3 A simple treatment of complexity: cosmological entropic boundary conditions on increasing complexity

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3.1 THE COMPLEXITY OF COMPLEXITY

Proud Biologist: "Life forms are more complex than stars"
Humble Astronomer: "You'd look simple too from a trillion miles away"

One of the central questions of evolutionary biology and cosmology is: is there a general trend towards increasing complexity? In order to answer that question, it would help to have a definition of complexity that can be quantified. Various definitions of complexity have been proposed (Gell-Mann, 1994, 1995; Kauffman, 1995; Adami, 2002; Gell-Mann & Lloyd, 2003; Fullsack, 2011). With useful oversight, Lloyd (2001) groups various conceptions of complexity into three groups based on (1) difficulty of description (measured in bits) (2) difficulty of creation (measured in time, energy or price) and (3) degree of organization (measured in . . . ? . . . , we're not sure). For more details see: Weaver, 1948; Traub *et al.*, 1983; Chaitin, 1987; Weber *et al.*, 1988; Wicken, 1988; Bennett, 1988; Lloyd & Pagels, 1988; Zurek, 1989; Crutchfield & Young, 1989; McShea, 2000; Adami *et al.*, 2000; Adami, 2002; Hazen *et al.*, 2008; Li & Vitanyi, 2008; McShea & Brandon, 2010.

I will not try to unify these mildly-compatible definitions of complexity, since such an effort would probably resemble the

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confusing attempts to one-dimensionalize the *N*-dimensional concept of human intelligence. However, a unifying feature of the effective complexity discussed here (Gell-Mann & Lloyd, 2003) is the intuitive notion that complexity lives somewhere in the continuum between complete order and random chaos (Crutchfield & Young, 1989; Gell-Mann, 1995; Adami, 2002). Complex systems are far-from-equilibrium-dissipative systems (Prigogine, 1978). Thus, we are far from equilibrium (i.e. far from random) but we are also far from order. If simplicity is the opposite of complexity, then both random chaos and complete order are simple. In a cosmological context, order can be understood as the low entropy condition of homogeneously distributed matter in the early universe.

The fact that life forms are complex and that our DNA contains information about the environment (past and present) seems obvious and has been emphasized by various authorities:

> *[Organisms] encode the predictable occurrence of nature's storms in the letters of their genes.*
>
> *(Wilson, 1992)*
>
> *genes embody knowledge about their niches.*
>
> *(Deutsch, 1997)*

Adami *et al.* (2000) identify genomic complexity with the amount of information DNA sequences store about their environments. I want to emphasize that not only is the information in DNA *about* the environment, but that the information in DNA *came* from the environment (see Krakauer, Chapter 10). In any naturalistic explanation for the origin and evolution of life, the non-adaptive complexity of the physical environment precedes, and is the source of, the adaptive complexity of life. The information in DNA comes from the environment and it has been put into the DNA by selection. Darwinian evolution involving selection of all kinds (natural, sexual, and artificial) is the channel through which the complexity and information of the environment creates and shapes the complexity and information of biological organisms (e.g. Spiegelman, 1971).

The complexity of the environment is in the spatial and temporal differences of such variables as temperature, density, pressure, chemistry, and the availability of energy, water, and nutrients. These detailed non-adaptive structural complexities and differences did not always exist (Zaikowski & Friedrich, 2008). These differences started out as small fluctuations about equilibrium and were amplified by gravitational collapse. Galactic clouds of hydrogen evolved into stars. Stars evolved layers and produced complicated patterns of isotopic abundances in the interstellar medium. Undifferentiated objects in proto-planetary disks irreversibly differentiated and became planets, with density-segregated layers (core/mantle/crust/oceans/atmospheres) whose surfaces are pockmarked information-rich palimpsests of the history of the solar system. Solar System The number of minerals on Earth has increased with time (Hazen *et al.*, 2008; Hazen & Eldredge, 2010).

A subset of these differences provides useful gradients from which free energy can be extracted – gradients of luminosity, redox chemistry, pH, temperature, humidity, density, gravity, etc. (Schneider & Sagan, 2006; Lineweaver & Egan, 2008, 2011). The maintenance of irreversible processes requires the dissipation of these gradients and their associated free energy (Ulanowicz & Hannon, 1987). Or, equivalently, the flow of free energy driven by these gradients produces dissipative structures which are maintained as long as the gradients persist (e.g. Lineweaver & Egan, 2008; Kleidon, 2012).

Various forms of irreversible processes and dissipative structures produce and maintain complexity. All forms of irreversible processes are subject to entropic boundary conditions – these include simple near-equilibrium structures such as cooling planets and cups of coffee, but there are also far-from-equilibrium dissipative structures of varying complexity such as convection cells, hurricanes, and life forms (Prigogine, 1978). We are most interested in life forms since that is what we are. We have a mechanism (inheritable coded molecules of DNA or RNA) for storing information about the environment inside ourselves and passing it on to descendants. Thus we are *adaptive* systems. Hurricanes don't do that. Non-biological far-from-equilibrium dissipative structures don't do that. They are *non-adaptive* systems. However, the limits on the availability of free energy are limits for all irreversible structures – both non-adaptive and adaptive – from hot coffee cups and hurricanes to life forms.

