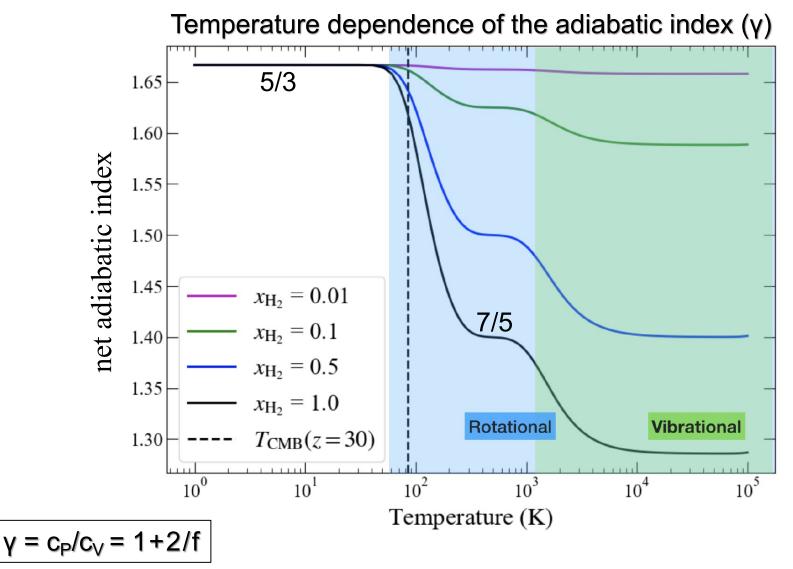
TODAY:

- The Equation of State (EOS) (→ recap notes)
- Implications/Modelling of the EOS (chemistry, heating, cooling)
- The validity of the gas/fluid approximation
- Gas/fluid instabilities (today: gravitational instability)

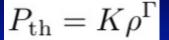
Adiabatic Index

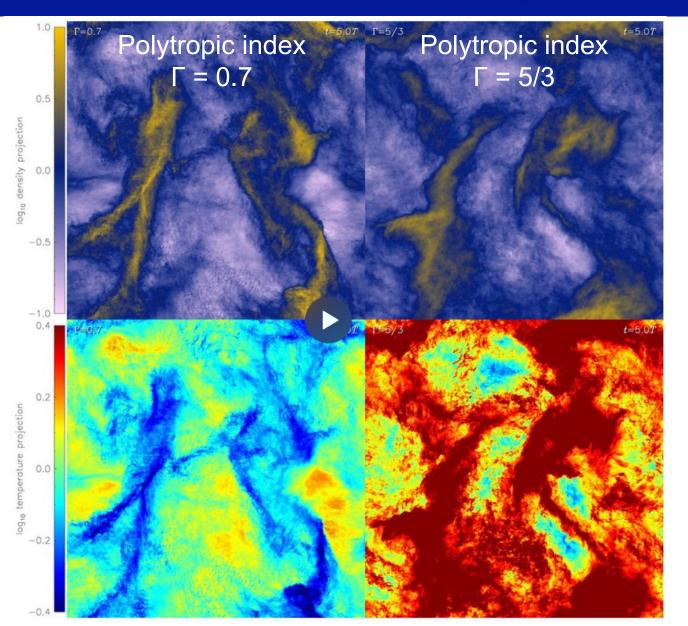


Number of degrees of freedom (f) depends on excitation of rotational and vibrational states (partition functions: T-dependence).

