Problem Set 2 - Cosmology–A3002 –Due on or before September 17th. Can be sent via internal mail (Brian Schmidt, RSAA) or handed into Ken Freeman.

Problem 1.1: (4pts)
We want to use supernovae as a standard candle to measure luminosity distance as a function of redshift in order to determine the Hubble Constant. From class we know that selection effects can bias our measurement. We want to simulate the effect using a Monte Carlo simulation. First, lets determine the bias as a function of redshif t for a sample of objects via a Monte Carlo calculation. To set up your simulation, assume that H_0 = 70 km/s/Mpc, and that the absolute magnitude of SN Ia =-19.5 mag, and that your supernova search will be able to detect a 100% of objects as faint as 18.5 magnitude, and none fainter. Using a computer program or excel spreadsheet, calculate the mean magnitude of the SNIa actually detected at z=0.05, and z=0.09 for 3 assumptions of the dispersion (Gaussian 1 std-deviation) of SNIa about their mean brightness of 0.1, 0.2, and 0.4 magnitudes. An excel hint: A Gaussian random variable with mean -19.5 and std deviation 0.2 in EXCEL is given by NORMINV(RAND(),-19.5,0.2). You are free to use any computer program to do the simulation.

Problem 1.2: (4pts)
The Monte Carlo simulation in 1.1 is not really necessary because the bias calculation can be expressed as an integral (which can be numerically evaluated). Given that a Gaussian probability distribution (That is the probability that an object will have a value within x and x+dx, for an object of mean, M, and std deviation, σ) is given by the expression

$$P(x)dx = \frac{1}{\sigma\sqrt{2\pi}} e^{-\left(\frac{(x-M)^2}{2\sigma^2}\right)} dx .$$

Write down the equation (involves an irreducible integral) that gives the mean absolute magnitude of a type Ia supernova in the observational scenario described in problem 1.1. Compare the bias you derive from this expression with your Monte Carlo values above.

Problem 1.3: (2pts)
To simulate the bias in measuring the Hubble Constant, we need to realistically select objects as a function of redshift. In the nearby Universe, we can assume that volume is proportional to z^3. If we want to randomly select objects in the volume between $0<z<z_{max}$, justify that the selection of a random redshift is given as

$$z(\text{random}) = (z_{max}\,\text{RANDOM})^{1/3}$$

where RANDOM is a random number between 0 and 1.

Problem 1.4: (4pts)
Now finish off your Monte Carlo simulation by selecting objects at random redshifts as per 1.3 from 0<z<0.1, assuming that they have a mean absolute

magnitude of M=-19.5 with a dispersion 0.2 magnitudes. Using only those objects which appear brighter than your limiting magnitude of 18.5, calculate the Hubble constant from your sample for all visible objects. How does this compare to your expected value of 70 km/s/Mpc? Do this for dispersions for SN Ia of 0.3 and 0.5 magnitudes as well and compare the resulting biases in the Hubble Constant.

Problem 1.5: (2pts)
Now imagine that you wanted to do this same calculation, not for the Hubble constant, but for measuring the cosmological parameters Ω and the associated equation of state parameters,w, for objects which are much fainter– m=24mag. What things would you need to take into account beyond our simple calculation done in Problem 1.4?

Problem 2:
In a flat Universe, the luminosity distance to an object at redshift z, in a universe made up of ***i*** forms of matter each with an equation of state $\boldsymbol{w_i}$ and density, Ω_i is given by

$$D_L = \frac{c}{H_0}(1+z)\int_0^z dz'\left[\sum_i \Omega_i (1+z')^{3+3w_i}\right]^{-1/2}$$

2.1 (5 pts).
Compute the luminosity distance at z=0.01, z=0.5, and z=1, for two plausible Universes.

Universe 1: H_0=70 km/s/Mpc, Ω_M=0.27, Ω_w=0.73, w=-1
Universe 2: H_0=70 km/s/Mpc, Ω_M=0.27, Ω_w=0.73, w=-0.9

You will need to write a computer program (or Excel spreadsheet) to do this integral. Attach your program/spreadsheet. What is the ratio of distances between the different Universes at z=0.5 and at z=1?