In summary, we are interested in the evolution of complexity in the universe. Just as biological complexity can be traced back to the complexity of the environment, environmental complexity can be traced back to free energy available due to an entropy gap between the initial low entropy of the universe and the maximum potential entropy of the universe. Complexity is limited by the availability of free energy and free energy is limited by the entropy gap.

3.2 EVOLUTION OF THE ENTROPY AND THE MAXIMUM POTENTIAL ENTROPY OF THE UNIVERSE

There is general agreement that the entropy of the universe ("S_{uni}" in Fig. 3.1), started out low and has been increasing ever since (Penrose, 1979, 2004; Davies, 1994; Lineweaver & Egan, 2008, 2011; Egan & Lineweaver, 2010; Carroll, 2010). All three panels in Fig. 3.1 have S_{uni} starting out low. An initial low entropy is not obvious since observations of the cosmic microwave background (Smoot *et al.*, 1992) revealed the conditions of the universe $\sim 400\,000$ years after the big bang. They revealed a nearly homogeneous, isotropic, isothermal, isobaric, iso-everything universe (at least at the level of a few parts in 10^5). There were no stars or planets or galaxies. These observations seem to suggest not a low initial entropy but a high initial entropy – a universe in equilibrium at its maximum possible value, S_{max}. This apparent equilibrium of the early universe is why $S_{\mathrm{uni}} = S_{\mathrm{max}}$ at early times in Fig. 3.1(a) & (b). But if the universe started out in equilibrium with $\Delta S = 0$, how did anything happen? What drove it out of equilibrium? Fig 3.1(c) does not have this problem because it includes in S_{uni} the low initial gravitational entropy of nearly homogeneously

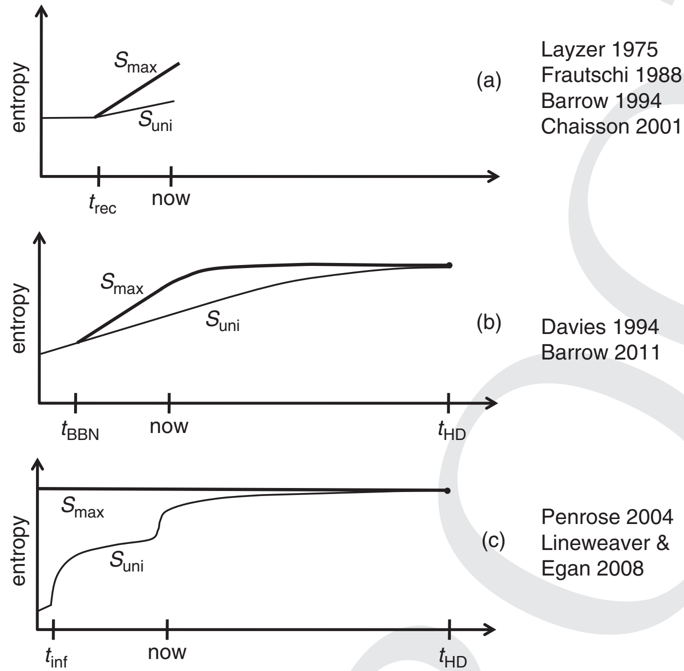

FIGURE 3.1 Three different views of the evolution of the entropy of the universe, $S_{\rm uni}$, and the maximum potential entropy of the universe, $S_{\rm max}$. In (a), $t_{\rm rec}$ is the time of recombination. In (b), $t_{\rm BBN}$ is the time of big bang nucleosynthesis. In (c), $t_{\rm inf}$ is the time of inflation. In (b) & (c), $t_{\rm HD}$ is the heat death of the universe. We would like to understand why these sketches are so different and which (if any) gives the most qualitatively correct picture. One important difference is what to include in $S_{\rm max}$ and $S_{\rm uni}$ (see text).

distributed matter. This gravitational entropy has been ignored in Fig. 3.1(a) & (b). Also, in Fig. 3.1(c), $S_{\rm max}$ is defined differently.

The entropy gap,

$$\Delta S = S_{\rm max} - S_{\rm uni} \tag{3.1}$$

shown in Fig. 3.2 is the difference between the maximum potential entropy and the actual entropy of the universe (Lineweaver & Egan, 2008). In Fig. 3.1, all three panels agree that both $S_{\rm uni}$ and the maximum potential entropy $S_{\rm max}$ cannot decrease – both obey