Sharda et al. (2019)

Equation of State – Polytropic EOS $P_{\rm th} = K \rho^{\Gamma}$





Density

Temperature

Federrath & Banerjee (2015)

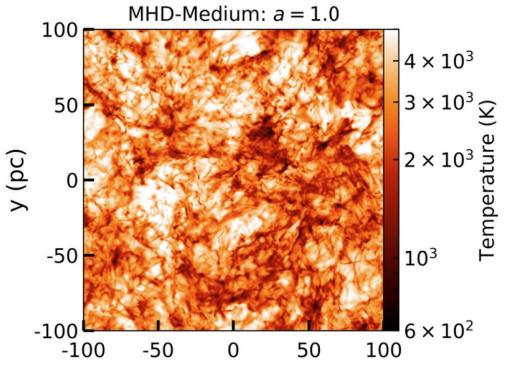
Movies available: http://www.mso.anu.edu.au/~chfeder/pubs/polytropic/polytropic.html

Energy equation with heating and cooling

$$\frac{\partial}{\partial t} e_{\text{tot}} + \nabla \cdot \left[(e_{\text{tot}} + P_{\text{tot}}) \boldsymbol{v} - \frac{1}{4\pi} (\boldsymbol{B} \cdot \boldsymbol{v}) \boldsymbol{B} \right] = \frac{1}{\rho} \left[\frac{\rho}{\mu m_{\text{H}}} \boldsymbol{\Gamma} - \left(\frac{\rho}{\mu m_{\text{H}}} \right)^2 \boldsymbol{\Lambda}(\boldsymbol{T}) \right]$$

Heating:

$$\Gamma = 2 \times 10^{-26} \,\mathrm{erg} \,\mathrm{s}^{-1}$$



x (pc)

Cooling:

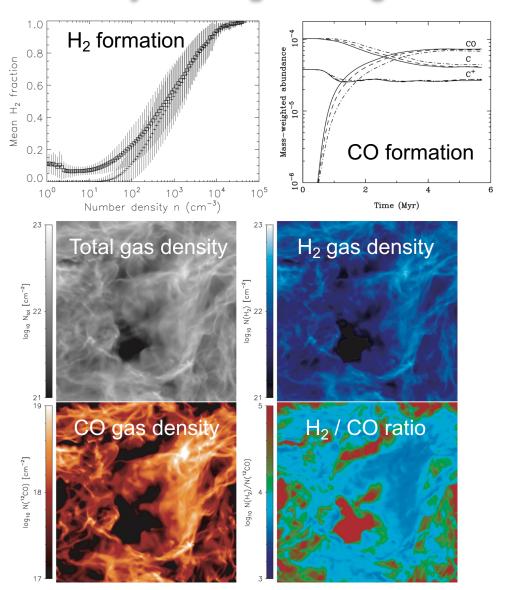
$$\frac{\Lambda(T)}{\Gamma} = 10^7 \exp\left(\frac{-1.184 \times 10^5}{T + 1000}\right)$$
$$+1.4 \times 10^{-2} \sqrt{T} \exp\left(\frac{-92}{T}\right) \text{ cm}^3$$

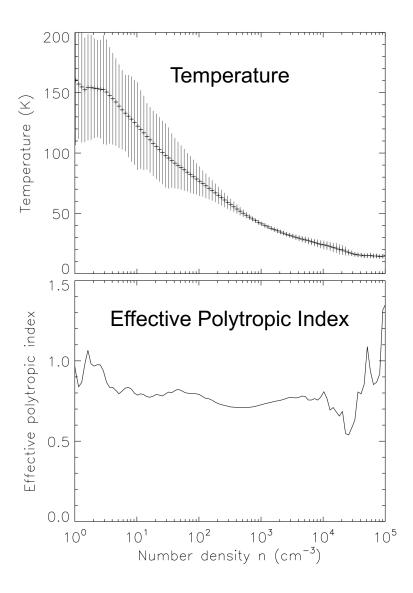
- Photoelectric heating from small grains and polycyclic aromatic hydrocarbons (PAHs)
- Heating and ionization from cosmic rays and X-rays
- H₂ formation and destruction
- Atomic and molecular line cooling

(Sutherland & Dopita 1993; Koyama & Inutsuka 2002; Vazquez-Semadeni et al. 2007)