2.2(2pts)
If we want to measure a luminosity distance with a supernova, its relative brightness (flux measured at Earth) is given by

$$\frac{F(z_1)}{F(z_2)} = \left(\frac{D_{L1}}{D_{L2}}\right)^{-2} K_{12},$$

where $\boldsymbol{K_{12}}$ is the K-correction – a correction which accounts for observing a different part of the spectrum of the supernova as it changes redshift. Calculate the relative brightnesses between z=0.01 and (z=0.5, z=1) for the two Universe models of a supernova, Using a value of for $K_{z=0.01-z=0.5}$=2 and $K_{z=0.01-z=1}$=3 (yes they make the object brighter than it would otherwise be)

2.3(3pts)
Most astronomical objects are fainter than the sky background -There are more photons in the background than in the object, and this object background must be subtracted off, to make a measurement. Show that, if the uncertainty in a

measurement (signal) of N photons is sqrt(N), and that the signal of the background dominates the signal of the object, the relative time to achieve the same signal/noise (that is signal divided by noise) of two objects of different fluxes is given by the expression $\frac{t_1}{t_2} = \left(\frac{F_2}{F_1}\right)^2$.

2.4(5pts)
We want to use supernovae to measure luminosity distance as a function of redshift. We have a choice, we can compare our nearby objects at z=0.01 with objects at z=1or at z=0.5. But we want to use the least amount of telescope time. We will assume that Gaussian statistics apply. That is, the ratio of the number of measurements required to achieve two different signals scales as

$$\frac{N_1}{N_2} = \left(\frac{Signal_1}{Signal_2}\right)^2$$

For example. If I have a method of measuring a person's height which has an uncertainty associated with it (which obeys Gaussian statistics), and I want to double my accuracy, I need to make 4 times more measurements.
Using your results in 9.1-9.4, calculate the ratio of time it will take to discriminate between Universe 1 and Universe 2 to any level of certainty, by comparing nearby objects to objects at z=0.5 or z=1.

Problem 3.
Assuming H_0=70km/s/Mpc, and that the Universe is flat for all of question 3. Express the answers to these problems in terms of billions of light years times the speed of light (c).

Problem 3.1 (2 pts):
We believe the CMB was emitted at t=100,000 years after the Big Bang(t=0). How far could light have travelled before that time? Assume the universe is radiation dominated the whole time. (Give answer in both commoving distance and in physical distance.)

Problem 3.2 (2 pts):
How far could light have travelled since the time of the Cosmic Microwave Background emission until now? Assume the universe has been matter dominated the whole time.

Problem 3.3(2 pts):
How big would one causally connected patch appear on the sky? The redshift of recombination is about z=1100.

Problem 4.1 (2 pt):
In class we described the scale at which material can collapse in the Universe as the Jeans' length. This scale is given approximately by the equation

$$L_{Jeans} \approx \sqrt{\frac{kT}{mG\rho}}.$$

Where the mass per particle is given as ***m***, ***T*** is temperature, ρ is density, and ***G*** and ***k*** are the gravitational and Stefan-Boltzmann constants, respectively. As the Universe expanded, a special time occurred when the Hydrogen and electrons recombined – photons and matter were no longer coupled, maybe allowing objects to freely gravitationally collapse. Given that the observed temperature of the CMB is 2.73K, and hydrogen recombines at T=3000K, approximately what redshift did this decoupling occur? What was the density of matter at that time? (using H_0=70 km/s/Mpc, Ω_M=0.27, Ω_w=0.73, w=-1)

Problem 4.2 (2 pt):
Determine the Jeans' length for this epoch? What is the Jeans' mass – i.e. the amount of mass within this scale? As the Universe evolves, how would this size and mass look today?

Problem 4.3 (2 pt):
Globular clusters have been proposed as the first objects to collapse in the Universe. What are the good and bad parts of the idea that globular clusters are the result of a Jeans' length collapse immediately after recombination?