ASTR4004/ASTR8004
Astronomical Computing
Lecture 09

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Fourier analysis

1 Fourier analysis

Fourier transformations are a powerful tool to analyse structure in signals. These signals can be of any kind, but the most common scientific applications involve time-domain (1D) or spatial-domain signals in 2D or 3D. Here we will focus first on a simple example of Fourier analysis of a 1D signal and then move to 2D spatial-mode analysis of column-density data.

1. First construct a discrete signal

$$f(x) = \sin(x) + \sin(10x) + 2 \quad \text{for} \quad x = [0, 2\pi], \tag{1}$$

   using `dindgen` with some 100 bins.

2. Plot the discretised version of $f(x)$. What are the characteristic modes in this data?

3. Now compute the discrete Fourier transform of the data, $\hat{f}(k)$, where $k = 2\pi/x$ is the wave number (use `fft`).

4. Plot the power in $\hat{f}$. What does this power spectrum tell us? What is the data structure?

5. Now filter the high-frequency component with a top-hat filter, compute the inverse Fourier transform of the Fourier-filtered data and plot it.

6. Create a similar test in 2D; make a function to display the 2D maps; apply different filters to the data and investigate the inverse Fourier transform of the filtered data.

7. Apply this to the column-density maps that we have analysed earlier, e.g., EXTREME_proj_xy_000300. See what can happen to the data when you apply different cutoff wave numbers for the top-hat filter. In particular, the filtered data may develop negative column densities, which is unphysical. Can you figure out a way to apply filtering that still preserves positivity after the inverse Fourier transform?

8. In general: what happens to the mean (average) of the data when you apply a Fourier filter?