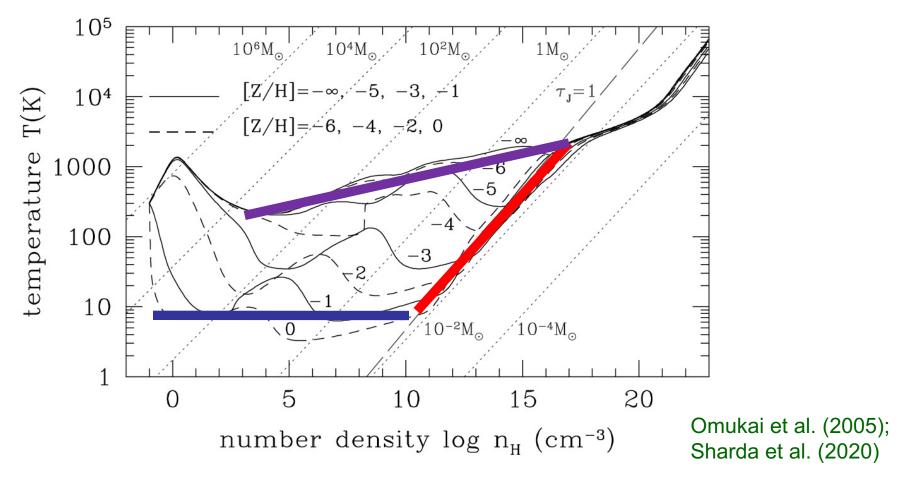
(Mandal et al. 2020)

Chemistry / Heating / Cooling: (Glover et al. 2007, 2010)





Chemistry / Heating / Cooling: (Glover+2007, 2010, Micic+2012, Clark+2012)



Molecule formation in high-density gas: t_{form}~ 1/n

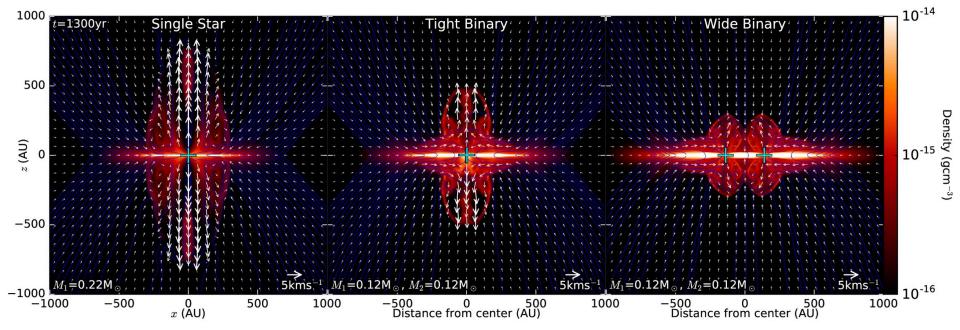
Micic et al. (2012), Hollenbach et al. (1971)

Equation of State – Polytropic EOS for Star Formation

$$P_{\rm th} = K \rho^{\Gamma} \quad \text{with} \quad \Gamma = \begin{cases} 1 & \text{for} \quad \rho \leq \rho_1 \equiv 2.50 \times 10^{-16} \, \mathrm{g \, cm^{-3}} \,, \\ 1.1 & \text{for} \, \, \rho_1 < \rho \leq \rho_2 \equiv 3.84 \times 10^{-13} \, \mathrm{g \, cm^{-3}} \,, \\ 1.4 & \text{for} \, \, \rho_2 < \rho \leq \rho_3 \equiv 3.84 \times 10^{-8} \, \, \mathrm{g \, cm^{-3}} \,, \\ 1.1 & \text{for} \, \, \rho_3 < \rho \leq \rho_4 \equiv 3.84 \times 10^{-3} \, \, \mathrm{g \, cm^{-3}} \,, \\ 5/3 & \text{for} \quad \rho > \rho_4 \,. \end{cases}$$

