

Growth of perturbations

- **What does it take for an object to Collapse in the Universe.**
- **We can estimate this by looking at the Gravitational Binding Energy of a spherical ball and comparing it to the thermal energy of the ball. When gravity dominates, the object can collapse. Scale where this happens is called the Jean's Length**

$$\frac{GM^2}{R} \approx \frac{M}{m} kT \quad E_{par} \approx kT \quad N_{par} = \frac{M}{m}$$

$$\frac{GM}{R} \approx \frac{kT}{m}$$

$$\frac{G(\frac{4}{3}\pi\rho R^3)}{R} \approx \frac{kT}{m}$$

$$R \approx \sqrt{\frac{kT}{mG\rho}} \equiv \lambda_J = c_s \sqrt{\frac{1}{G\rho}}$$

Growth of linear perturbations

- **The (non-relativistic) equations governing fluid motion under gravity can be linearized to give the following equation governing the growth of linear density perturbations:**

$$\ddot{\delta} + \frac{\dot{a}}{a} \dot{\delta} = \delta \left(4\pi G\rho_0 - \frac{c_s^2 k^2}{a^2} \right)$$

- **This has growing solutions for on large scales (small k) and oscillating solutions for for small scales (large k); the cross-over scale between the two is the Jeans length,**

$$\lambda_J = c_s \sqrt{\frac{\pi}{G\rho}}$$

where c_s is the sound speed, $c_s^2 = \partial p / \partial \rho$.

- **For $\lambda < \lambda_J$, sound waves cross an object on the same time-scale as the gravitational collapse, so pressure can counter gravity.**
- **In an expanding universe, λ_J varies with time; perturbations on some scales swap between growing and oscillating solutions.**

Peculiar Velocity and Linear Growth

Peebles, (1976) demonstrated in the linear regime (i.e. acceleration Due to a mass concentration is constant – unaffected by the growth of the mass concentration) the following relationship holds.

$$\frac{v_{pec}(r)}{H_0 r} = -\frac{1}{3} \Omega_M^{0.6} \delta(r) \quad \text{BIAS: } \delta_{gal}(r) = b \delta(r)$$

$$\frac{v_{pec}(r)}{H_0 r} = -\frac{1}{3} \frac{\Omega_M^{0.6}}{b} \delta_{gal}(r)$$

So... We think $\Omega_M = 0.3$,

between us and the Virgo Cluster the density of galaxies we see over the background is a factor of 2 in that sphere,

$H_0 = 70$ km/s

Distance to Virgo cluster is 16 Mpc...

$$v_{pec}(r) = -\frac{H_0 r}{3} \frac{\Omega_M^{0.6}}{b} \delta_{gal}(r)$$

$$v_{pec}(r) = -\frac{70 * 16}{3} \frac{0.3^{0.6}}{b} 2 = 362 \text{ km/s}$$

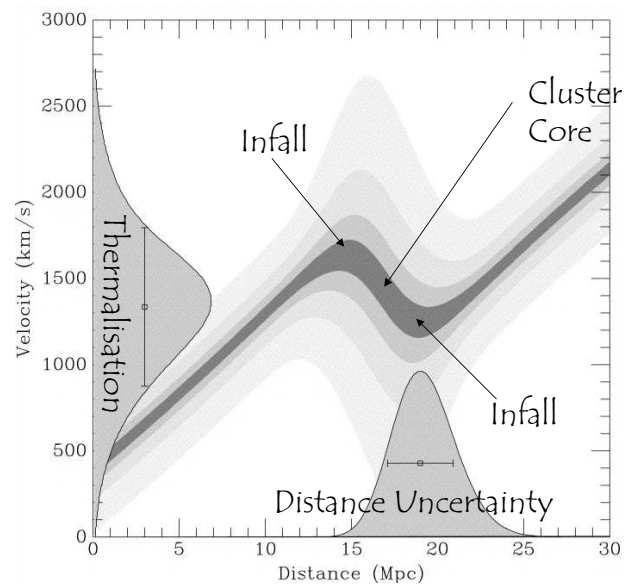
Bias: light vs mass

- ***Gravitational instability theory applies to the mass distribution but we observe the galaxy distribution - are these 1-to-1?***
- ***A bias factor b parameterises our ignorance: $\delta_g = b \delta_M$, i.e. fractional variations in the galaxy density are proportional to fractional variations in the mass density (with ratio b).***
 - ***What might produce a bias? Do galaxies form only at the peaks of the mass field, due to a star-formation threshold?***
 - ***Variation of bias with scale. This is plausible at small scales (many potential mechanisms), but not at large scales.***
 - ***Observed variation with galaxy type. The ratio E:Sp is 10:1 in clusters ($\delta_g \gg 1$) but 1:10 in field ($\delta_g \leq 1$).***

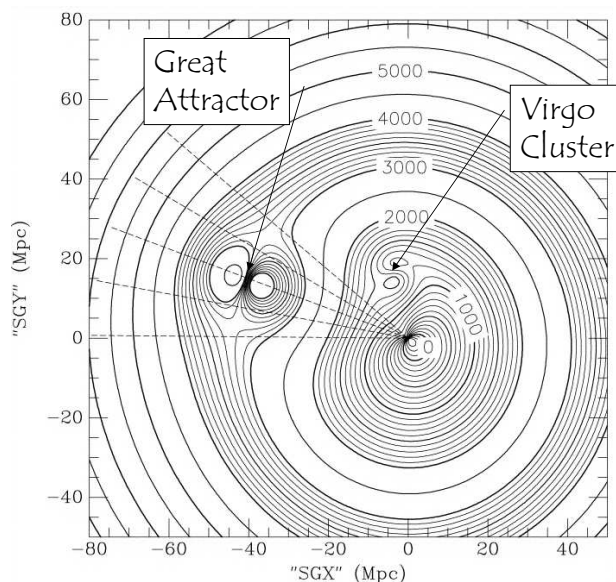
Non-Linear Growth

- ***Eventually structures grow and this causes their Mass to increase, and the linear regime to breakdown***
- ***Galaxies start to interact with each other and thermalise (Called Virialisation)***