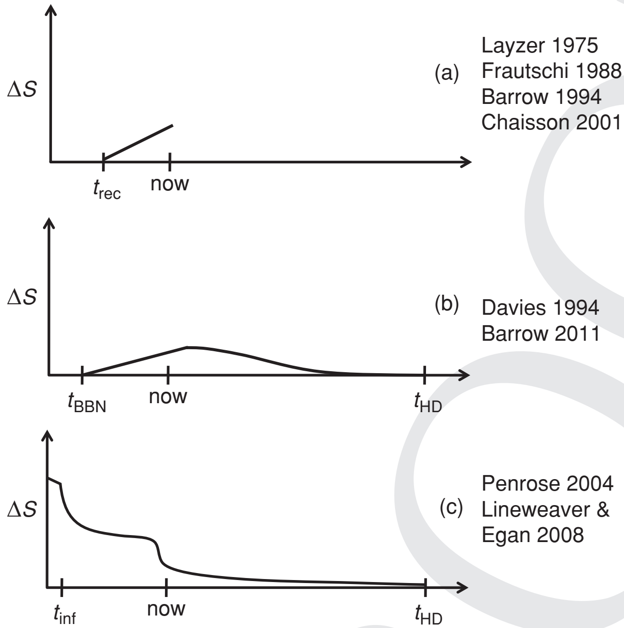

FIGURE 3.2 Same as Fig. 3.1 except the y-axis is now the difference $\Delta S - S_{max} - S_{uni}$ (Eq. (3.1)) from each panel of Fig. 3.1. ΔS is important because it is a measure of free energy (Eq. (3.4)), which is the only thing that can maintain existing complexity or drive increasing complexity. Panel (a) suggests that ΔS continues to increase indefinitely, thus potentially allowing for an unlimited increase of free energy and complexity. In panels (b) and (c), $\Delta S \to 0$ at the heat death of the universe, suggesting the dissipation of all free energy and the reduction and disappearance of complexity. The significant disagreement between panel (a) and the other two needs to be resolved if we are to resolve the long term fate of the complexity of the universe.

= and non-subscripted minus sign

the second law of thermodynamics. In panel (c), S_{max} is a constant set at our estimated value of the highest entropy the universe will ever reach, through known dissipative processes such as black hole formation and evaporation (Egan & Lineweaver, 2010). However, in panels (a) and (b), S_{max} represents a time-dependent maximum potential entropy that *would* be produced if all the free energy at time t could somehow be dissipated through instantaneous and unknown dissipation mechanisms into the coldest heat sink available at time t.

The reason $S_{\max}$ continues to rise in panel (a) is discussed in the next section.

The Helmholtz free energy is $F = U - TS$ (e.g. Bejan, 2006). With constant energy U and steady state temperatures, we can write any change in the available free energy as

$$dF = -T dS, \tag{3.2}$$

i.e. when entropy increases, the amount of free energy decreases (Lineweaver & Egan 2008, 2011). Since this is a simple treatment of complexity, the controversial caveats about applying the equations of thermodynamics to non-equilibrium conditions are ignored (e.g. Jaynes, 1989; Kleidon & Lorenz, 2005; Rubi, 2008). Thus,

$$\int_{F_{\max}}^{F_{\min}} dF = -T \int_{S_{\mathrm{uni}}}^{S_{\max}} dS, \tag{3.3}$$

$$\Delta F = T \cdot \Delta S. \tag{3.4}$$

The entropy gap ΔS is a measure of the amount of free energy left in the universe. Free energy is the only kind of energy able to drive irreversible processes such as life and any increase in complexity. Whatever complexity is, it cannot increase without a supply of free energy since all forms of complexity involve irreversible processes. Just as life (or biological complexity) depends on the free energy available from an environment out of equilibrium (Schrödinger, 1944), the increase of any kind of complexity cannot happen without a supply of free energy. The availability of free energy is a necessary but not sufficient requirement for complexity since ΔF and ΔS are not measures of complexity, but are measures of the potential for complexity.

> *Thermodynamic potentials such as entropy or free energy measure capacity for irreversible change, but do not agree with subjective complexity. A human body is more complex than a vat of nitroglycerine, but has lower free energy.*
>
> *(Bennett, 1994)*

The complexity of animal bodies has more to do with the evolved complexity associated with its ability to tap into a flow of free energy than with the free energy value of their contents. It is hard to say more about the relationship between free energy flow and complexity without a definition of complexity.

As $\Delta S \to 0$ (as it does in Fig. 3.2(b) & (c)), we have $\Delta F \to 0$, and therefore all food for life and the ability to maintain complexity goes to zero. We are not suggesting that large ΔS and large ΔF are equivalent to high complexity. The low-complexity early universe with large ΔS (according to Fig. 3.1, panel (c)) is probably the best example of how a large entropy gap is not a good measure of complexity.

Whatever measure of complexity we use, there was little of it in the first tens of millions of years after the big bang when ΔF was large (Fig. 3.1(c)). The first stars formed only after a few hundred million years and the first terrestrial planets formed about a billion years later (Lineweaver, 2001). Thus the early universe was not complex according to any current definition of complexity.

A non-zero entropy gap ΔS is a necessary, but possibly not a sufficient, condition to produce complexity. If it is necessary *and* sufficient, then there must be a considerable time lag before available free energy produces complexity. This is plausible since it takes time for power to spread through the system from primary sources of free energy to secondary sources. This concept is central to Bennett's (1988) slow growth law, under which it takes time for a low entropy universe to evolve the dissipative systems that produce complexity. Complex adaptive systems have a tendency to give rise to other complex adaptive systems (Gell-Mann, 1995), but this requires dissipation, the decrease of free energy, and time. The evolution of the complexity of non-adaptive systems also takes time. For example, it takes a few hundred million years for star formation to access the free energy of nuclear potential of hydrogen. And it takes millions of years of accretion for black holes to access the dominant amount of the gravitational potential of the more homogeneously distributed matter around it. For examples of how solar power spreads through the subsystems of the Earth, see Kleidon (2010) and Lineweaver (2010).

