ASTR4004/ASTR8004
Astronomical Computing
Lecture 20

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Sedov-Taylor Expansion with FLASH

1 Physical Description

Stars in the interstellar medium explode, i.e., supernova explosions (Fig. 1). The sudden release of large amount of energy (from a very small-volume) into the interstellar medium gives rise to a strong explosion, which is characterized by shock waves. The primary question is how fast will the shock travel. This problem provides a very useful test for hydrodynamical schemes and also shows the power of dimensional analysis. Assuming a point explosion and a symmetric shock wave (in two-dimensions), as shown in Fig. 2, the solution for the radius of the expanding shell as function of time can be derived using dimensional analysis.

The radius R in the Sedov-Taylor phase can only depend on the following three quantities: energy of the explosion E, density of the medium ρ_0 and time t. So,

$$R = CE^a \rho_0^b t^c,$$

where C is a constant of proportionality and a, b, c are powers to be determined.
Dimensionally,

$$[L] = [M]^a[L]^{2a}[T]^{-2a}[M]^b[L]^{-2b}[T]^c$$
$$[L] = [M]^{a+b}[L]^{2a-2b}[T]^{c-2a}$$

Equating powers,

$$a + b = 0 \implies a = -b.$$
$$2a - 2b = 1 \implies 2a - (-2a) = 1 \implies 4a = 1 \implies a = 1/4, b = -1/4.$$
$$c - 2a = 0 \implies c = 2a = 1/2.$$

Thus,

$$R = C\left(\frac{Et^2}{\rho_0}\right)^{1/4}, \; R \propto t^{1/2}.$$

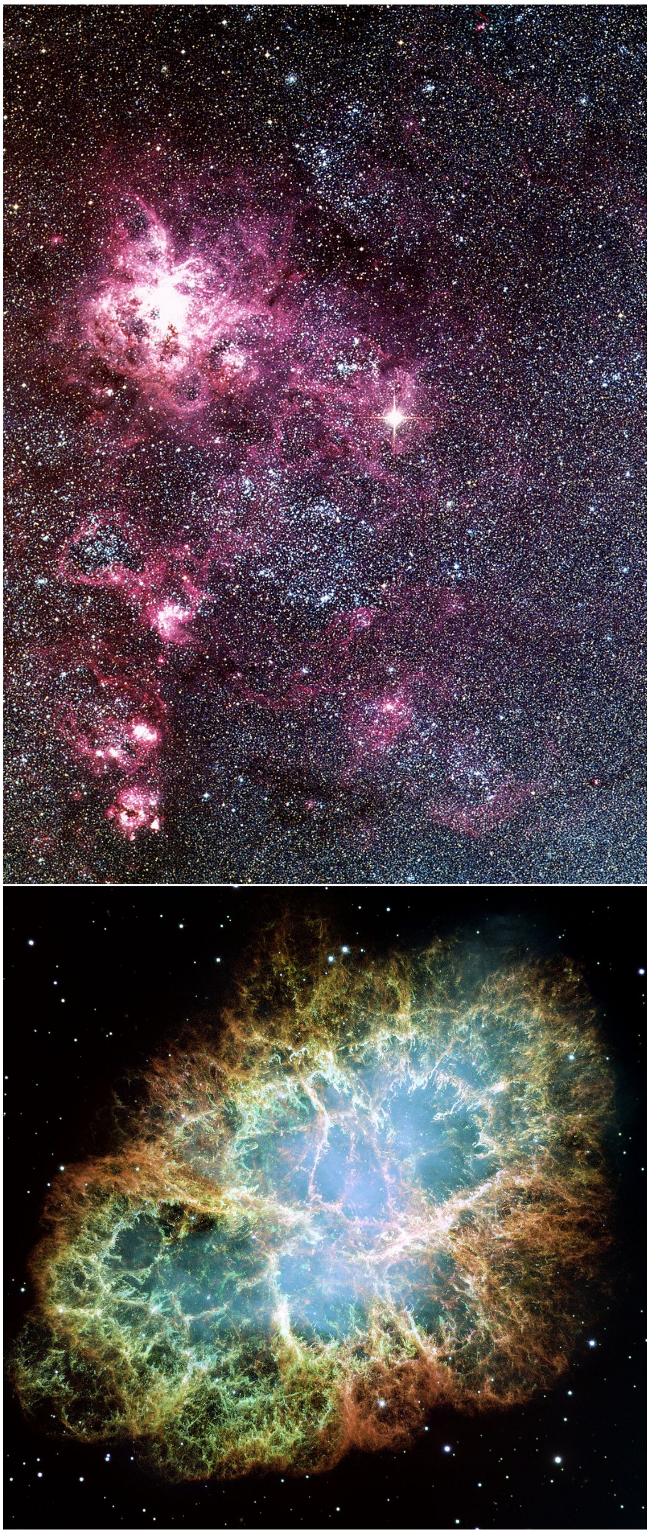

Figure 1: Top: SN1987A, exploding star. Bottom: Crab nebula, supernova remnant.

