

Magnetic pressure and tension

The magnetic force is

$$\vec{F}_L = -\frac{1}{8\pi} \nabla \vec{B}^2 + \frac{1}{4\pi} (\vec{B} \cdot \nabla) \vec{B}$$

Rewrite with

$$\vec{B} = B \vec{S},$$

where $\vec{S}$ is a unit vector pointing in the direction of $\vec{B}$.
Then, the Lorentz force becomes

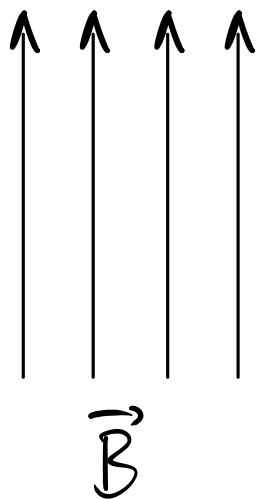
$$\begin{aligned} -\frac{1}{8\pi} \nabla \vec{B}^2 + \frac{1}{4\pi} (\vec{B} \cdot \nabla) \vec{B} &= -\frac{1}{8\pi} \nabla B^2 + \frac{1}{8\pi} \vec{S} (\vec{S} \cdot \nabla B^2) \\ &\quad + \frac{B^2}{4\pi} (\vec{S} \cdot \nabla) \vec{S} \\ &\quad \underbrace{\frac{1}{4\pi} (B \vec{S} \cdot \nabla) (B \vec{S})}_{\text{product}} \\ &= -\frac{1}{8\pi} \nabla_{\perp} B^2 + \frac{B^2}{4\pi} (\vec{S} \cdot \nabla) \vec{S}, \end{aligned}$$

where $\nabla_{\perp} = \nabla - \vec{S} (\vec{S} \cdot \nabla)$ is the projection of the gradient operator in the direction perpendicular to $\vec{S}$, and $\vec{S} (\vec{S} \cdot \nabla) = \nabla_{\parallel}$ is the grad. op. along $\vec{S}$.

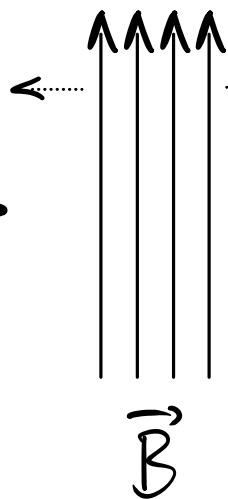
Thus, the first term, $-\frac{1}{8\pi} \nabla_{\perp} B^2$, is the **magnetic pressure**, always acting in the direction **perpendicular to $\vec{B}$** .

Since $\vec{s} \cdot \nabla$ is the projection of the grad. op. along $\vec{s}$, **$(\vec{s} \cdot \nabla) \vec{s}$** is the rate of change of $\vec{s}$ along its own direction. Thus, the second term, $\frac{B^2}{4\pi} (\vec{s} \cdot \nabla) \vec{s}$, describes the magnetic force due to **curvature of $\vec{B}$** $\Rightarrow$ **tension**.

Pressure



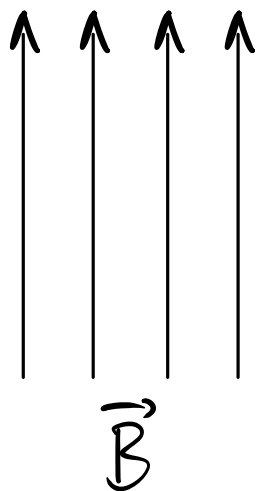
Compression $\rightarrow$



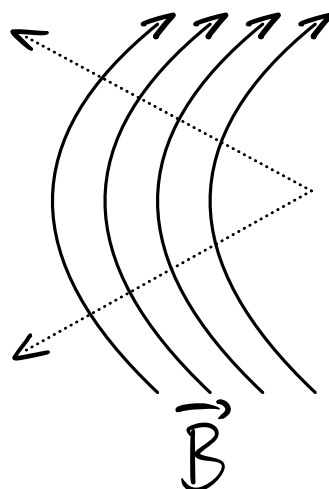
pressure force
resists compression

$$\left(-\frac{1}{8\pi} \nabla B^2 \right)$$

Tension



Curvature $\rightarrow$



magnetic tension
force resists bending

$$\left(\frac{1}{4\pi} (B \cdot \nabla) \vec{B} \right)$$