3.3 EXISTING EQUILIBRIUM, OPENING AN ENTROPY GAP, AND THE CONCEPTUAL PROBLEM OF THE MAXIMUM POTENTIAL ENTROPY OF THE UNIVERSE

The central conceptual problem in Fig. 3.1(a) & (b) is this: once the universe is at equilibrium, what processes can get it out? Equilibrium is a state around which the universe can fluctuate (Evans & Searles, 1994), but the universe cannot ratchet its way out of equilibrium without violating the second law. Based on the second law I argue that if it appears that an entropy gap is opening, then that is because there is some unrecognized free energy that should have been included in $S_{\rm max}$ but was not. In Fig. 3.1(a) & (b), Layzer (1975), Frautschi (1988), Barrow (1994, 2011), Chaisson (2001), and Davies (1974, 1994), start the universe in equilibrium, at maximum entropy, with $S_{\rm uni} = S_{\rm max}$. The process that is claimed to open the entropy gap in Fig. 3.1(a) & (b) is the expansion of the universe driving the universe out of equilibrium as $S_{\rm max}$ increases faster than $S_{\rm uni}$. As the universe expands and cools, components that had once been in thermal equilibrium with each other fall out of thermal equilibrium with each other. For example, gravitons decoupled at the Planck time from the rest of the universe. Two seconds later, neutrinos decoupled from the cosmic background radiation, such that today, gravitons, neutrinos, and cosmic background photons co-exist at different temperatures: ~ 0.6 K, 1.95 K and 2.7 K respectively (Egan & Lineweaver, 2010). However, this does not open an entropy gap ΔS that can be inserted into Eq. (3.4) because no work can be extracted from the decoupled fluids.

In Fig. 3.1(a), consider the opening of the entropy gap at recombination. The reason for this opening is supposed to be the emergence of a temperature difference between photons and matter. Such a temperature difference can arise in two ways. One way is that, after recombination, the temperature of the photons scales as $1/a$ while the temperature of the matter scales as $1/a^2$ (where a is the scale factor of the universe). Thus, as a increases, the temperature of the matter cools faster than the temperature of the radiation – the temperatures of the photons and matter diverge. Temperature differences in general are often associated with the ability to drive winds or convection cells and do work (e.g. steam engines or internal combustion engines). However, this cosmological photon/matter temperature difference is between two *decoupled, non-interacting fluids*, both of which are ubiquitous and co-spatial. No work or free energy can be extracted from the temperature difference of decoupled fluids any more than Maxwell's demon can extract work by spatially separating fast and slow particles. No winds or convection cells are produced. Analogous statements can be made about any two decoupled, co-spatial fluids such as between the present 2.7 K cosmic microwave background and the 1.95 K neutrino background – or between either of these and the < 1 K graviton background. Thus, the expansion of the universe and the decoupling of matter from photons cannot open an entropy gap that could be a source of free energy in Eq. (3.4). However, the decoupling of matter and photons is not instantaneous nor complete. A small amount of heat can flow from the hotter photons to the cooler matter because of this residual coupling. This produces some entropy and is known as bulk viscosity (e.g. Zimdahl & Pavon, 2001). But because the hot and cold are not separated by any macroscopic boundary, that is, because there is no macroscopic temperature gradient, no work can be extracted and no dissipative structure can form.

The other way to open up a temperature difference between matter and photons is to heat the matter with an extra source of UV photons, ionizing the matter to temperatures $\sim 10^4$ K. This is what happened during the epoch of re-ionization at a redshift $z \sim 12$. However, the free energy source of the photons was the gravitational accretion energy from either active galactic nuclei or shocks (Dopita *et al.*, 2011) or nuclear fusion in population III stars. This free energy existed before the temperature difference. In other words, some previously existing source of free energy ($\Delta F > 0$ and $\Delta S > 0$) drove re-ionization and the temperature difference. It was not the expansion of the universe or the increase of the scale factor a.

Beyond the entropy produced from bulk viscosity, the expansion of the universe does not increase the entropy of a comoving volume of relativistic energy or non-relativistic matter (Lineweaver & Egan, 2008; Egan & Lineweaver, 2010). Nor does the expansion of the universe increase the entropy of the universe when one species of particle of mass m decouples at equilibrium ($mc^2 \sim kT$), even though at a later epoch, as the photon temperature T decreases, we have $mc^2 > kT$. It is only if the massive particle is unstable and decays under the conditions $mc^2 > kT$ that we have an entropy increase attributable to the expansion of the universe. But if these unstable particles are homogenous and microscopically mixed with all the other particles, no work can be extracted.

