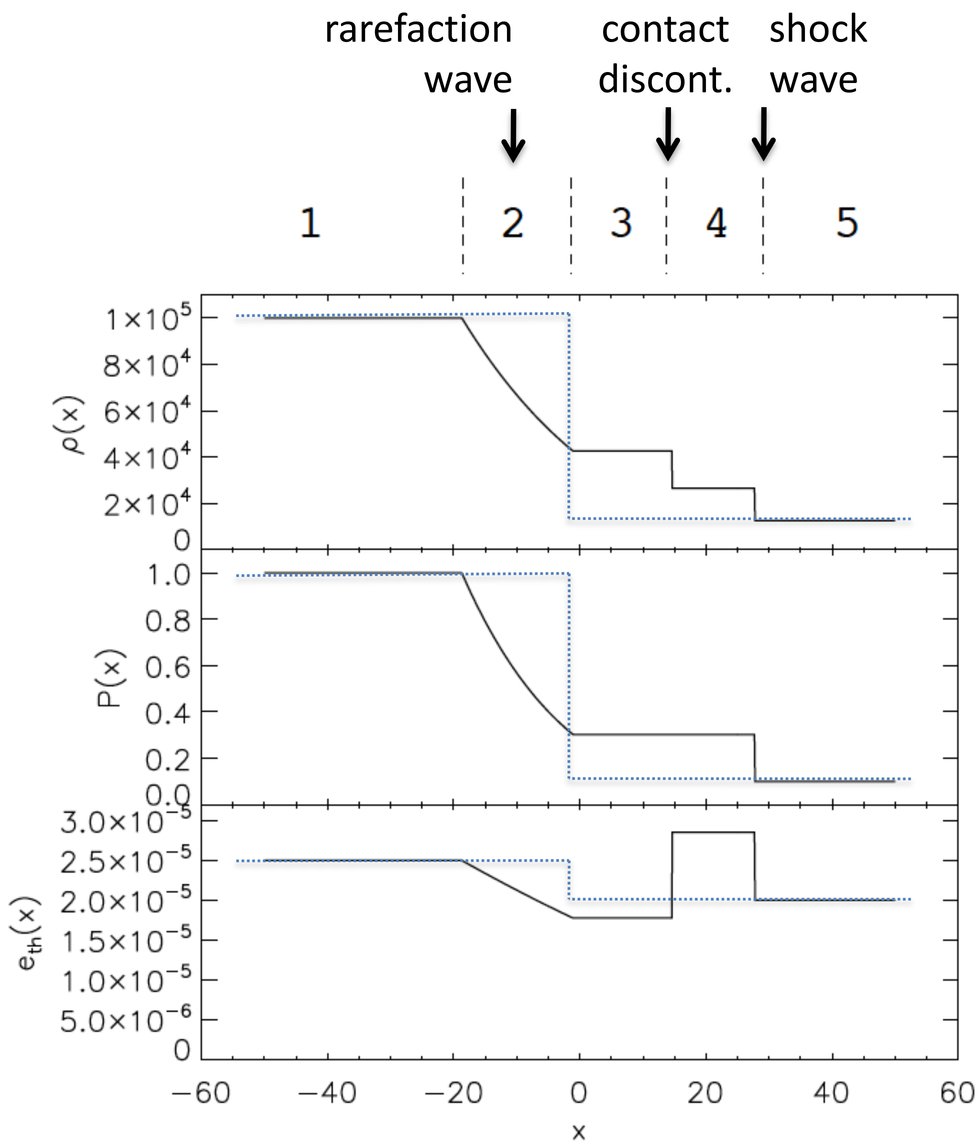
(Sod 1978)



IDL> shocktube_test

Sod shocktube test: $\rho_l = 10^5, P_l = 1$ $\rho_r = 1.25 \times 10^4$ and $P_r = 0.1$

(Sod 1978)