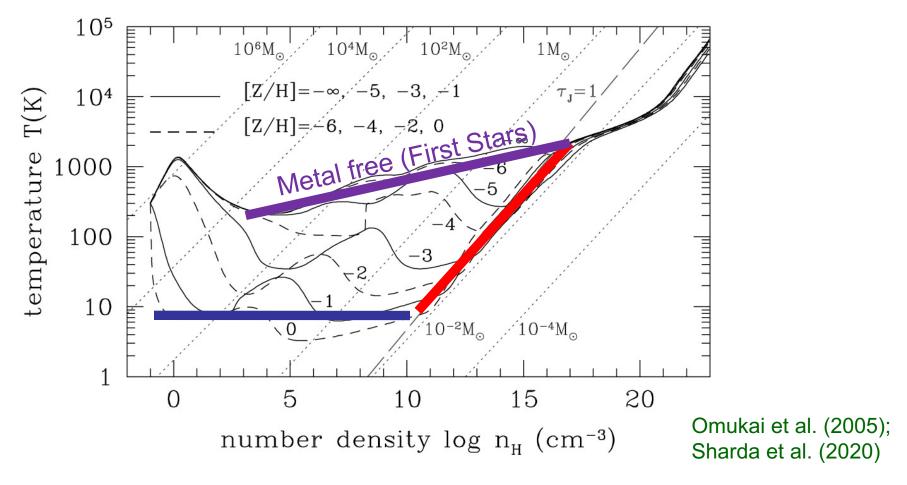
Movies available:

https://www.mso.anu.edu.au/~chfeder/pubs/binary_jets/binary_jets.html



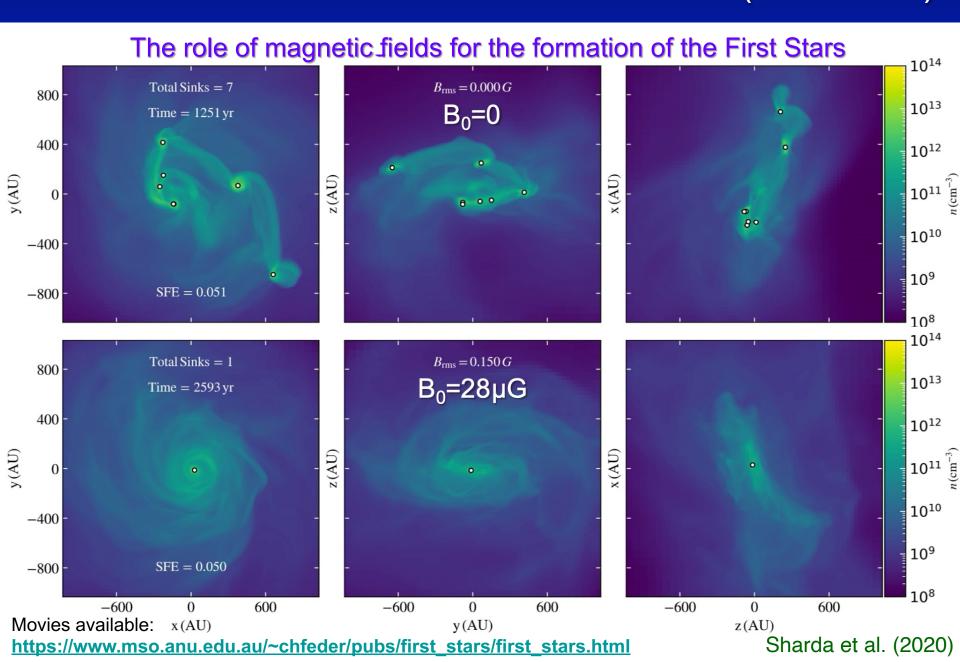
Kuruwita et al. (2017); Gerrard et al. (2018)

