ASTR4012/ASTR8002
Astrophysical Gas Dynamics
Assignment 1

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due Friday, August 12, 2022

1 The hydrodynamical equations

1. Write down the hydrodynamical equations (mass, momentum, energy conservation) in Eulerian form.

   (3 points)

2. Reform the energy equation to write it in Lagrangian form, using the co-moving Lagrangian derivative

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla \,. \tag{1}$$

   Important: Construct two forms of the energy equation (both in Lagrangian form): one for e_{tot} and one for ρe_{tot}. Which one of the two describes the evolution of the conserved variable? Please do not just write down the end result, but show a few intermediate steps.

   (5 points)

3. In case of a one-dimensional (1D) hydrodynamical problem, how many equations do we have to describe mass, momentum and energy conservation? Write them down in 1D. How many unknowns are in these equations and which are they? Is the system of equations closed? If not, how can we close it?

   (2 points)

2 Conservation of energy in an external gravitational potential

The energy equation of hydrodynamics is a statement of energy conservation for an adiabatic gas, subject only to pressure forces. Show that the analogous statement in the presence of a given static external gravitational potential $\Phi(\mathbf{x})$ is

$$\frac{\partial}{\partial t}\left(\rho e_{\mathrm{tot}} + \rho\Phi\right) + \nabla \cdot \left[\left(\rho e_{\mathrm{tot}} + P + \rho\Phi\right)\mathbf{v}\right] = 0, \tag{2}$$

where $e_{\rm tot} = \mathbf{v}^2/2 + e_{\rm int}$ is the sum of specific kinetic and internal energy, and $\rho e_{\rm int} = P/(\gamma - 1)$ for an adiabatic gas. Hint: start with the velocity equation (recall how we obtained that from the momentum equation), add $-\nabla\Phi$ as the source term for gravity and then follow the derivation of the energy equation from the lecture notes.

(10 points)

Please submit your solutions via Turnitin by the assignment deadline.