Using the Rankine-Hugoniot shock jump conditions

Using the Rankine-Hugoniot shock jump conditions to derive

the density variance – Mach number relation of supersonic turbulence

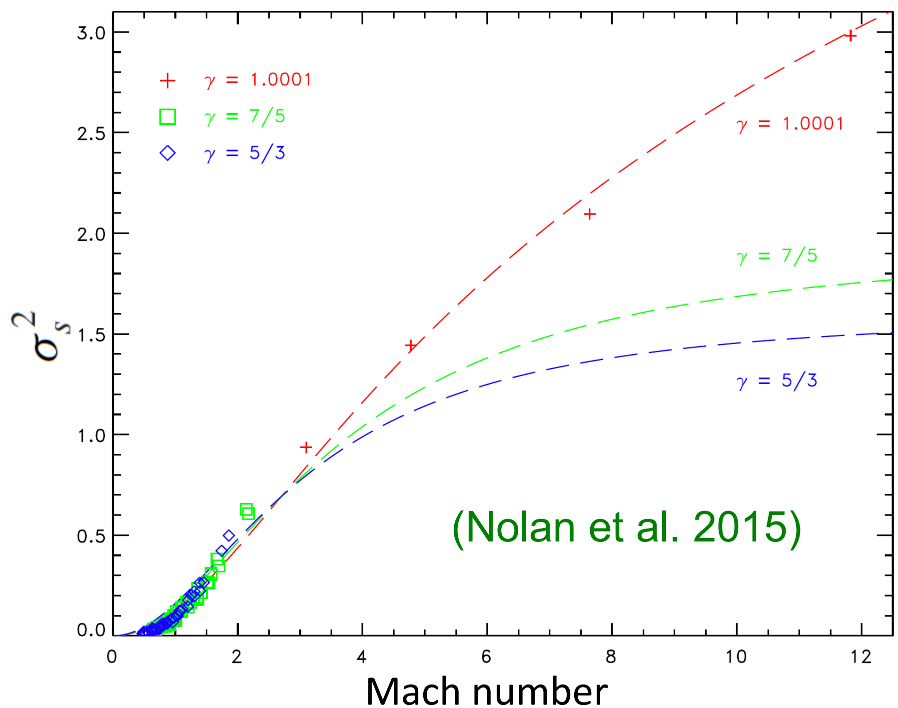
$$\frac{\rho}{\rho_0} = \frac{v_0}{v} = \frac{(\gamma+1)b^2\mathcal{M}^2}{(\gamma-1)b^2\mathcal{M}^2+2}$$

(Nolan et al. 2015)

with $\sigma^2_{\rho/\rho_0} = \frac{1}{V}\int_V \left(\frac{\rho}{\rho_0} - 1\right)^2 \mathrm{d}V$ (Padoan & Nordlund 2011)

gives $\sigma_s^2 = \ln\left(1 + \frac{(\gamma+1)b^2\mathcal{M}^2}{(\gamma-1)b^2\mathcal{M}^2+2}\right)$

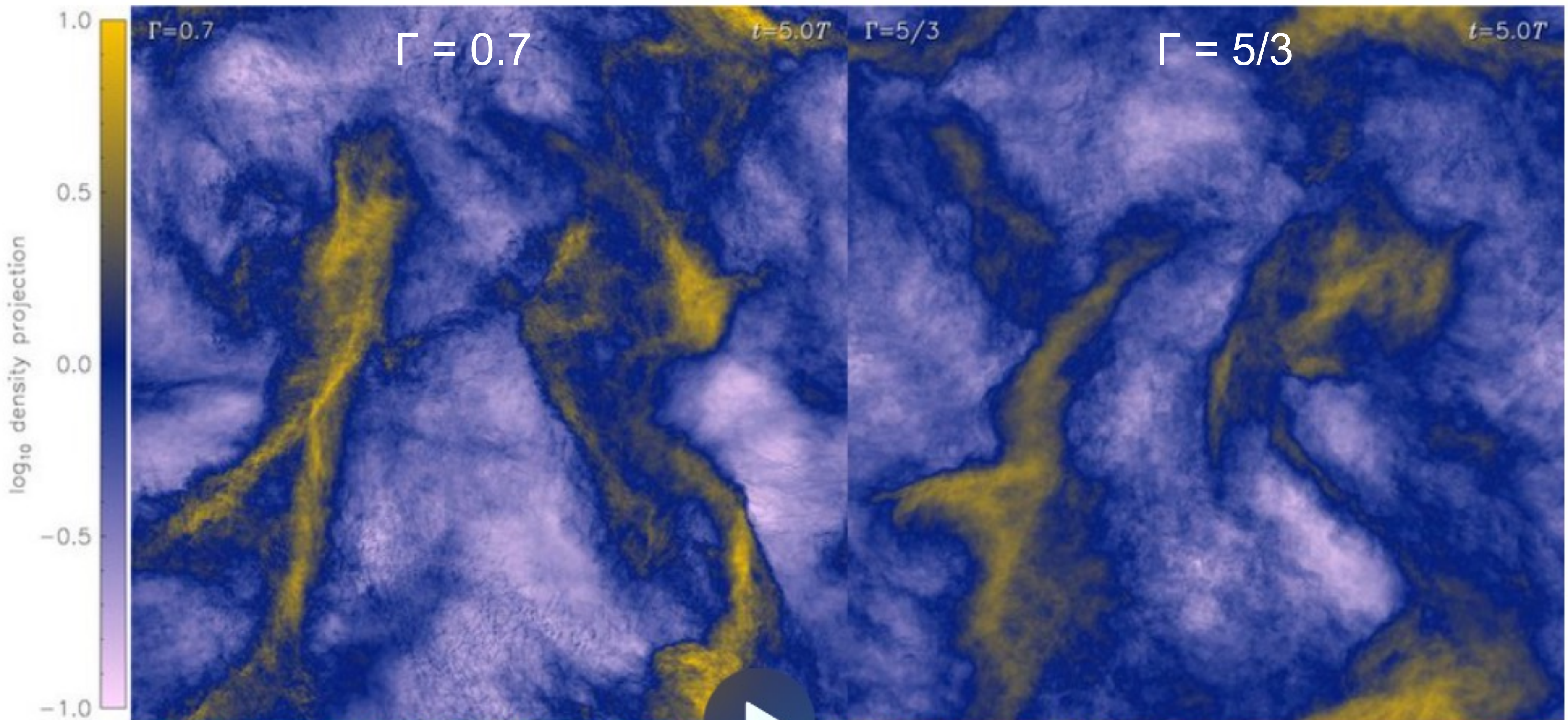
Density variance – Mach number relation
for supersonic **adiabatic** turbulence