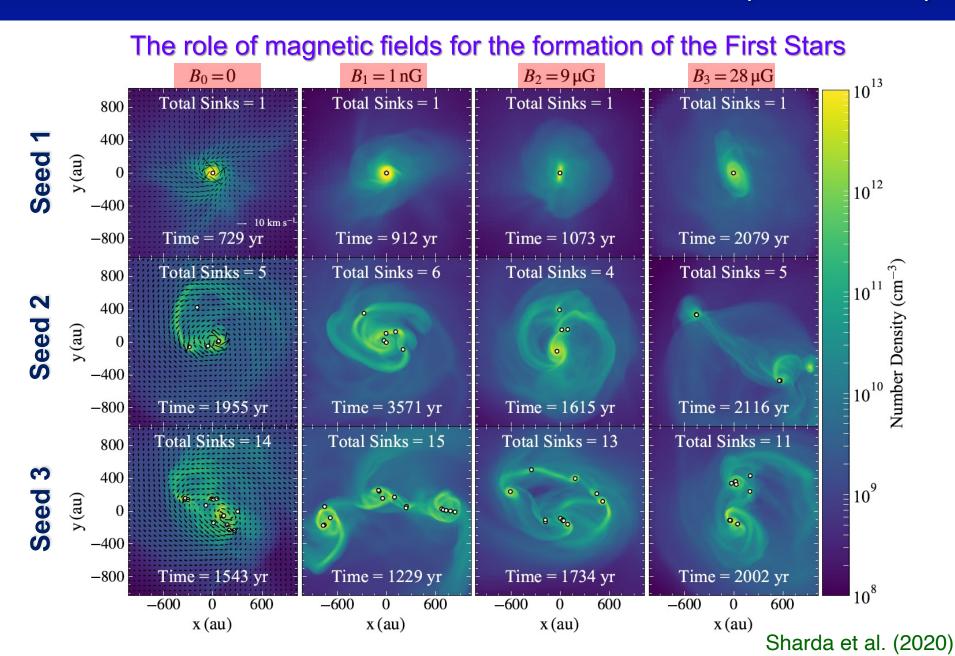
Chemistry / Heating / Cooling: (Glover+2007, 2010, Micic+2012, Clark+2012)



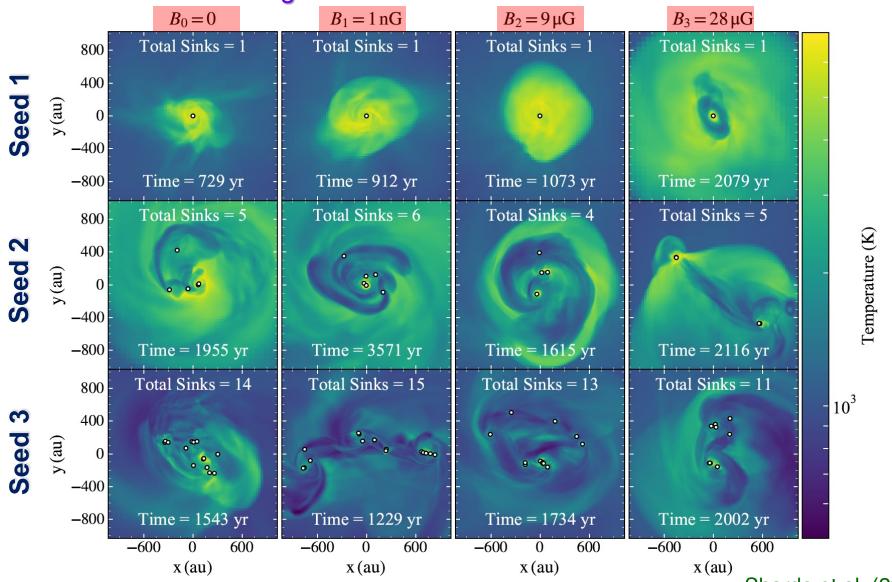
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Micic et al. (2012), Hollenbach et al. (1971)