Virgo Cluster as Measured with Surface Brightness Fluctuations

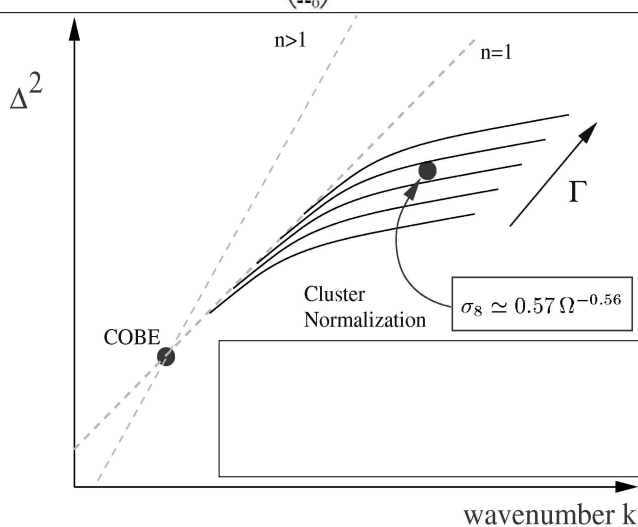


Map of velocities in nearby Universe-SBF



$$\Gamma_{\text{eff}}(k) = \Omega_0 h \left[\alpha_{\Gamma} + \frac{1 - \alpha_{\Gamma}}{1 + (0.43ks)^4} \right], \quad (30)$$

$$\alpha_{\Gamma} = 1 - 0.328 \ln(431 \Omega_0 h^2) \frac{\Omega_b}{\Omega_0} + 0.38 \ln(22.3 \Omega_0 h^2) \left(\frac{\Omega_b}{\Omega_0} \right)^2. \quad (31)$$



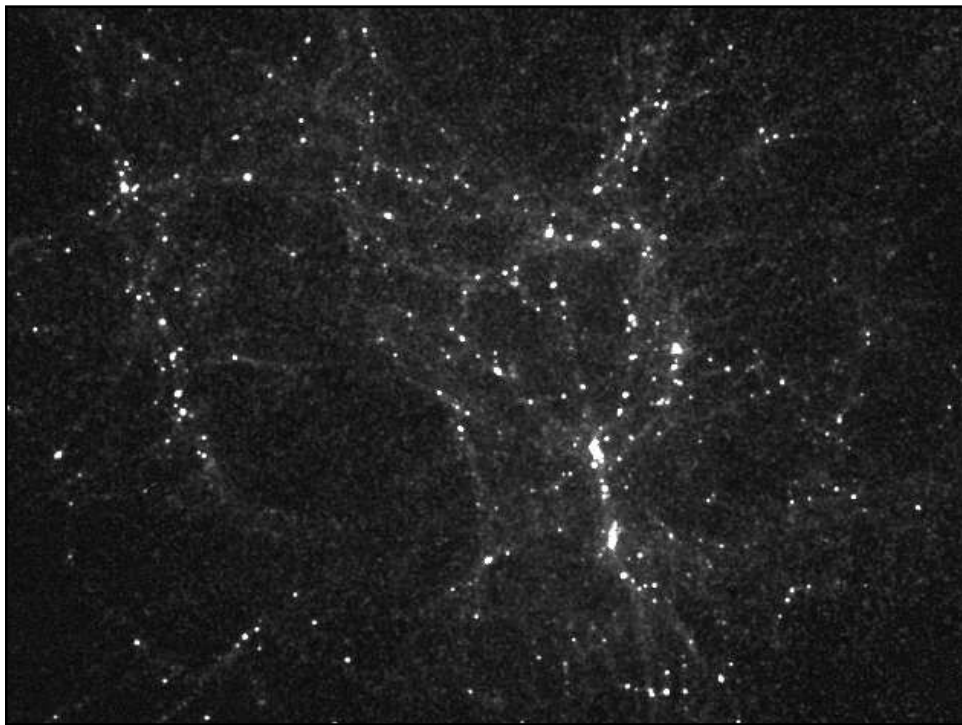
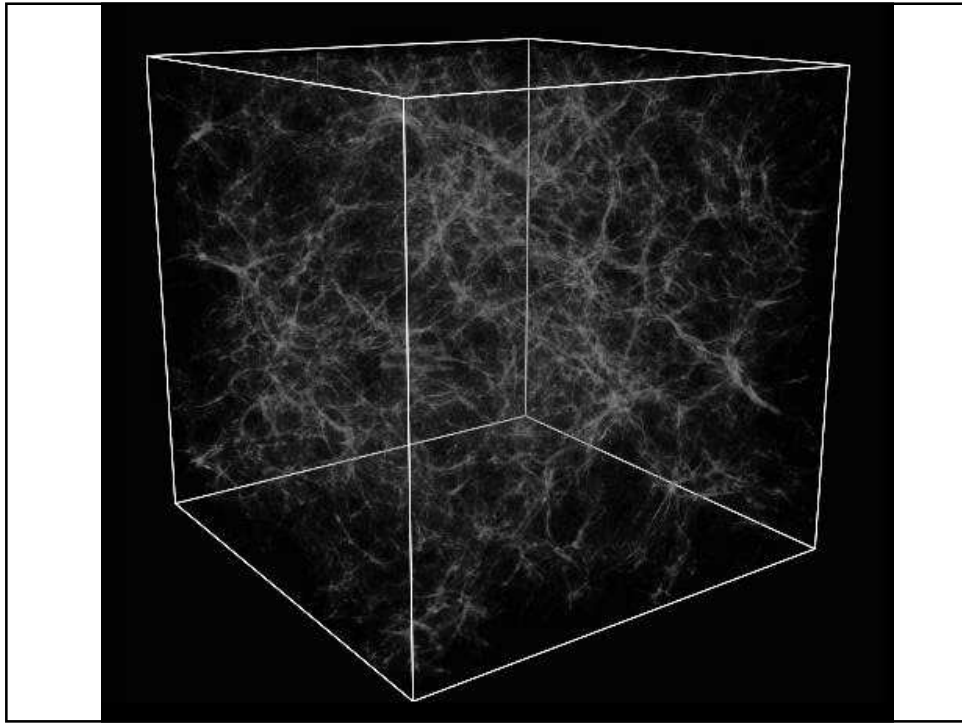
The CDM Power Spectrum