(Nolan et al. 2015)

Equation of State – Polytropic EOS $P_{\mathrm{th}} = K\rho^{\Gamma}$

Using the Rankine-Hugoniot shock jump conditions to derive
the density variance – Mach number relation of supersonic turbulence



For supersonic **polytropic** turbulence (Federrath & Banerjee 2015)

Movies available: http://www.mso.anu.edu.au/~chfeder/pubs/polytropic/polytropic.html

Using the Rankine-Hugoniot shock jump conditions

Using the Rankine-Hugoniot shock jump conditions to derive

the density variance – Mach number relation of supersonic turbulence

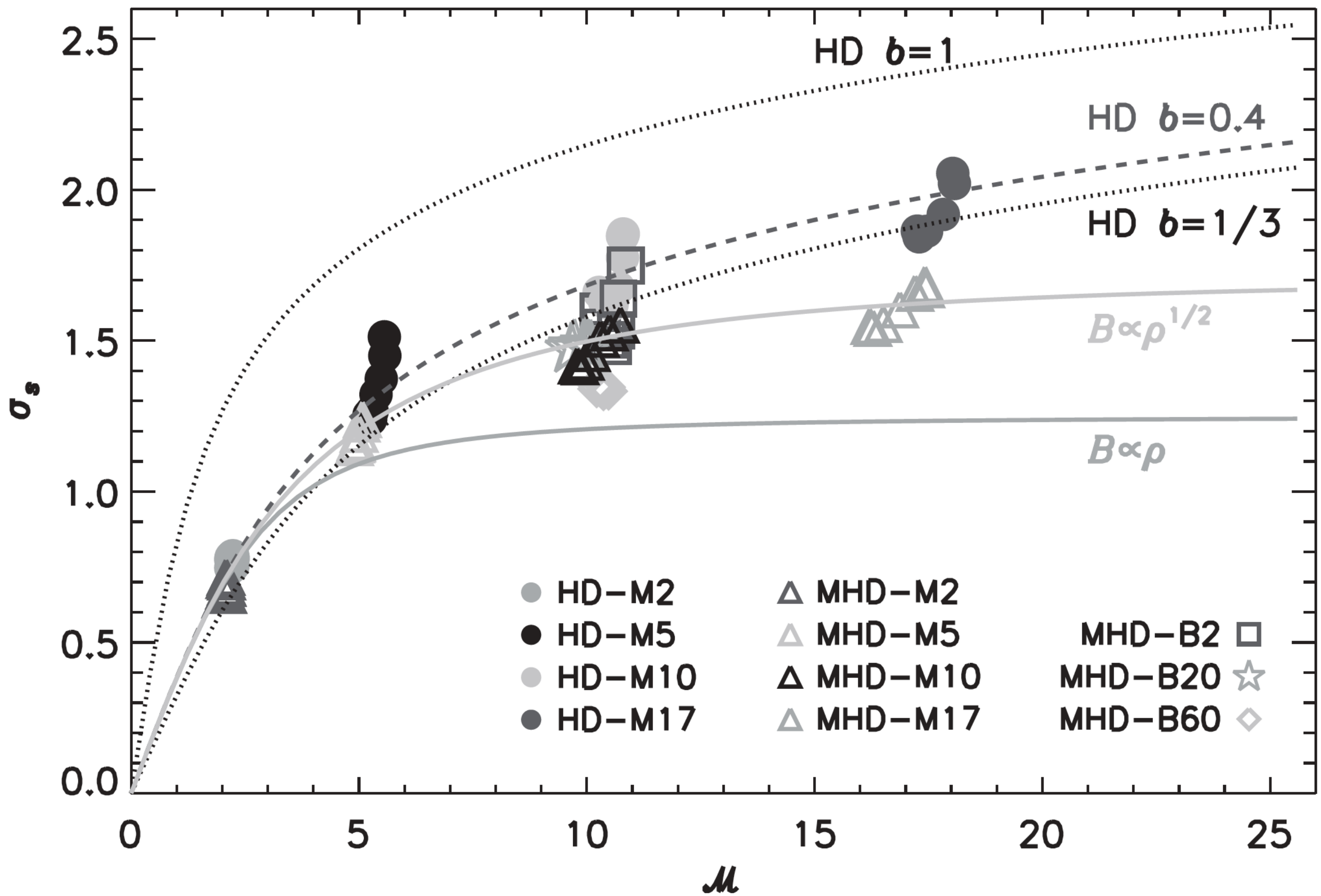
With magnetic field (Molina et al. 2012)

$$\rho_1 \left(v_{\|,1}^2 + \frac{c_{\mathrm{s},1}^2}{\gamma_1} + \frac{v_{\mathrm{A}\perp,1}^2}{2} \right) = \rho_2 \left(v_{\|,2}^2 + \frac{c_{\mathrm{s},2}^2}{\gamma_2} + \frac{v_{\mathrm{A}\perp,2}^2}{2} \right)$$