Consider the opening and closing of the entropy gap in Fig. 3.1(b). Davies (1994) wrote about it:

> *It is important to realize that the crucial effect of the expansion was in the early universe – hence the sudden widening of the gap early on. Today it seems likely (though I haven't checked) that the gap is narrowing: the universe produces copious quantities of entropy at a rate which I imagine is faster than the (now rather feeble) expansion raises the maximum possible entropy. The actual entropy will presumably asymptote towards the maximum possible entropy in the very far future.*

The idea behind the opening of the gap after t_{BBN} in Fig. 3.1(b) is that before t_{BBN} the temperature is too high for free-energy-yielding nuclear fusion to occur. For example, nuclear fusion cannot produce free energy when the entire universe is a quark–gluon plasma. The first three minutes was not hot enough or dense enough or long enough (due to the rapid expansion of the universe) to complete fusion and release all potential nuclear binding energy. Big bang nucleosynthesis (BBN) and the expansion of the universe left lots of hydrogen, deuterium, and helium that had not been burned to iron (cf. Fig. 4 of Lineweaver & Egan, 2008). If heated up later in the cores of stars at high density, hydrogen can fuse into heavier elements and eventually into iron. The view taken in Fig. 3.1(b) is that before t_{BBN}, this potential nuclear free energy should not be included in the entropy gap, but that after t_{BBN} this potential nuclear free energy should be included. One could argue that in order for nucleosynthesis to open up an entropy gap that could be associated with free energy and work, the hydrogen has to have collapsed into stars and this doesn't happen until a few hundred million years after the big bang. The time at which the "potential" for such gravitational collapse appeared is not well defined. It could be before t_{BBN} when an excess of matter over antimatter appeared in the universe. Or it could be when the universe transitioned from radiation dominated to matter dominated, allowing cold dark matter to collapse to form the seeds of large scale structure. Or it could be at recombination, when baryonic matter decoupled from photons and began to clump in the over-dense cold dark matter haloes. Or it could be a few hundred million years later when hydrogen began to fuse into helium in the first stars. Contrafactual "potential" is a slippery concept.

Similarly, arbitrary S_{max} budgeting produces similar confusion with the accounting of the entropy of black holes. As black holes form, the entropy of the universe S_{uni} increases. But when and how are we to include potential black hole formation into the budget of S_{max}? What does it mean to include in S_{max} the entropy of black holes that *could* form, but *never will form*? There are well-discussed entropy bounds in the literature, for example, which envisage all the matter in the observable universe collapsing into a black hole (Susskind, 1995). But we live in a Λ-dominated universe in which the acceleration of the universe has been shutting off the growth of structure for the past billion years. Thus, the entropic bound of all matter in the observable universe collapsing into a black hole cannot give us an attainable or plausible value for S_{max}. If we are concerned with values of S_{max} that can be inserted into Eqs. (3.1) and (3.4) to give us the currently most plausible estimate of the amount of free energy that eventually becomes available to produce complexity during the evolution of the universe, then the most meaningful $S_{\max}$ seems to be the one in Fig. 3.1(c).

Another reason why (contrary to what is shown in Fig. 3.1(a) & (b)) S_{uni} cannot be equal to $S_{\max}$ for all $t < t_{\text{BBN}}$ is that the asymmetry between matter and antimatter ("baryogenesis") had its origin in conditions that required thermodynamic disequilibrium or $\Delta S > 0$ (Sakharov, 1967; Kolb & Turner, 1990; Quinn & Nir, 2008).

With the establishment in ~ 1998 of the cosmological constant as the dominant form of energy in the universe, Barrow's (1994) views changed from Fig. 3.1(a) to Fig 3.1(b) (Barrow, 2011, personal communication). Also, in contrast with Davies (1994), Barrow sees the entropy gap opening at the Planck time, about 3 minutes (~ 45 orders of magnitude) earlier than at the t_{BBN} shown in Fig. 3.1(b).

3.4 SOURCES OF FREE ENERGY

In the standard ΛCDM big bang model with an early epoch of inflation tacked on at the beginning (e.g. Liddle & Lyth, 2000), the sources of free energy in the early universe are:

(1) vacuum energy. At the end of inflation there was a transition from a false vacuum to a true vacuum. The potential energy of the false vacuum was dumped into the universe in the form of radiation, matter, and antimatter;
(2) the disequilibrium that produced more matter than antimatter (Sakharov, 1967) and is a source of free energy in that, if there were no asymmetry, the universe would contain only radiation and there would be no matter that could collapse;
(3) the gravitational potential energy of the homogeneous distribution of the excess matter.

After the energy of the vacuum was dumped into the universe, this energy was out of equilibrium in at least two ways. Firstly, it had to be in disequilibrium to even produce a matter–antimatter asymmetry. Secondly, homogeneous matter can clump into inhomogeneities. The mutual annihilation of matter and antimatter was a source of energy but was not itself a source of free energy for the same reason that the heat transfer from photons to baryons after recombination was not a source of free energy: no macroscopic boundary between source and sink.

The transition from unclumped matter to clumped matter is still going on today, creating gradients of density, pressure, and temperature, and which, at the center of stars, is permitting access to the unburned nuclear binding energy of hydrogen and helium. All of this makes the entropy of the universe increase. As matter clumps, the entropy increases, but there is not yet an equation linking the parameters of large scale structure formation to gravitational entropy. The high gravitational potential energy and the associated low gravitational entropy of initially unclumped matter is the main fact upon which Penrose (1979, 2004) and Lineweaver & Egan (2008, 2011) claim that the universe started out at low entropy and that initial ΔS was at a maximum. We hypothesize that the low entropy origin of the universe was due to the highly homogeneous matter far from gravitational equilibrium, despite it being near thermal and chemical equilibrium.