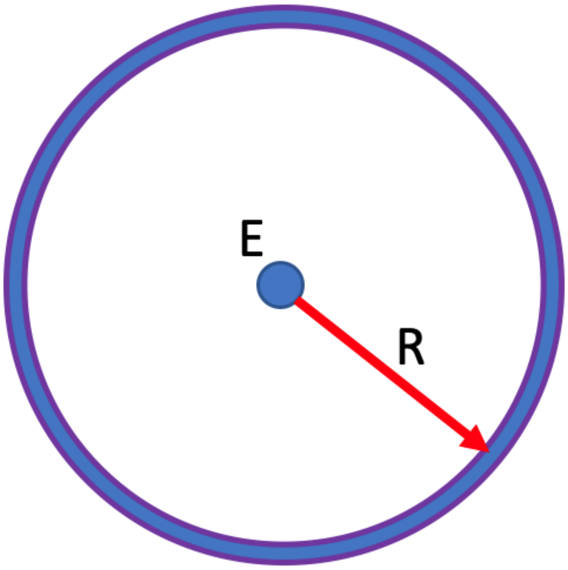

Figure 2: Symmetric shockwave with radius R carrying energy E deposited at the centre by explosion.

2 Numerical Solution

To test the above obtained solution numerically (also also estimate the constant of proportionality), we use the FLASH code as described below.

1. Login to the mash server of mso:

```
ssh -Y mash
```

2. Create a folder for today in your home directory:

```
mkdir astrocodeweek20
```

3. Go to /data/mash/astroflash directory and access the code:

```
cd /data/mash/astroflash
cd code
```

4. Setup the code for Sedov-Taylor expansion:

```
./setup Sedov -auto -2d --gridinterpolation=native
-objdir=/home/amitseta/astrocodeweek20/sedov2d
```

5. Go to the setup folder in the home directory:

```
cd /home/amitseta/astrocodeweek20/sedov2d
```

6. Compile the code:

```
make
```

7. Make a directory for running the code and cd to the folder:

```
mkdir ../sedovrun
cd ../sedovrun
```

8. Copy the executable from the compilation folder and
copy the parameter file from `/data/mash/astroflash` (Check flash.par!):

```
cp ../sedov2d/flash4 .
cp /data/mash/astroflash/flash.par .
```

9. Run the code:

```
./flash4
```

10. Add latex to your path (only required on mso servers):

```
PATH=$PATH:/usr/local/texlive/2017/bin/x86_64-linux
```

11. Copy the analysis file from `/data/mash/astroflash` (Check files!):

```
cp /data/mash/astroflash/sedov_analysis.py .
```

12. Make a folder to save all data plots

```
mkdir plots
```

13. Run the analysis file:

```
python sedov_analysis.py
```

14. Parts of the analysis code:

2d density plots:

```
X, Y = np.meshgrid(np.arange(0.0,1.0, (1.0/n)), np.arange(0.0,1.0,(1.0/n)))
plt.figure()
dmap = plt.imshow(dens[:,:,0], extent=(0.0,1.0,0.0,1.0),
               interpolation='none',cmap='Spectral',
               norm=colors.LogNorm(vmin=0.005, vmax=5.0))
plt.title('t=%1.4f'%(t))
plt.xlabel(r'$x$')
plt.ylabel(r'$y$')
plt.xlim([0.0,1.0])
plt.ylim([0.0,1.0])
plt.xticks([0.0,0.5,1.0])
plt.yticks([0.0,0.5,1.0])
plt.minorticks_on()
```