Using the Rankine-Hugoniot shock jump conditions

Using the Rankine-Hugoniot shock jump conditions to derive the density variance – Mach number relation of supersonic turbulence

With magnetic field (Molina et al. 2012)



The Star Formation Rate – Magnetic fields

Statistical Theory for the Star Formation Rate:

$$p(s) = \frac{1}{\sqrt{2\pi\sigma_s^2}} \exp\left(-\frac{(s-s_0)^2}{2\sigma_s^2}\right)$$

$$s = \ln(\rho/\rho_0) \qquad t_{\mathrm{ff}}(\rho) = \left(\frac{3\pi}{32G\rho}\right)^{1/2}$$

SFR ~ Mass/time

$$\mathrm{SFR}_{\mathrm{ff}} = \epsilon \int_{s_{\mathrm{crit}}}^{\infty} \underbrace{\frac{t_{\mathrm{ff}}(\rho_0)}{t_{\mathrm{ff}}(\rho)}}_{\text{freefall time}} \underbrace{\frac{\rho}{\rho_0}}_{\text{mass fraction}} p(s)\,\mathrm{d}s = \epsilon \int_{s_{\mathrm{crit}}}^{\infty} \exp\left(\frac{3}{2}s\right) p(s)\,\mathrm{d}s$$

$$= \frac{\epsilon}{2} \exp\left(\frac{3}{8}\sigma_s^2\right)\left[1 + \mathrm{erf}\left(\frac{\sigma_s^2 - s_{\mathrm{crit}}}{\sqrt{2\sigma_s^2}}\right)\right]$$

MAGNETIC FIELD:

$$P_{\mathrm{th}} \rightarrow P_{\mathrm{th}} + P_{\mathrm{mag}} \qquad \mathcal{M} \rightarrow \mathcal{M}\left(1+\beta^{-1}\right)^{-1/2}$$

$$s_{\mathrm{crit}} \propto \ln\left(\alpha_{\mathrm{vir}}\,\mathcal{M}^2 \frac{\beta}{\beta+1}\right)$$

$$\sigma_s^2 = \ln\left(1 + b^2 \mathcal{M}^2 \frac{\beta}{\beta+1}\right)$$

$$\mathrm{SFR}_{\mathrm{ff}} = \mathrm{SFR}_{\mathrm{ff}}\left(\alpha_{\mathrm{vir}}, b, \mathcal{M}, \beta\right)$$

(Padoan & Nordlund 2011; Molina et al. 2012)

$2E_{kin}/E_{grav}$ **forcing** **Mach number** **plasma** $\beta = P_{th}/P_{mag}$

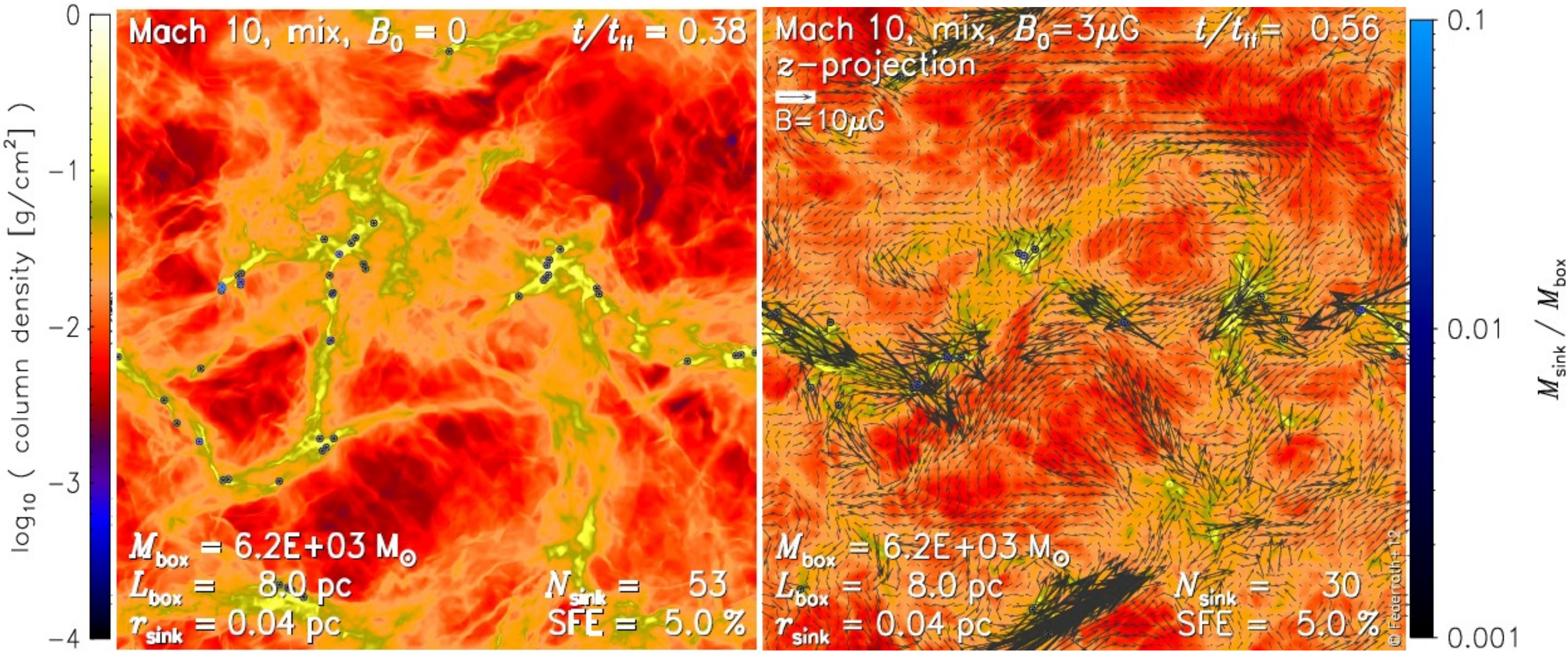
Federrath & Klessen (2012)