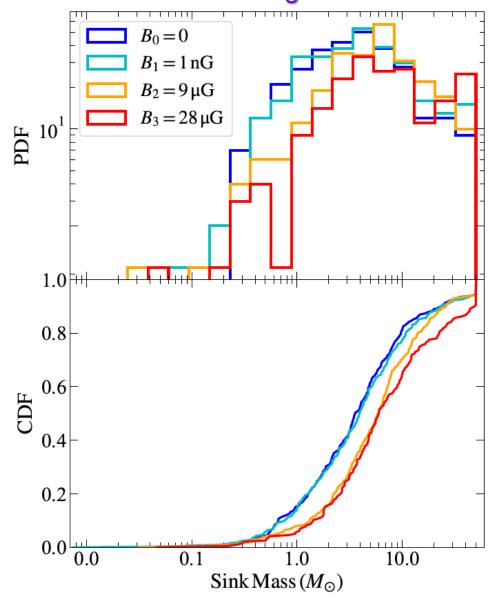






Sharda et al. (2020)

The role of magnetic fields for the formation of the First Stars



→ B field tends to suppress fragmentation, but also stochasticity

Sharda et al. (2020)

TODAY:

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Validity of the Gas/Fluid Approach

Gas/Fluid approximation only valid if particle mean free path much less than scales of interest

(particle collisions are sufficiently frequent on microscopic level)

Medium	Particle mean free path	Size scale
Water	9 x 10 ⁻⁹ cm	
Air	5 x 10 ⁻⁶ cm	
Solar core	2 x 10 ⁻⁸ cm	$\sim R_{sol}/4 \sim 2 \times 10^{10} \text{ cm}$
Solar corona	1x10 ⁸ cm	$\sim R_{sol} \sim 7 \times 10^{10} \text{ cm}$
Solar wind	1x10 ¹³ cm	$\sim AU \sim 1.5 \times 10^{13} \text{ cm}$
Interstellar medium	1x10 ⁵⁻¹⁵ cm	$\sim pc \sim 3 \times 10^{18} cm$
Galaxy cluster (intracluster medium)	1x10 ²³ cm	~ Mpc ~ 10 ²⁴ cm

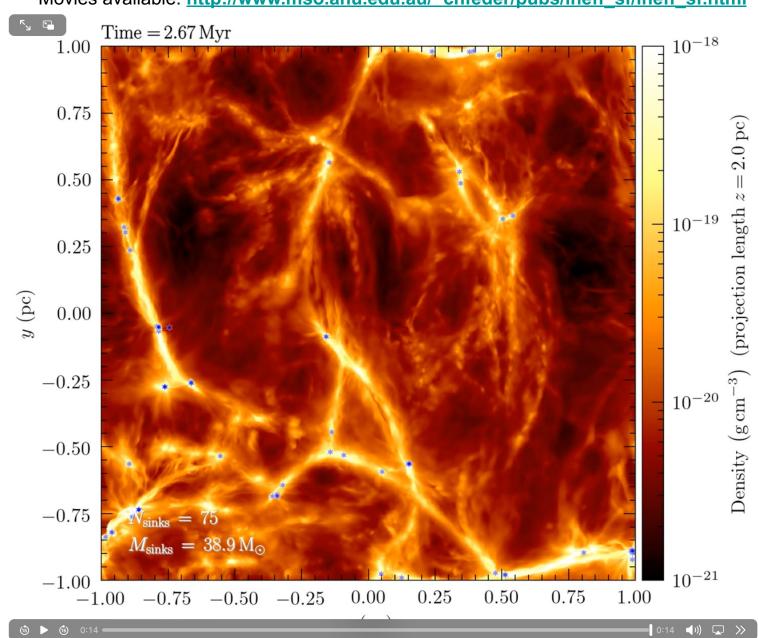
If the gas/fluid approximation breaks down, kinetic treatment is required.

