ASTR4004/ASTR8004
Astronomical Computing
Lecture 10

Christoph Federrath

26 August 2019

Monte Carlo error propagation, Python startup

1 Monte Carlo error propagation

Uncertainty analysis of measurements and derived quantities in astrophysics is extremely important, and published measurement results should always have a proper error analysis associated with them. Here we learn how to do Monto Carlo (MC) error propagation, which goes beyond standard analytic error propagation and can handle non-Gaussian distributions for propagating uncertainties.

1. Suppose you have measured two Gaussian random variables,

$$a = 10 \pm 1 \qquad \text{and} \qquad b = 1.0 \pm 0.1, \tag{1}$$

and you want to calculate the derived quantity

$$c = \frac{a^2}{b^2}. \tag{2}$$

2. First, calculate the uncertainty of c using standard analytic methods of error propagation.

3. Now write a program that does the MC error propagation. First make Gaussian random numbers (`randomn`) and define a and b based on these Gaussian random distributions. Then define c based on a and b.

4. Now plot the PDFs of a, b, and c. What is special about the PDF of c? Try also a log-y axis version.

5. Compute the mean and standard deviation of c based on the PDF of c and compare to the analytic estimate. Make sure to implement bin-centered binning for the numerical integration of the PDF, in order to recover the mean and standard deviation more accurately than from the staggered bins.

6. Get the mode (most probable value) of c and the 16th and 84th percentile values.

7. Explore what happens when you replace a with $a = 1.0 \pm 0.5$. Think about the interpretation of writing $\pm$ (standard deviation) when quoting errors/uncertainties, relative to the mathematically correct range for values of c.

8. Try changing the sample size of the Gaussians and the number of bins used for the PDFs, in order to study numerical convergence of the results.