The Star Formation Rate – Magnetic fields

Numerical experiment for Mach 10 and $\alpha_{vir} \sim 1$

Movies available: http://www.mso.anu.edu.au/~chfeder/pubs/sfr/sfr.html

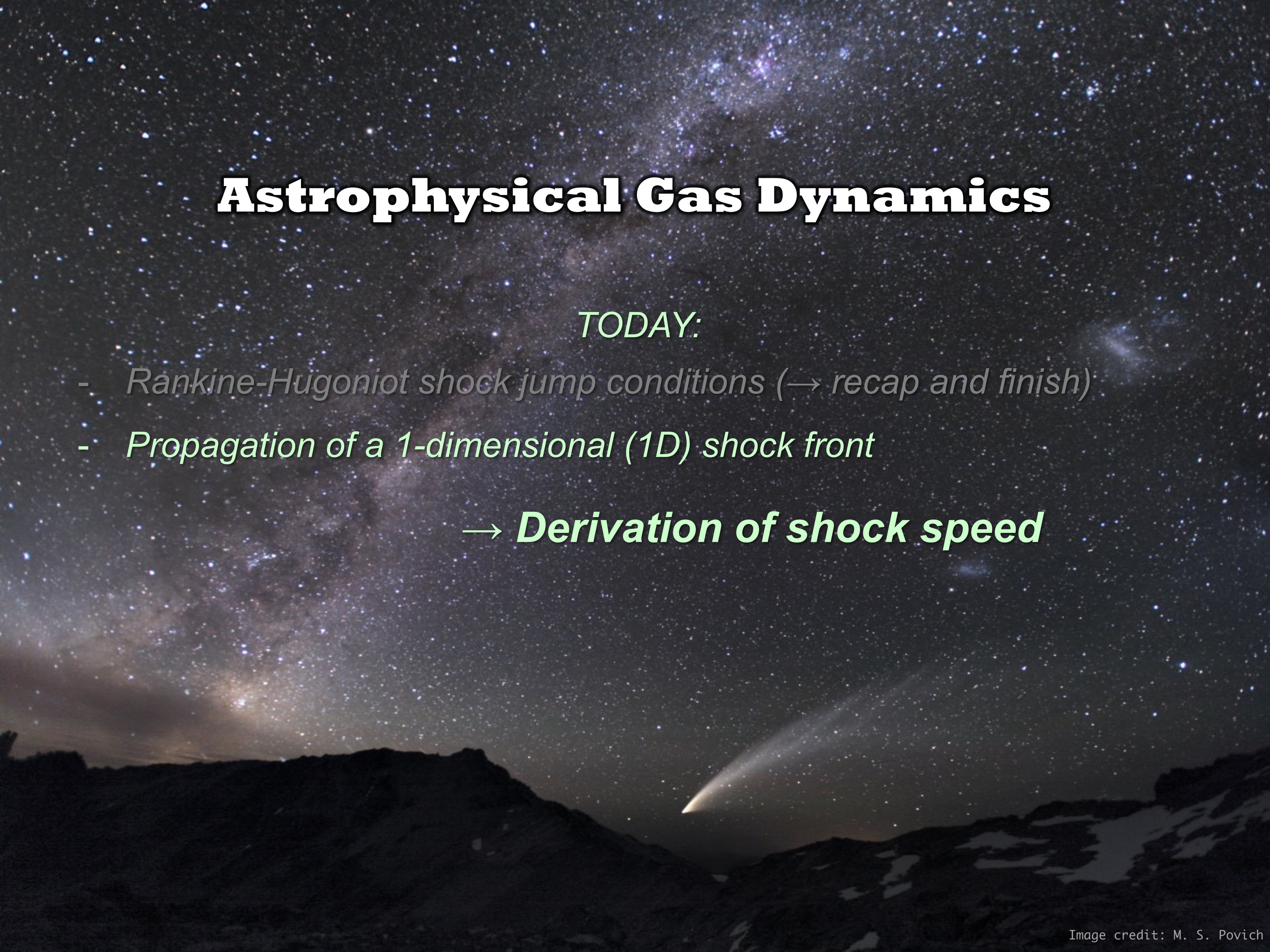
B=0 ($M_A=\infty$, $\beta = \infty$) B=3μG (M_A=2.7, β = 0.2)