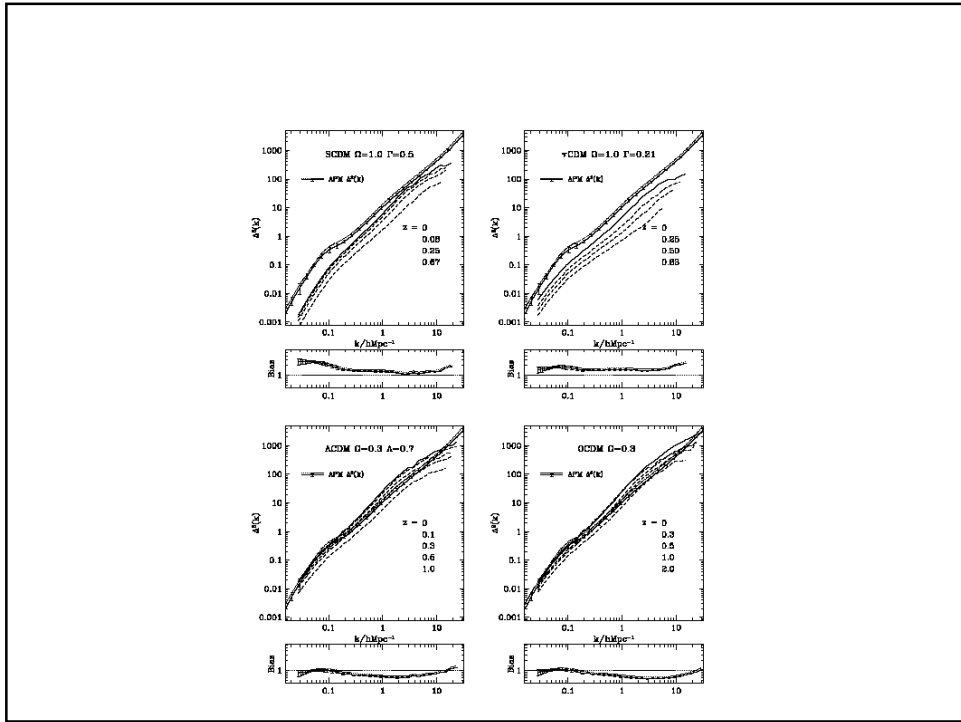
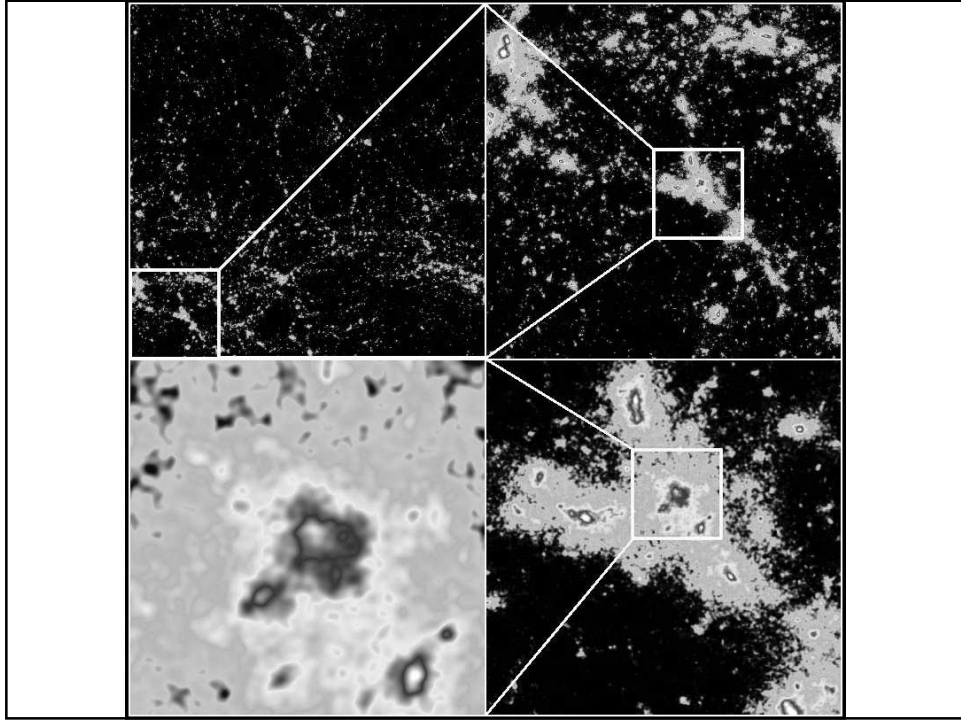
Fluctuations in the density grow as $\delta(a) = a f(\Omega|a)$

Scale of break in power spectrum relates to baryons suppressing growth of CDM

In practice, get shape parameter $\Gamma \approx \Omega_m h$

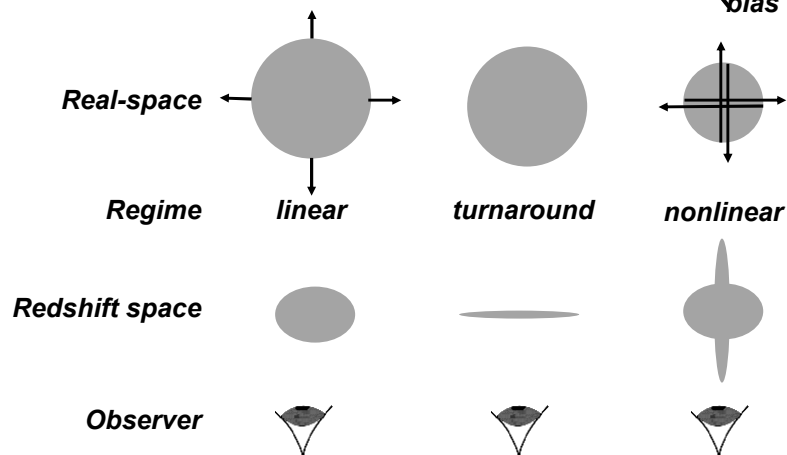
Γ is a shorthand way of fitting the actual power spectra coming out of Nbody models





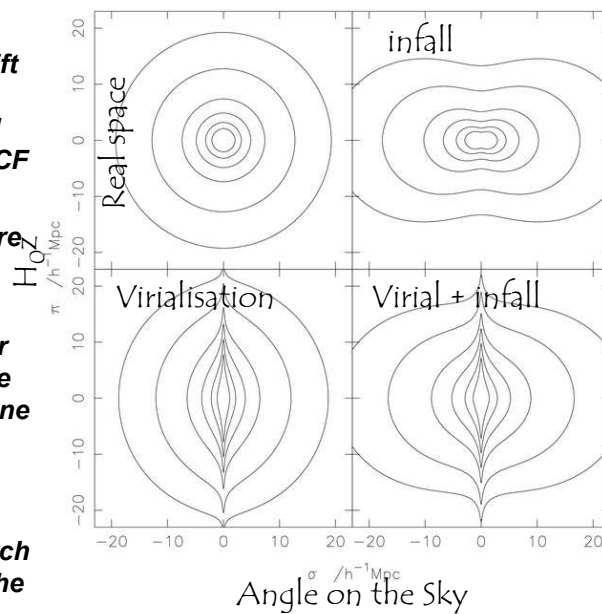
Redshift-space distortions

$$z_{\text{obs}} = z_{\text{true}} + v_{\text{pec}}/c \quad \text{where} \quad v_{\text{pec}} \propto \Omega^{0.6} \delta\rho/\rho = (\Omega^{0.6}/b) \delta n/n$$



Redshift-space distortions

- Because of peculiar velocities, the redshift space Correlation Function is distorted w.r.t. the real-space CF
- In real space the contours of the CF are circular.
- Coherent infall on large scales (in linear regime) squashes the contours along the line of sight.
- Rapid motions in collapsed structures on small scales stretch the contours along the line of sight.