One source of the conceptual confusion underlying the differences in the sketches of Fig. 3.1 is whether one should assign a large initial potential $S_{\max}$ to this unclumped matter or not. The disagreement is not about how or when matter clumps, but about how much of its not-yet-clumped potential to assign to $S_{\max}$ and when to assign it. Should a metastable local minimum of energy be considered equilibrium or disequilibrium (Fig. 3.3)? Trying to estimate ΔS from how much matter could clump now (but hasn't) seems to be a difficult or even unaddressable issue which we circumvent by estimating the ultimate extent to which matter will clump. Thus in Egan & Lineweaver (2010) we chose to use a constant $S_{\max}$ set by the degree to which matter eventually clumps (under our current assumptions about the far future of the universe).

Consider Fig. 3.3. If the universe starts out in a false vacuum at ϕ_1 and then tunnels into a true vacuum at ϕ_2, then the potential energy difference $\Delta V_{12} = V_1 - V_2$ is dissipated during reheating and

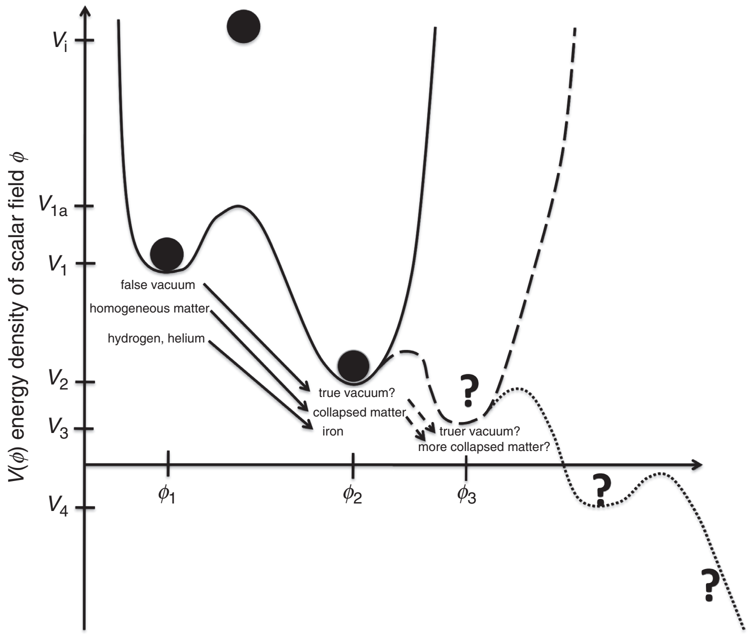

FIGURE 3.3 Generic sketch of transitions and potential transitions in the early universe that are the sources of free energy ($\Delta V \sim \Delta F$ of Eq. (3.4)). In order to have a ΔS that increases without bounds (Fig. 3.2(a)) we require an infinite cascade of phase transitions beyond the supposedly true vacuum of ϕ_2, similar to the three transitions described between ϕ_1 and ϕ_2, i.e. false to true vacuum, unclumped to clumped matter, and hydrogen to iron.

the entropy of the universe goes up. Therefore the universe in the false vacuum before the end of inflation is in disequilibrium, not at $S_{\text{uni}} = S_{\text{max}}$ as is assumed in Fig. 3.1(a) and (b). During reheating, matter was dumped homogeneously into the universe. The matter could then begin to collapse. Thus, a universe with homogeneously distributed matter is in disequilibrium with respect to gravity, because the matter has not all collapsed. When matter does collapse and large-scale structure forms, heat is released and entropy increases. Thus, our early universe with homogeneously distributed matter was in disequilibrium, not at $S_{\text{uni}} = S_{\text{max}}$.

The same reasoning goes for hydrogen and iron. A universe filled with hydrogen and helium is filled with a fuel, at a local minimum of nuclear binding energy (e.g. ϕ_1 of Fig. 3.3). We interpret this local minimum of the energy as a metastable recoverable disequilibrium source of free energy that has existed since very early in the evolution of the universe when the expansion rate and density and therefore the eventual incompleteness of BBN were determined (see Fig. 4 of Lineweaver & Egan, 2008). We consider nuclear potential free energy as being recoverable because it will eventually burn in our universe and be taken out of this local minimum by the high temperatures and densities at the centers of stars. If we lived in a universe in which no stars formed, then hydrogen and helium would not be a source of free energy and we would not include this source of entropy in S_{max}. Thus, the transitions from a false vacuum to a true vacuum, from unclumped matter to clumped matter, and from hydrogen to iron are sources of disequilibrium that exist initially and are the reasons why ΔS in Fig. 3.1 panel (c) is so large.

One disadvantage of the constant S_{max} approach shown in Fig. 3.1(c) is that one would like ΔS to be a measure of how much free energy is available at time t to drive the dissipative processes that are occurring at time t. This is not the case for our constant S_{max} and the ΔS derived from it, which are measures of the free energy that will eventually become available at some time during the evolution of the universe. For example, although nuclear fusion of hydrogen into helium is included in our ΔS at early times (i.e. $t <$ few hundred million years), the first stars, giving access to temperatures and densities which permit fusion, do not exist until a few hundred million years after the big bang.