SFR_{ff} (simulation) = **0.46**	x **0.63**	SFR_{ff} (simulation) = **0.29**
SFR_{ff} (theory) = **0.45**	x **0.40**	SFR_{ff} (theory) = **0.18**

Magnetic field reduces SFR and fragmentation (by factor 2) → **IMF**

Federrath & Klessen (2012)



Astrophysical Gas Dynamics

TODAY:

- *Rankine-Hugoniot shock jump conditions (→ recap and finish)*
- *Propagation of a 1-dimensional (1D) shock front*

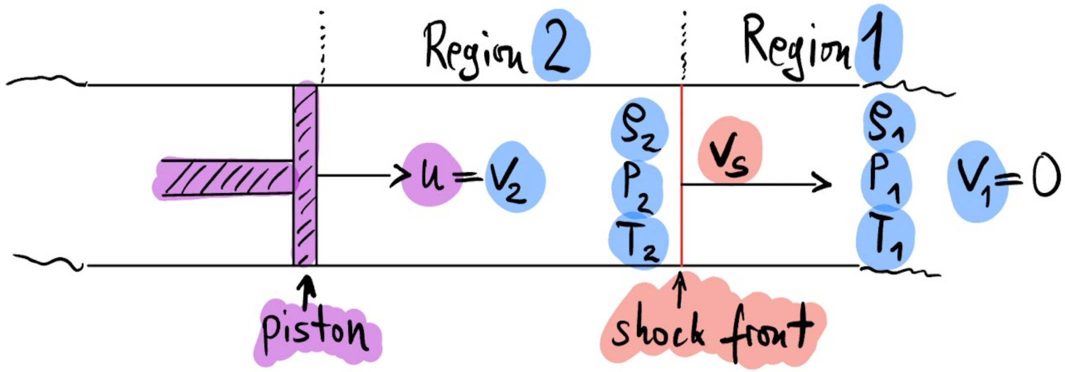
→ Derivation of shock speed

Image credit: M. S. Povich

- Python program to solve 1D hydro equations: hydro.py
- > ./hydro.py -h
- > ./hydro.py -sim shockpiston_test