Some Relevant questions in Large Scale Structure

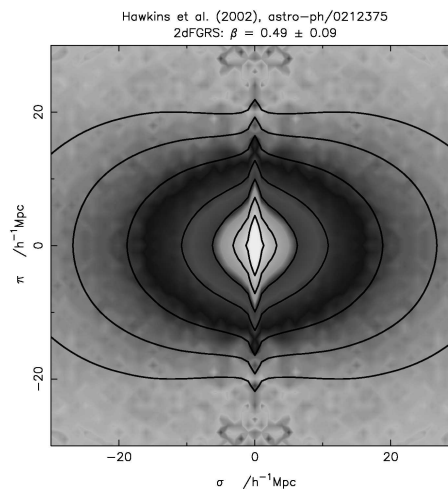
- *What is the shape of the power spectrum?
what is the value of $\Gamma = \Omega h$?*
- *Mass and bias:*
 - *what is the value of $\beta \approx \Omega^{0.6}/b$?*
 - *can we obtain Ω and b independently of each other?*
 - *what are the relative biases of different galaxy populations?*
- *Can we check the gravitational instability paradigm?*
- *Were the initial density fluctuations random-phase (Gaussian)?*

Measuring β from $P(k)$

- *z-space distortions produce 'Fingers of God' on small scales and compression along the line of sight on large scales.*
- *Or can measure the degree of distortion of ξ_s in σ - π plane from ratio of quadrupole to monopole:*
- $$\begin{aligned} P_2^s(k) &= 4/3\beta + 4/7\beta^2 \\ P_0^s(k) &= 1 + 2/3\beta + 1/5\beta^2 \end{aligned}$$

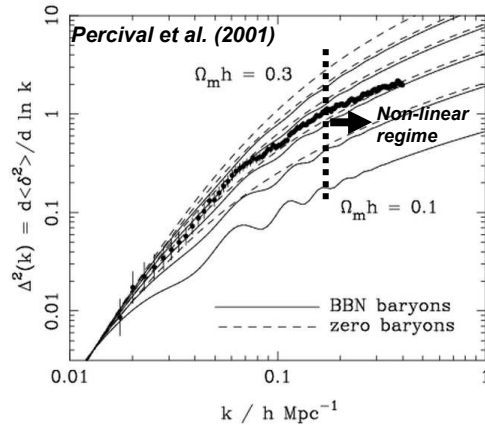
$$(\Omega_M)^6 = b * 0.49$$

$$\Omega_M = .30 * b^{5/3}$$



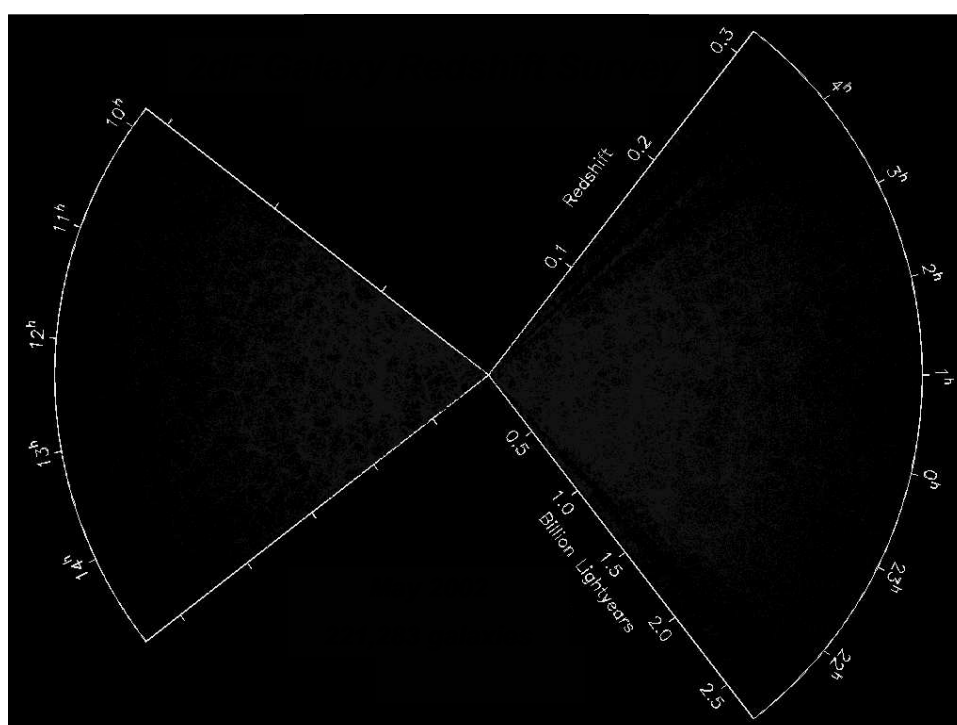
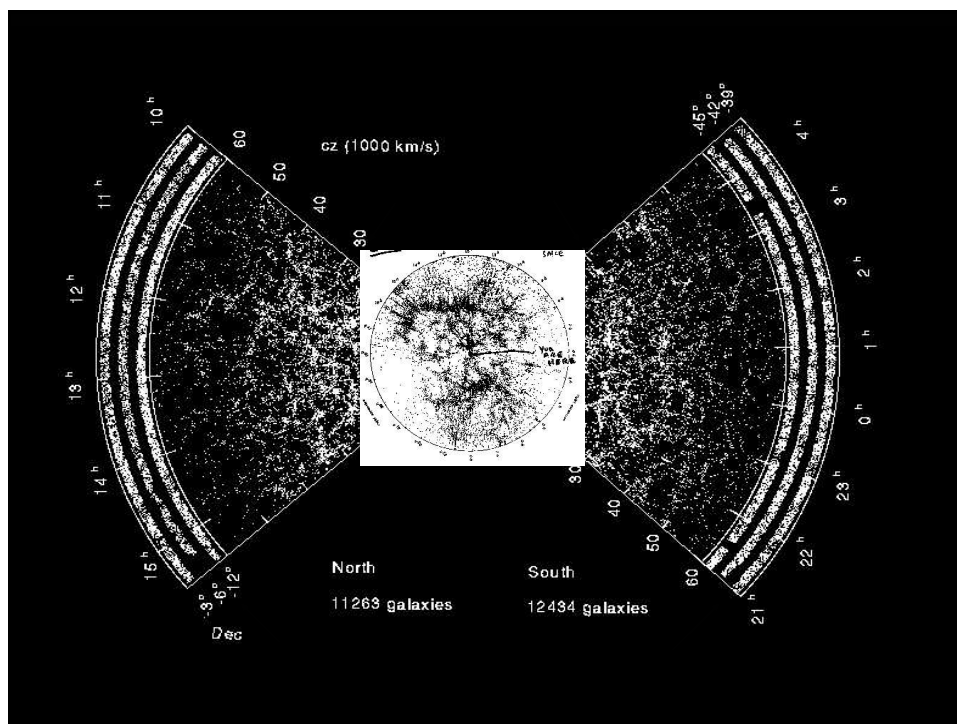
Large scales - P(k)