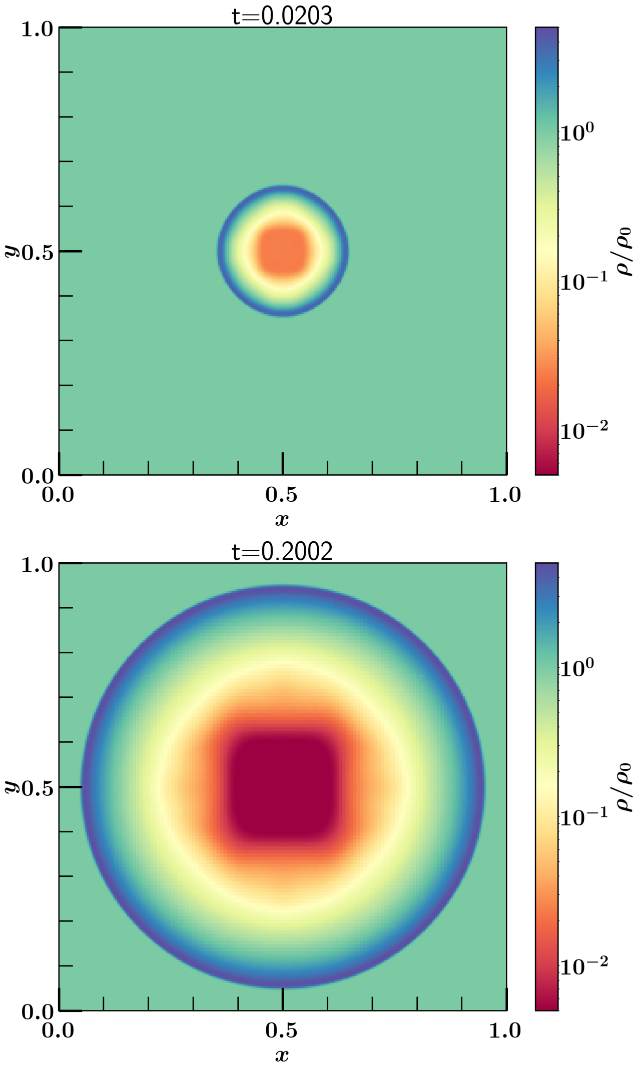

Figure 3: 2d density plots at $t = 0.02$ and $t = 0.2$.

```
    plt.tick_params('both', length=50, width=5, which='major', direction='in')
    plt.tick_params('both', length=30, width=3, which='minor', direction='in')
    plt.tick_params(axis='x', which='major', pad=20)
    plt.tick_params(axis='y', which='major', pad=8)
    cbar = plt.colorbar(dmap,label=r'$\rho/\rho_0$')
    plt.show()
    plt.close()
```

1d density profiles in a single plot:

```
plt.figure() # for 1d profile
x = np.linspace(0.0,1.0,n) # x extent

rad = np.array([]) #empty array to store radius
time = np.array([]) #empty array to store time
for f in glob.glob('sedov_hdf5_plt_cnt_*'): #loop over all files
     t,dens = luigi(f) # read data
     plt.plot(x,dens[:,n/2,0],lw=3,c='r',label='t=\%1.4f'\%(t))
```