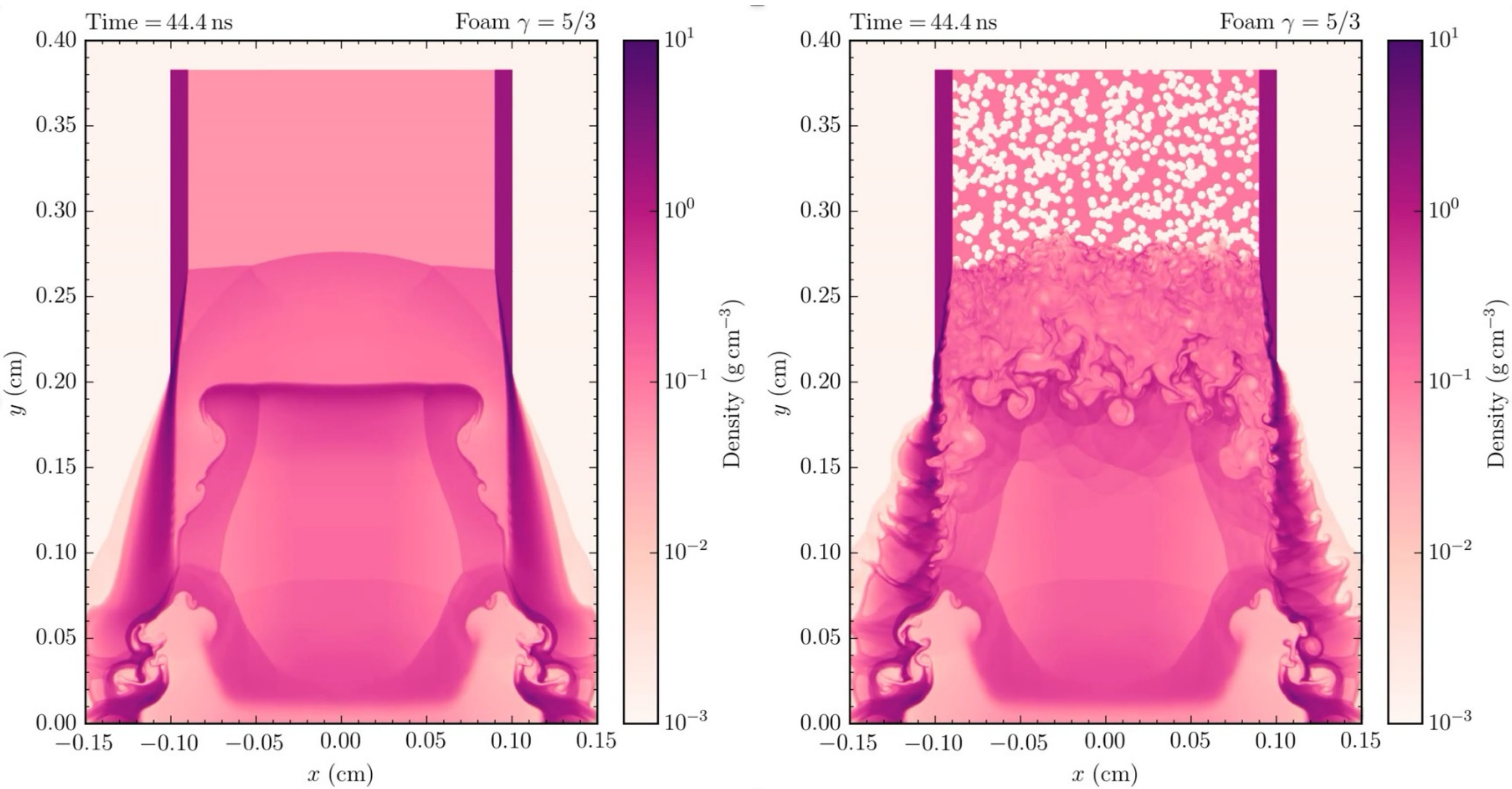
Test the derived shock speed v_s

(try for different gamma)



This code uses cftools:
https://www.mso.anu.edu.au/~chfeder/teaching/astr_4012_8002/codes/cftools/

Astrophysical Gas Dynamics - Shocks

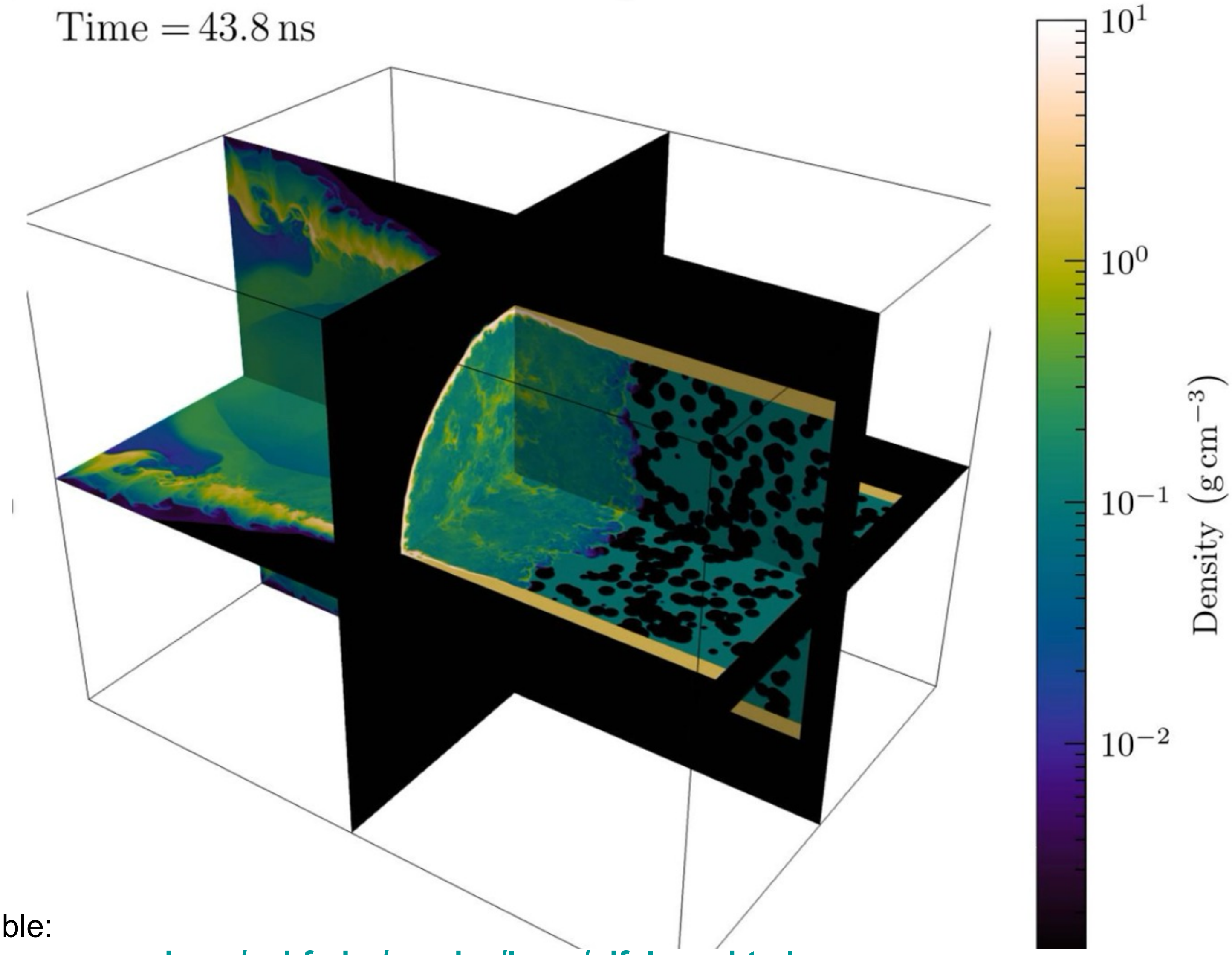


Movies available:
https://www.mso.anu.edu.au/~chfeder/movies/laser/nif_laser.html