- **P(k) is preferred to $\xi(r)$ on large scales: it is more robust to compute, the covariance between scales is simpler, and the error analysis is easier.**
- **Fits to P(k) give $\Gamma \approx 0.2$, implying $\Omega \approx 0.3$ if $h \approx 0.7$, but the turnover in P(k) around $200 h^{-1} \text{Mpc}$ (the horizon scale at matter-radiation equality) is not well determined.**

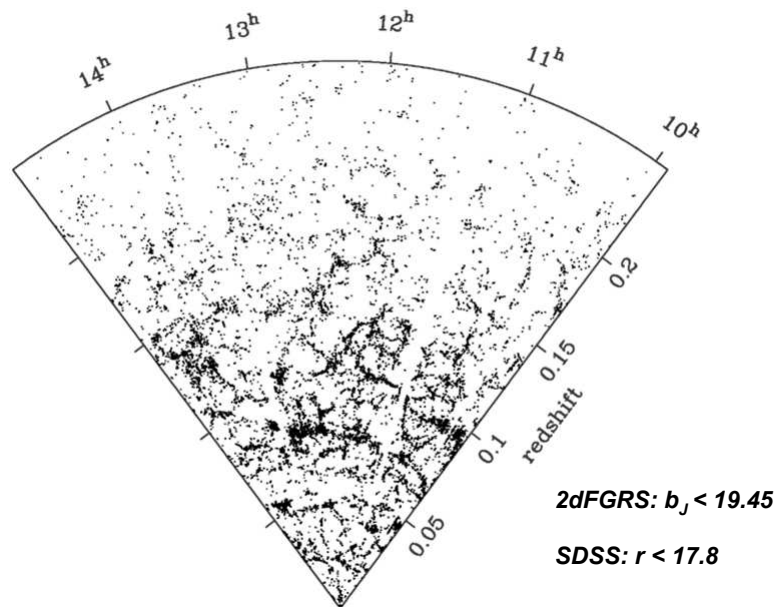


Major new Large Scale Structure Surveys

- **Massive surveys at low z (10^5 - 10^6 galaxies $\langle z \rangle \approx 0.1$):**
 - 2dF Galaxy Redshift Survey and Sloan Digital Sky Survey
 - high-precision Cosmology: measure P(k) on large scales and β from z -space distortions to give Ω_M and b .
 - low- z galaxy population: Φ and ξ as joint functions of luminosity, type, local density and star-formation rate
- **Massive surveys at high redshift ($\langle z \rangle \approx 0.5$ -1.0 or higher):**
 - VIMOS and DEIMOS surveys (and others)
 - evolution of the galaxy population
 - evolution of the large-scale structure
- **Mass and motions survey (6dF Galaxy Survey):**
 - NIR-selected z -survey of local universe, together with...
 - measurements of σ for 15000 E/S0 galaxies...
 - ⇒ masses and distances from Fundamental Plane
 - ⇒ density/velocity field to 15000 km/s ($150 h^{-1} \text{Mpc}$)

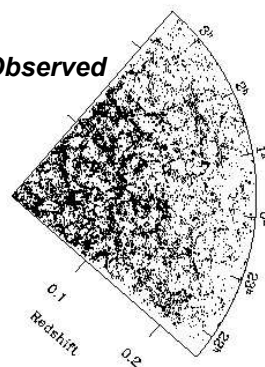


Fine detail: 2-deg NGP slices (1-deg steps)

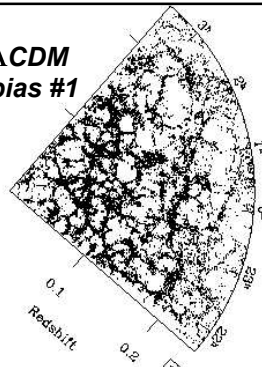


***Cosmology
by eye!***

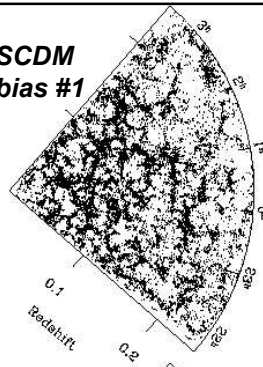
Observed



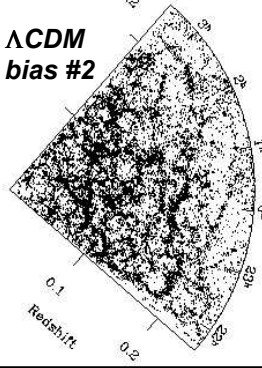
**Λ CDM
bias #1**



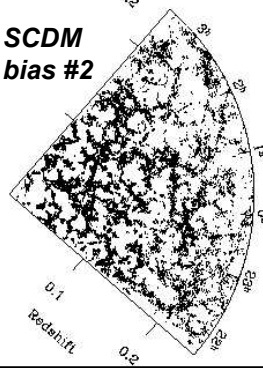
**SCDM
bias #1**



**Λ CDM
bias #2**



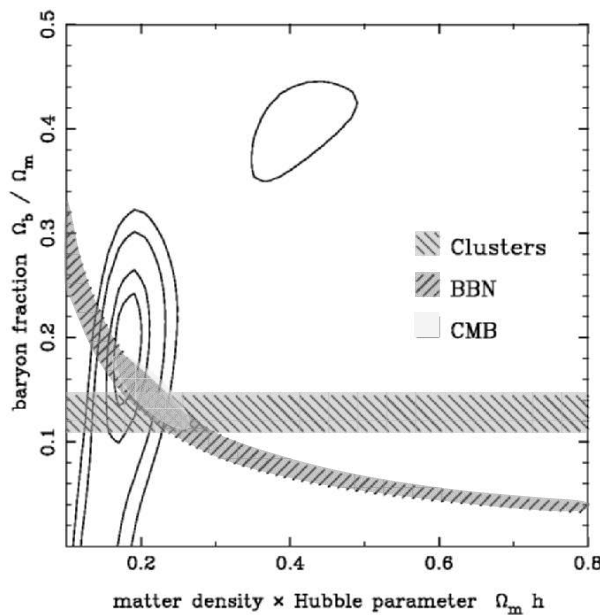
**SCDM
bias #2**



2dFGRS: LSS + Cosmology Highlights

- *The most precise determination of the large-scale structure of the galaxy distribution on scales up to $600 h^{-1}$ Mpc.*
- *Unambiguous detection of coherent collapse on large scales, confirming structures grow via gravitational instability.*
- *Measurements of Ω_m (mean mass density) from the power spectrum and redshift-space distortions: $\Omega = 0.27 \pm 0.04$*
- *First measurement of galaxy bias parameter: $b = 1.00 \pm 0.09$*
- *An new upper limit on the neutrino fraction, $\Omega_\nu/\Omega < 0.13$, and a limit on the mass of all neutrino species, $m_\nu < 1.8$ eV.*