One way to make the concept of maximum entropy more useful is to distinguish between the constant S_{max} of Fig. 3.1(c) and a time-dependent $S'_{\mathrm{max}}(t)$ that depends on the rate at which entropy is being produced. For example, we could define it as

$$S'_{\mathrm{max}}(t) = S_{\mathrm{uni}}(t) + (\alpha/H)\mathrm{d}S_{\mathrm{uni}}/\mathrm{d}t, \tag{3.5}$$

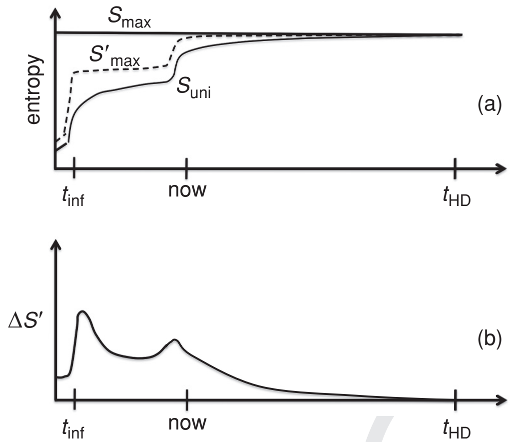

FIGURE 3.4 Same as Figs. 3.1(c) and 3.2(c) except showing $S'_{max}(t)$ in (a) and the resulting new $\Delta S'$ in (b). See Eq. (3.5)–(3.8).

where H is Hubble's constant and the dimensionless constant α is constrained by the condition

$$S_{uni}(t) \leq S'_{max}(t) \leq S_{max}, \tag{3.6}$$

which ensures that in the far future, as the universe approaches a heat death, that $S'_{max}(t = t_{HD}) = S_{max}$. With this new maximum entropy we can define a new entropy gap,

$$\Delta S' = S'_{max}(t) - S_{uni}(t) = (\alpha/H)\mathrm{d}S_{uni}/\mathrm{d}t \tag{3.7}$$

and a modified version of Eq. (3.4),

$$\Delta F' = T\Delta S' \tag{3.8}$$

that reflects the free energy $\Delta F'$ available as a function of time and depends on the instantaneous rate at which the entropy of the universe is increasing, $\mathrm{d}S_{uni}/\mathrm{d}t$. In a cash-flow analogy, Eq. (3.8) amounts to trying to estimate how much a customer has to spend now ($\Delta F'$) by measuring how much they are spending ($\Delta S'$). This is different from the total amount a customer will ever be able to spend (ΔF of Eq. (3.4)). Figure 3.4 sketches what $S'_{max}(t)$ and $\Delta S'$ might look like. Notice that, in contrast to ΔS of Fig. 1(c), $\Delta S'$ increases when free energy becomes available (e.g. as stars fuse hydrogen). In contrast with ΔS of Fig. 3.1(b), $\Delta S'$ does not depend on non-existent instantaneous dissipation mechanisms or subjective estimates of the potential for dissipation. $\Delta S'$ depends on a measurable quantity, $\mathrm{d}S_{\mathrm{uni}}/\mathrm{d}t$.

3.5 DOES COMPLEXITY INCREASE?

> *Does complexity increase in the course of evolution? Or does it decrease? According to the Second Law of Thermodynamics the latter should be the case. But looking just superficially at the richness of nature, one comes to believe in an ongoing and open-ended emergence of increasingly complex structures that stabilize further and further from thermodynamic equilibrium – with humans and their creations possibly being the latest manifestations of this.*
>
> *(Fullsack, 2011)*

To answer the question "is complexity increasing?" we need to disambiguate it. Are we talking about the average complexity of the universe or about the complexity of the most complex object? Are we talking about a current increasing trend (that could be quite ephemeral and last for only a million or a billion years)? Or are we talking about an ultimate enduring trend? In this chapter we use the connection between complexity and entropy to conclude that the ultimate trend of the average complexity must be to decrease.

Since all laws of physics are time-reversible except for the second law, if there is a secular change in some quantity, such as complexity, then it will have a deep connection with the only law that has a direction for time, the second law. This is the connection we have used in this chapter.

Complexity relies on $\Delta F > 0$. However, the combination of the cosmological constant and the second law of thermodynamics requires $\Delta S \rightarrow 0$ and therefore $\Delta F \rightarrow 0$ (Eq. (3.4)). This entails the heat death of the universe, the fading of complexity like a flashlight with a dying battery, the extinction of all life, and the disappearance of all structure, leaving us in the simplicity of equilibrium, forever.

This conclusion seems to be in disagreement with Dyson (1979) but in agreement with Krauss & Starkman (2000). And it leaves room for comforting statements like:

> *[W]e will still ultimately lose the battle against degeneration. But the second law does not mandate a steady degeneration. It quite happily coexists with spontaneous development of order and complexity.*
>
> *(Rubi, 2008)*

Rubi

The happy coexistence of the second law and complex objects that the author is referring to is based on a continuous but unsustainable expulsion of high entropy material by the most complex objects. Even in a universe in which the average complexity is decreasing, the complexity of the most complex objects can increase for a while. This is certainly the niche we identify with, but like burning fossil fuels, it is not sustainable.

