Shock Waves

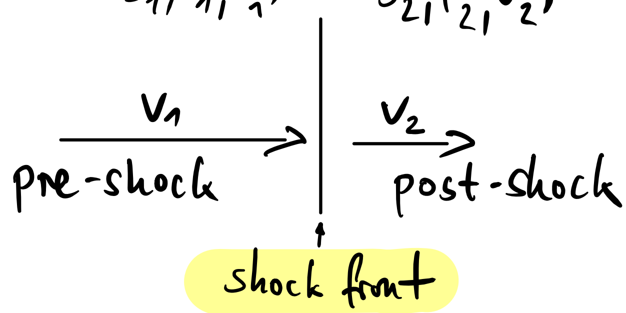
- Looked at the Parker wind and Bondi accretion: both had a transition point from subsonic ($v < c_s$) to supersonic ($v > c_s$) flows ($\rightarrow$ sonic point).
- What is a shock wave? $\rightarrow$ for example:
  - supersonic flow hits an obstacle (jets into dense gas)
  - object moves with supersonic speeds through gaseous medium with sound speed c_s (Bondi-Hoyle accretion for Mach > 1)

$\rightarrow$ No pressure perturbations (normal sound waves) can propagate upstream.

The Rankine-Hugoniot shock jump conditions

- Consider flow from left to right $\rightarrow$ contact plane between Region 1 and 2

(ρ_1, P_1, v_1) (ρ_2, P_2, v_2)



- When we derived the hydro equations, we identified fluxes leading to conservation of mass, momentum and energy.
- These fluxes are

$\rho \vec{v}$ mass flux

$\rho v_i v_j + P \delta_{ij}$ momentum flux

$\left(\frac{1}{2} \rho v^2 + \rho e_{int} + P\right) \cdot \vec{v}$ energy flux

$\left(\text{ideal gas } e_{int} = \frac{P}{\rho} \frac{1}{\gamma - 1}\right)$

$\Longrightarrow$ So the energy flux can be written as

$$\rho \vec{v} \left(\frac{1}{2} v^2 + \frac{P}{(\gamma-1)\rho} + \frac{P}{\rho} \right) = \rho \vec{v} \left(\frac{1}{2} v^2 + \underbrace{\frac{P \gamma}{\rho (\gamma - 1)}}_{= h \text{ (enthalpy)}} \right)$$

- Because of these three conservation laws, we must have across the shock:

$$\rho_1 v_1 = \rho_2 v_2$$

$$\rho_1 v_1^2 + P_1 = \rho_2 v_2^2 + P_2$$

$$\left(\frac{1}{2} v_1^2 + h_1\right) \rho_1 v_1 = \left(\frac{1}{2} v_2^2 + h_2\right) \rho_2 v_2$$

- Writing the velocity left of the discontinuity as

$$\boxed{v_1 = \mathcal{M}\, c_s} \quad \left(\text{Mach number } \mathcal{M} = \frac{v_1}{c_s}\right)$$

these three conservation equations can be combined to yield the „Rankine-Hugoniot" shock jump conditions (where we assumed a polytropic EOS):

$$\boxed{\frac{\rho_2}{\rho_1} = \frac{v_1}{v_2} = \frac{(\gamma+1)\,\mathcal{M}^2}{(\gamma-1)\,\mathcal{M}^2 + 2}}$$

(density and velocity jump)

$$\boxed{\frac{P_2}{P_1} = \frac{1-\gamma + 2\gamma\,\mathcal{M}^2}{\gamma+1}}$$

(pressure jump condition)

- From ideal gas ($P \sim \rho T$):

$$\boxed{\frac{T_2}{T_1} = \frac{P_2}{P_1} \cdot \frac{\rho_1}{\rho_2}}$$

(temperature jump)

- By construction, the Mach number is greater than 1, $\mathcal{M} > 1$, which implies $\rho_2 > \rho_1$; $v_2 < v_1$

$P_2 > P_1$; $T_2 > T_1$

- For an isothermal shock, we get

$$\boxed{\frac{\rho_2}{\rho_1}} = \frac{v_1}{v_2} = \frac{P_2}{P_1} = \boxed{\mathcal{M}^2}$$

(this can happen, if shock can radiate excess compression heat, e.g. in molecular clouds)

- For adiabatic gas, e.g. $\gamma = 5/3$, and in the limit of hypersonic flow ($\mathcal{M} \to \infty$):

$$\boxed{\lim_{\mathcal{M} \to \infty} \frac{\rho_2}{\rho_1} = \frac{\gamma + 1}{\gamma - 1}},$$

which leads to the maximum density compression ratio of

$$\boxed{\frac{\gamma+1}{\gamma-1} = \frac{8/3}{2/3} = 4 \quad (\text{for } \gamma = 5/3)}$$