Confidence Limits on Γ and Ω_b/Ω_m



If no constraint on n :

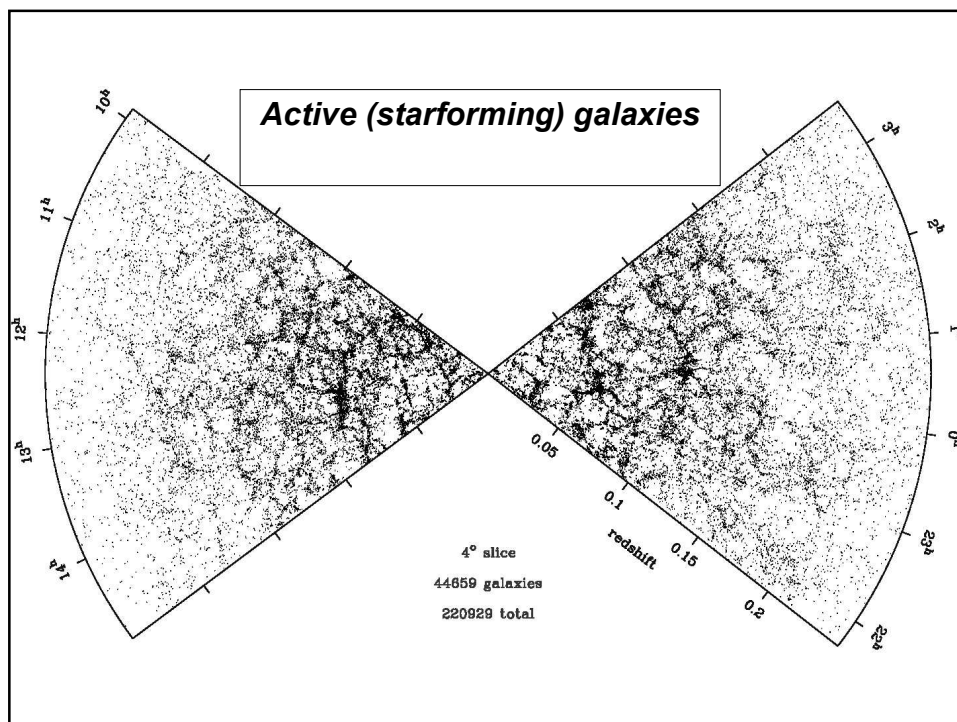
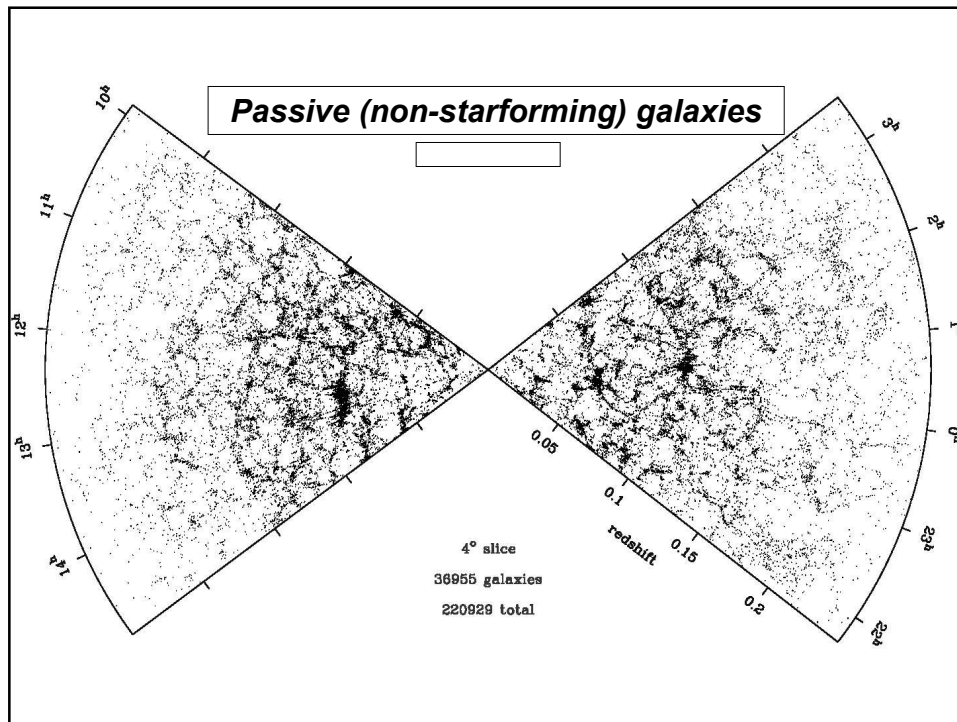
$$\Gamma = \Omega_m h = 0.20 \pm 0.03$$