Simulation of a laser-induced shock running into foam

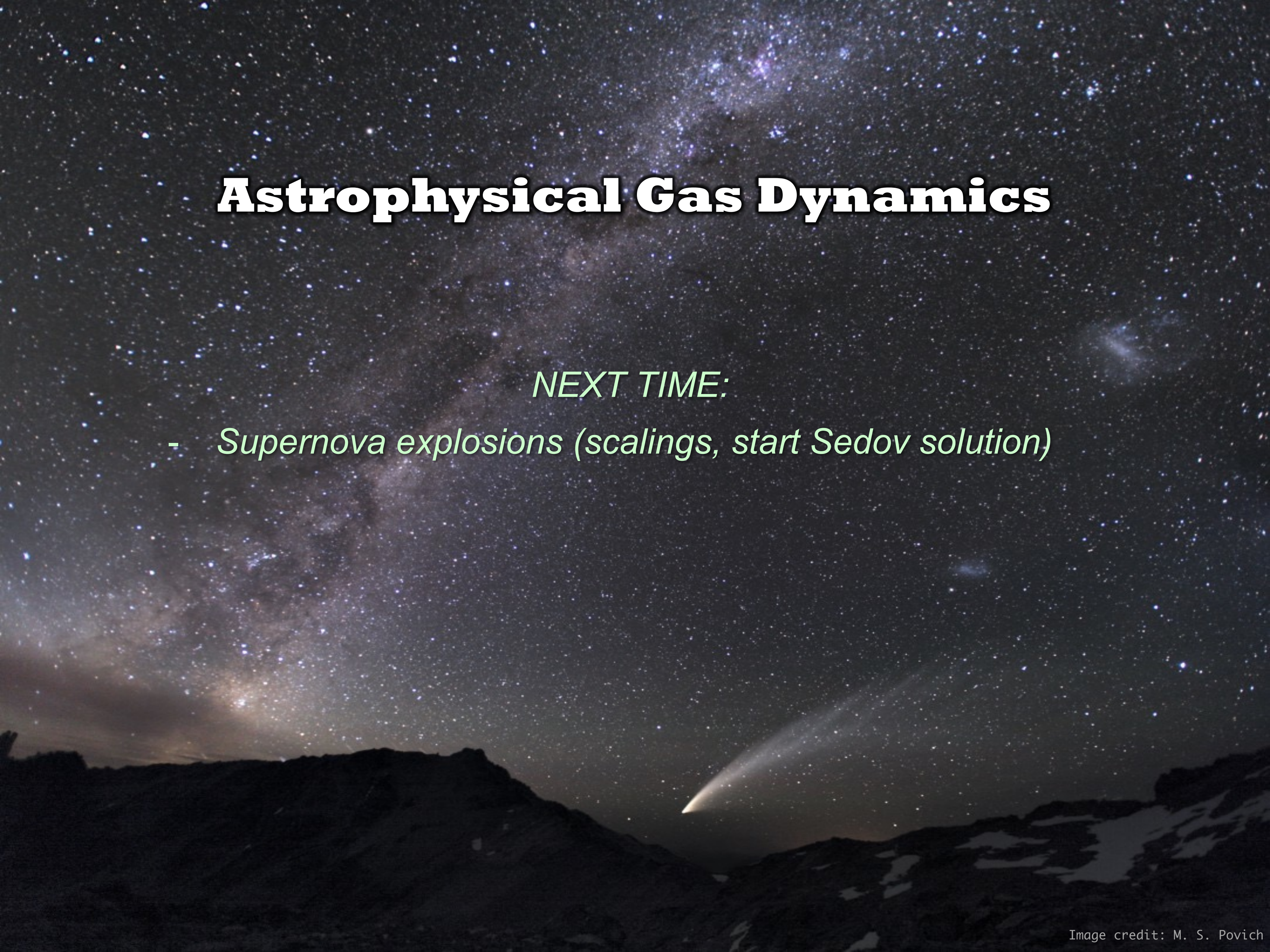
(Dhawalikar et al. 2022)

Astrophysical Gas Dynamics - Shocks



Movies available:
https://www.mso.anu.edu.au/~chfeder/movies/laser/nif_laser.html

Simulation of a laser-induced shock running into foam



Astrophysical Gas Dynamics

NEXT TIME:

- *Supernova explosions (scalings, start Sedov solution)*

Image credit: M. S. Povich