Long-term patterns of biological evolution are not exempt from the second law. Adami *et al*. (2000) describe how biological evolution acts like a Maxwell demon. Maxwell's demon (Maxwell, 1888) lowers the entropy of molecules in a box by letting only hot molecules pass from one side to the other – thus separating the hot from the cold molecules, and apparently violating the second law. Natural selection can also be thought of as a Maxwell's demon selecting fitter phenotypes (~ "hotter molecules") whose DNA contains more information about the environment. But just as Szilard (1929) and Bennett (1987) have pointed out that Maxwell's demon is not a perpetual motion machine, the Maxwell demon of natural selection is not a perpetual motion machine. There is a source of free energy that both provides an information-rich environment and the energy that sustains life. Driven by free energy, natural selection turns the crank that ratchets up and preserves the accumulation of information in DNA.

> *Darwinian selection is a filter, allowing only informative measurements (those increasing the ability for an organism to survive) to be preserved. In other words, information cannot be lost in such an event because a mutation corrupting the information is purged due to the corrupted genome's inferior fitness.*
>
> *(Adami* et al.*, 2000)*

We have argued that a complex environment is the result of the initial low gravitational entropy of the early universe and the resulting gravitational collapse of galaxies, stars, and planets. The ultimate driver of complexity is the dissipation of free energy. This does not seem to be a well-accepted point of view. Gell-Mann (1995) does not associate the evolution of complexity with low gravitational entropy, but wants to explain the complexity of life as the result of "frozen accidents, giving rise to regularities" with little or no connection with free energy.

> *As the universe grows older and frozen accidents pile up, the opportunities for effective complexity to increase keep accumulating as well. Thus there is a tendency for the envelope of complexity to expand.*
>
> *(Gell-Mann, 1995)*

> *The second law of thermodynamics, which requires average entropy (or disorder) to increase, does not in any way forbid local order from arising through various mechanisms of self-organization, which can turn accidents into frozen ones producing extensive regularities.*
>
> *(Gell-Mann, 1995)*

Note the similarity between this statement and Miguel Rubi's statement above. Both refer to the uncontroversial increase in the local order. However, the crucial ingredient not mentioned in the recipe is that these "extensive regularities" or complexities (like the ΔT produced by Maxwell's demon) come at a price of higher entropy elsewhere. And since sources of free energy are always decreasing, the trend toward local order and complexity, like a civilization built on fossil fuel, can only be temporary.

Does local order keep increasing? It can until the exported entropy fills up the universe. For example, air conditioners and refrigerators work as long as the heat they generate can be removed . . . as long as there is a sink. Segregation can continue, but will not last forever since the amount of free energy is limited and without free energy there can be no segregation, no export of high entropy, leaving the low entropy behind.

We know that life forms are not unusual statistical fluctuations or Boltzmann brains because we persist in ways that 1000 sigma statistical fluctuations do not. When the molecules in this room pile into a corner at random, they immediately pile out. They are not frozen. The reason regularities can be frozen into life is because of a constant free energy supply which supplies the electricity to the freezer. This persistence requires a flow of free energy whose dissipation is the price of our persistence. You cannot freeze accidents for free.

If we find that we are living in a false vacuum and that protons and other seemingly stable particles decay, then these will be new sources of free energy and the universe will be able to evolve to the right in Fig. 3.3. If an infinite number of such free energy sources are identified then the universe can keep evolving to the right in Fig. 3.3 forever. On the other hand, if we have already identified all the sources of free energy in the universe, then the acceleration of the expansion of the universe and its asymptotic approach to a vacuum state will lead to the heat death of the universe, the dissipation of all free energy and the reduction and disappearance of complexity as shown in Fig. 3.2, panel (c).

3.6 SUMMARY

In any naturalistic explanation for the origin and evolution of life, the non-adaptive complexity of the physical environment precedes, and is the source of, the adaptive complexity of life. However, the complexity of the physical environment did not always exist. 400 000 years after the big bang there were no stars, planets, or life. The complexity of the physical environment is the result of irreversible processes driven by the dissipation of free energy – initially gravitational free energy associated with the initial low entropy of the universe. Since the amount of free energy decreases as the entropy of the universe increases, cosmological estimates of entropy yield upper limits on physical complexity and therefore biological complexity. I used the concept of an entropy gap, $\Delta S = S_{\text{max}} - S_{\text{uni.}}$, between the maximum entropy and the actual entropy of the universe to quantify the available free energy and the potential for complexity in the universe. Previous estimates of ΔS were compared and found to differ because of different assumptions about S_{max}, equilibrium and free energy. I have clarified some of these differences. I found that the combination of the cosmological constant and the second law of thermodynamics requires $\Delta S \to 0$. This entails the heat death of the universe, the decrease of complexity like the fading glow of a flashlight with a dying battery, the extinction of all life, the disappearance of all structure – leaving us in the simplicity of equilibrium, forever and ever. Amen.

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