$$\text{Baryon fraction} = \Omega_b / \Omega_m = 0.15 \pm 0.07$$

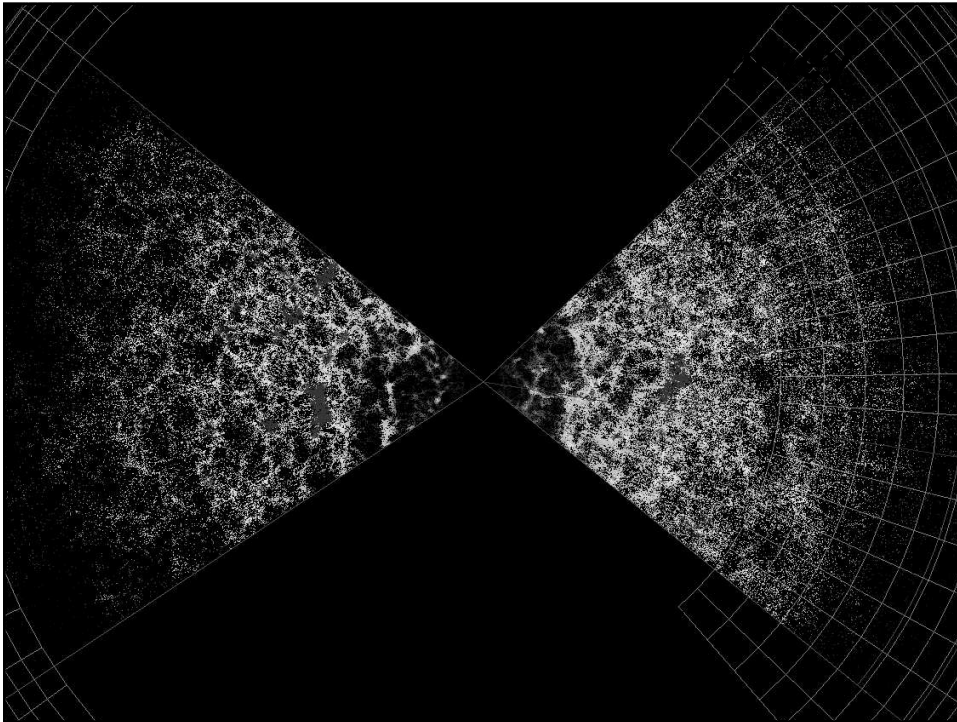
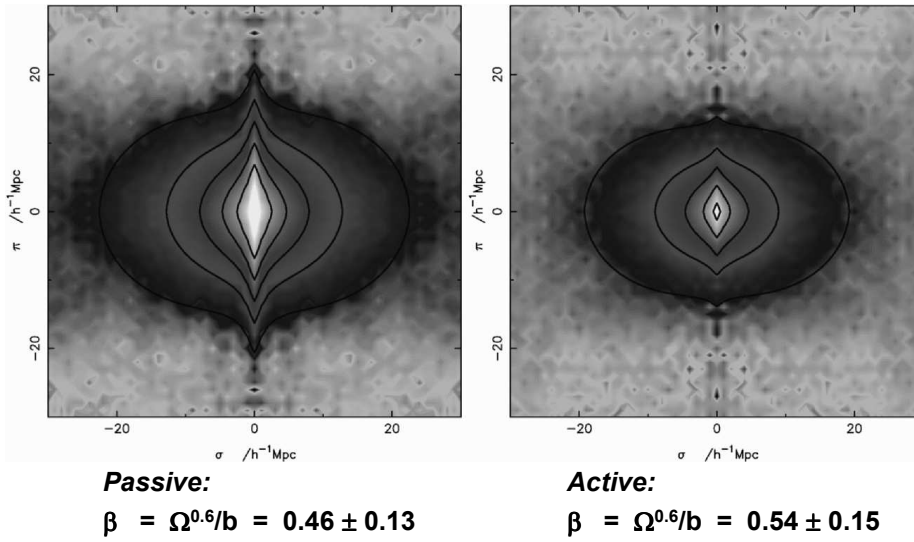
If require $n=1$:

$$\Gamma = \Omega_m h = 0.18 \pm 0.02$$

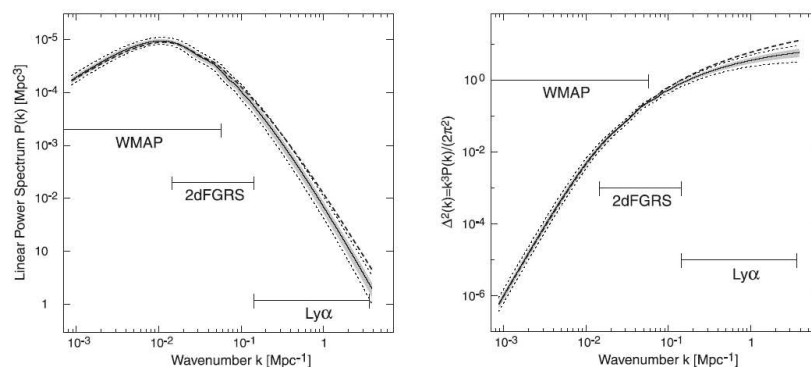
$$\text{Baryon fraction} = \Omega_b / \Omega_m = 0.17 \pm 0.06$$