TODAY:

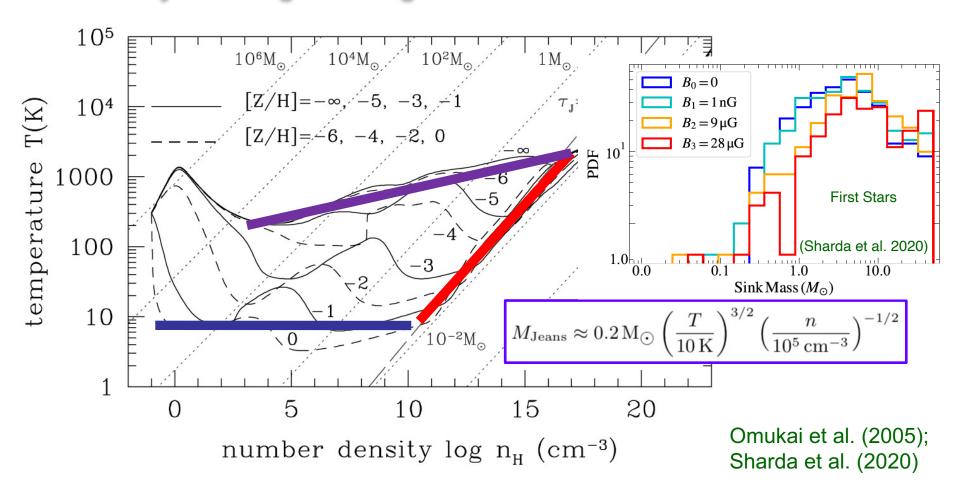
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Jeans fragmentation

Movies available: http://www.mso.anu.edu.au/~chfeder/pubs/ineff_sf/ineff_sf.html



Chemistry / Heating / Cooling: (Glover+2007, 2010, Micic+2012, Clark+2012)



Molecule formation in high-density gas: t_{form}~ 1/n

Micic et al. (2012), Hollenbach et al. (1971)

Toomre (gravitational disc) instability

Condition for disc instability

$$\frac{c_s \kappa}{\pi G \Sigma} < 1$$

 c_s is the speed of sound κ is the epicyclic frequency Σ is the surface density

(Toomre 1964; Romeo et al. 2010)

For Jeans instability, we basically compare thermal pressure versus gravity. For Toomre, we compare thermal pressure + rotation versus gravity.

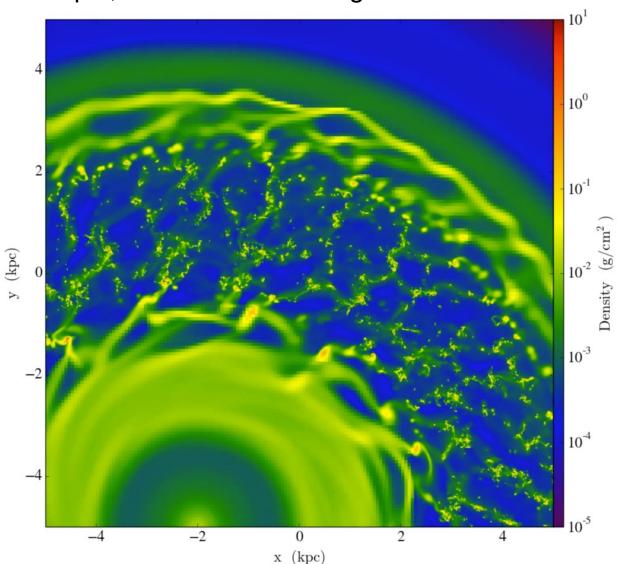
Notes:

- Surface density can have gas and star contribution
- Sound speed could be modified to include turbulence and/or magnetic pressure
 (e.g., turbulent, magnetic Jeans length)

(Federrath & Klessen 2012)

Toomre (gravitational disc) instability

Movies available: https://www.mso.anu.edu.au/~chfeder/pubs/turb_driv_gal/turb_driv_gal.html
For example, simulations of disc galaxies:



(Jin et al. 2017)

NEXT TIME:

Gas/fluid instabilities: Kelvin-Helmholtz instability