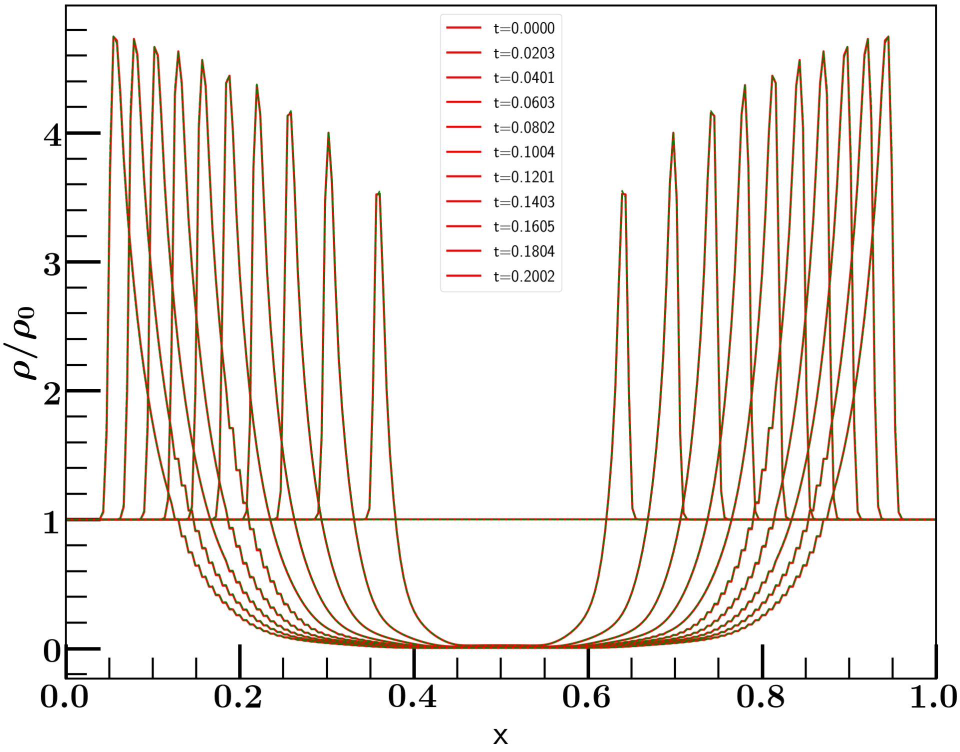

Figure 4: 1d density profiles at various times.

```
    #plotting density profile along x (y=half the total length)
    plt.plot(x,dens[n/2,:,0],lw=2,ls='--',c='g')
    # plotting density profile along y (x=half the total length)

    maxat = x[np.where(dens[:,n/2,0]==np.max(dens[:,n/2,0]))]
    #finding maxima along profile to get diameter
    if(np.size(maxat)==2): # to check if the circle is defined
       rad = np.append(rad,(maxat[1]-maxat[0])/2.0) # saving radius
       time = np.append(time,t) # saving time
plt.xlabel('x')
plt.ylabel(r'$\rho/\rho_0$')
plt.xlim([0.0,1.0])
plt.minorticks_on()
plt.tick_params('both', length=50, width=5, which='major', direction='in')
plt.tick_params('both', length=30, width=3, which='minor', direction='in')
plt.legend(loc='best',fontsize=24)
plt.show()
```

Fitting the data to obtain the function which represents radius as function of time:

```
def fitfunc(x, a, b): #function to determine the fit
    return a*(x**b)

params = curve_fit(fitfunc, time, rad)
a,b = params[0]

plt.figure()
```

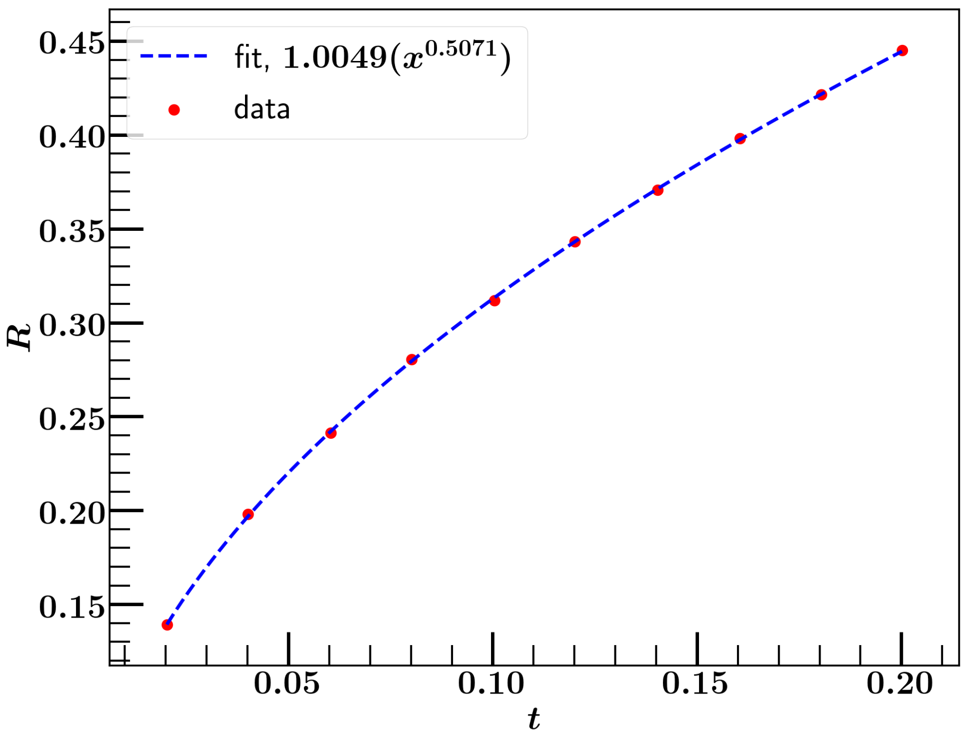

Figure 5: Radius as a function of time from the simulation and the fit. The fitted function agrees with the analytical result, $R \propto t^{1/2}$.

```
plt.scatter(time,rad,c='r',s=250,label='data') #plotting raw data
x = np.linspace(min(time),max(time),100) # plotting fit
plt.plot(x,a*(x**b),c='b',lw=5,ls='--',label='fit, $%1.4f (x^{%1.4f})$'%(a,b))
plt.xlabel(r'$t$')
plt.ylabel(r'$R$')
plt.minorticks_on()
plt.tick_params('both', length=50, width=5, which='major', direction='in')
plt.tick_params('both', length=30, width=3, which='minor', direction='in')
plt.legend(loc='best')
plt.show()
```

15. Movie, copying plots to laptop (ffmpeg not installed in mso servers, so for the movie, copy data to own laptop), so on your laptop:

```
mkdir sedovmovie
scp amitseta@msossh1.anu.edu.au:
/home/amitseta/astrocodeweek20/sedovrun/plots/*.png .
ffmpeg -i sedov_hdf5_plt_cnt_%04d.png movie.mpeg
```

3 Optional assignment

Derive the dependence of the radius of the shock wave on time analytically for a *spherically symmetric three-dimensional shock wave* and then confirm it numerically.