Redshift-space distortions and galaxy type

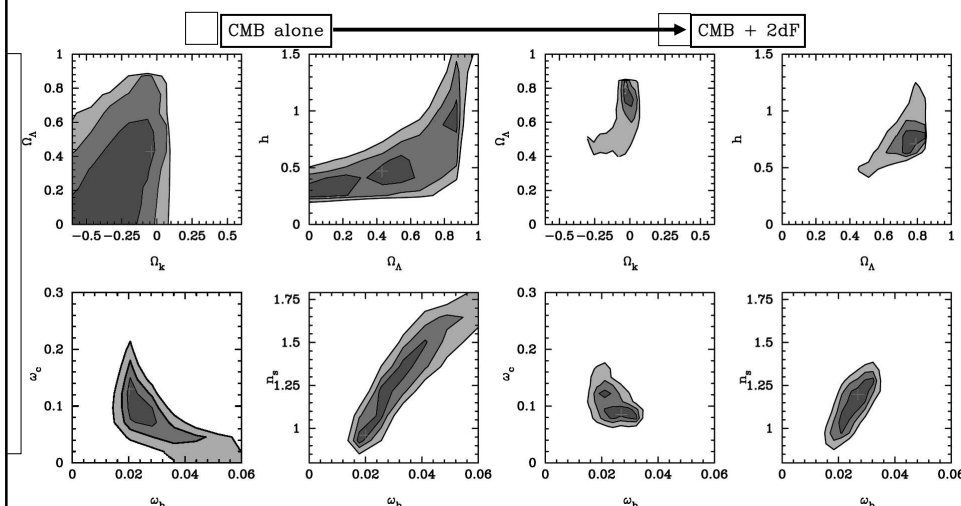


The Power Spectrum from all sources



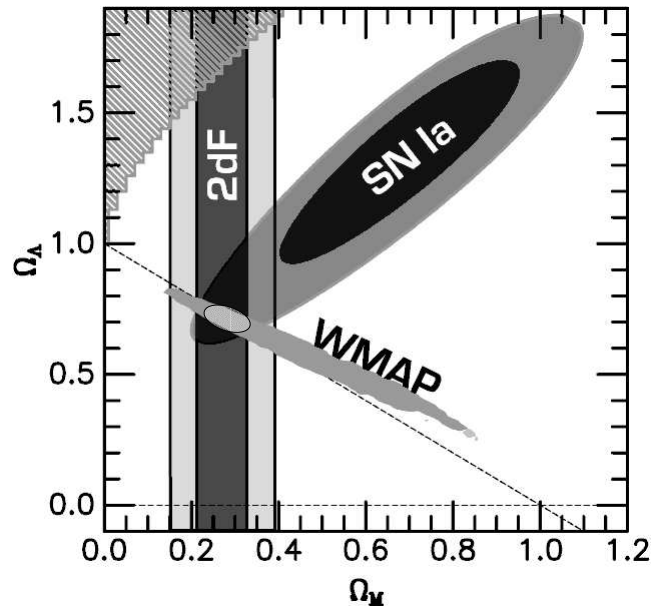
Spergel et al. 2003

Cosmology from 2dFGRS + CMB



The Cosmic Timeline

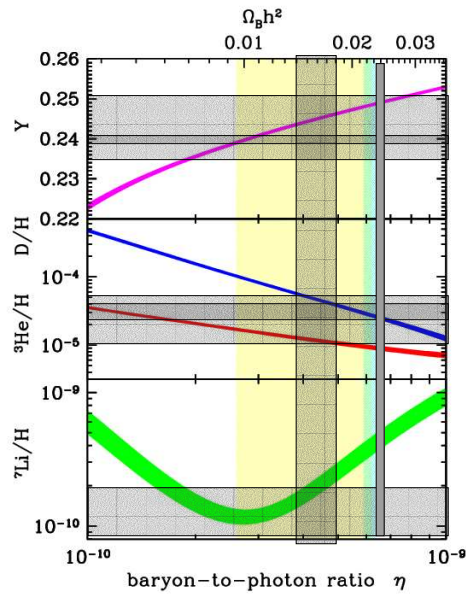
- **CMB last scattering surface:**
(WMAP)
 $t_{\text{dec}} = 379 \pm 8 \text{ kyr}$
($z_{\text{dec}} = 1089 \pm 1$)
- **Epoch of re-ionization?: WMAP and QSOs seem to disagree –**
 $t_r = 100 - 1000 \text{ Myr}$
- **Age of the universe today: SN + WMAP agree very well**
 $t_0 = 13.7 \pm 0.2 \text{ Gyr}$
- **Hubble constant: (CMB+2dF, or Cepheids + SN Ia)**
 $H_0 = 71 \pm 4 \text{ km/s/Mpc}$



The 'Concordance model'

(WMAP +
2dFGRS + SN
Ia + HST KP)

Description	Symbol	Value	+ uncertainty	- uncertainty
Total density	Ω_{tot}	1.02	0.02	0.02
Equation of state of quintessence	w	< -0.78	95% CL	—
Dark energy density	Ω_Λ	0.73	0.04	0.04
Baryon density	$\Omega_b h^2$	0.0224	0.0009	0.0009
Baryon density	Ω_b	0.044	0.004	0.004
Baryon density (cm^{-3})	n_b	2.5×10^{-7}	0.1×10^{-7}	0.1×10^{-7}
Matter density	$\Omega_m h^2$	0.135	0.008	0.009
Matter density	Ω_m	0.27	0.04	0.04
Light neutrino density	$\Omega_\nu h^2$	< 0.0076	95% CL	—
CMB temperature (K) ^a	T_{cmb}	2.725	0.002	0.002
CMB photon density (cm^{-3}) ^b	n_γ	410.4	0.9	0.9
Baryon-to-photon ratio	η	6.1×10^{-10}	0.3×10^{-10}	0.2×10^{-10}
Baryon-to-matter ratio	$\Omega_b \Omega_m^{-1}$	0.17	0.01	0.01
Fluctuation amplitude in $8h^{-1}$ Mpc spheres	σ_8	0.84	0.04	0.04
Low- z cluster abundance scaling	$\sigma_8 \Omega_m^{0.5}$	0.44	0.04	0.05
Power spectrum normalization (at $k_0 = 0.05 \text{ Mpc}^{-1}$) ^c	A	0.833	0.086	0.083
Scalar spectral index (at $k_0 = 0.05 \text{ Mpc}^{-1}$) ^f	n_s	0.93	0.03	0.03
Running index slope (at $k_0 = 0.05 \text{ Mpc}^{-1}$) ^c	$dn_s/d \ln k$	-0.031	0.016	0.018
Tensor-to-scalar ratio (at $k_0 = 0.002 \text{ Mpc}^{-1}$)	r	< 0.71	95% CL	—
Redshift of decoupling	z_{dec}	1089	1	1
Thickness of decoupling (FWHM)	Δz_{dec}	195	2	2
Hubble constant	h	0.71	0.04	0.03
Age of universe (Gyr)	t_0	13.7	0.2	0.2
Age at decoupling (kyr)	t_{dec}	379	8	7
Age at reionization (Myr, 95% CL))	t_r	180	220	80
Decoupling time interval (kyr)	Δt_{dec}	118	3	2
Redshift of matter-energy equality	z_{eq}	3233	194	210
Reionization optical depth	τ	0.17	0.04	0.04
Redshift of reionization (95% CL)	z_r	20	10	9
Sound horizon at decoupling (°)	θ_A	0.598	0.002	0.002
Angular diameter distance to decoupling (Gpc)	d_A	14.0	0.2	0.3
Acoustic scale ^d	ℓ_A	301	1	1
Sound horizon at decoupling (Mpc) ^d	r_s	147	2	2

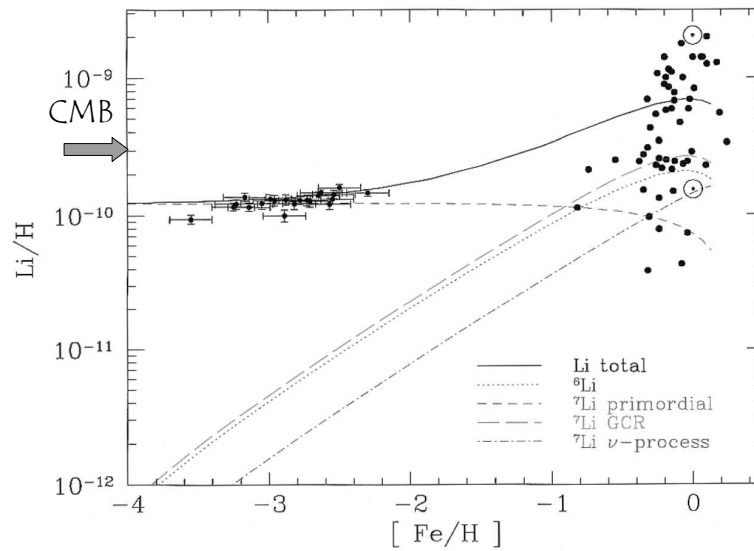


OK

OK

Yuk!

Lithium does evolve slowly due to SN? Or other sources..



Choose stars without convective envelopes...Measure...