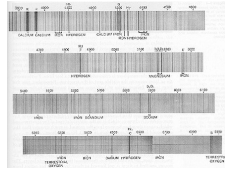


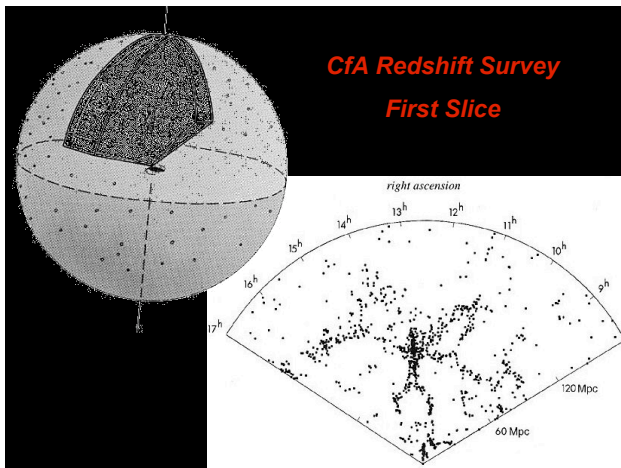
Redshift surveys

- A z-survey is a systematic mapping of a volume of space by measuring redshifts: $z = \lambda_i/\lambda_0 - 1 = a-1$
- Redshifts as distance coordinates...
 $H_0 D_L = c(z + (1-q_0)z^2/2 + \dots)$
 ...this is the viewpoint in low-z surveys of spatial structure.
- For low-z surveys of structure, the Hubble law applies:
 $cz = H_0 d$ (for $z \ll 1$)
- For pure Hubble flow, redshift distance = true distance, i.e. $s=r$, where s and r are conveniently measured in km/s.
- But galaxies also have 'peculiar motions' due to the gravitational attraction of the surrounding mass field, so the full relation between z-space and real-space coordinates is:
 $s = r + v_p/r = r + v_p$ (for $s \ll c$)

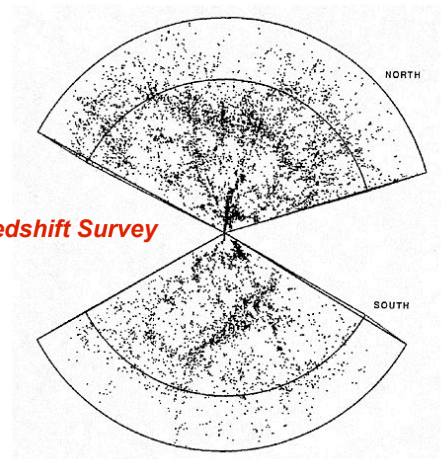


Uses of z-surveys

- Three (partial) views of redshift:**
 - z measures the distance needed to map 3D positions
 - z measures the look-back time needed to map histories
 - $cz - H_0 d$ measures the peculiar velocity needed to map mass
- Three main uses of z-surveys:**
 - to map the large-scale structures, in order to...
 - do cosmography and chart the structures in the universe
 - test growth of structure through gravitational instability
 - determine the nature and density of the dark matter
 - to map the large-velocity field, in order to
 - 'see' the mass field through its gravitational effects
 - to probe the history of galaxy formation, in order to...
 - characterise the galaxy population at each epoch
 - determine the physical mechanisms by which the population evolves

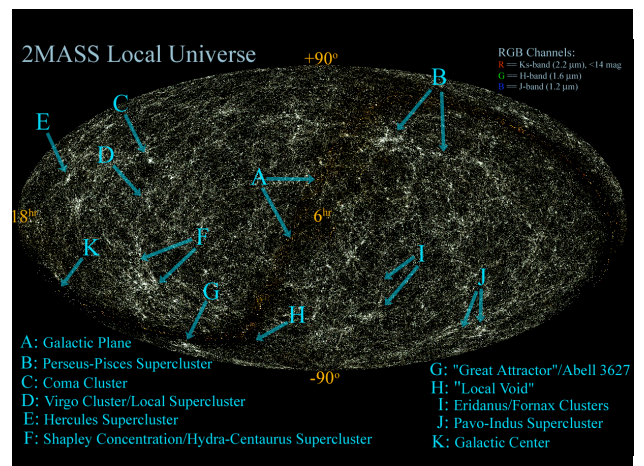


CfA Redshift Survey



Cosmography

- The main features of the local galaxy distribution include:
 - Local Group:** Milky Way, Andromeda and retinue.
 - Virgo cluster:** nearest significant galaxy cluster, LG→Virgo.
 - Local Supercluster (LSC):** flattened distribution of galaxies $cz < 3000$ km/s; defines supergalactic plane (X,Y,Z).
 - 'Great Attractor':** LG/Virgo→GA, lies at one end of the LSC, $(X,Y,Z) = (-3400, 1500, \pm 2000)$.
 - Perseus-Pisces supercluster:** $(X,Y,Z) = (+4500, \pm 2000, -2000)$, lies at the other end of the LSC.
 - Coma cluster:** nearest very rich cluster, $(X,Y,Z) = (0, +7000, 0)$; a major node in the 'Great Wall' filament.
 - Shapley supercluster:** most massive supercluster within $z < 0.1$, at a distance of 14,000 km/s behind the GA.
 - Voids:** the Local Void, Sculptor Void, and others lie between these mass concentrations.
- Yet larger structures are seen at lower contrast to $> 100 h^{-1}$ Mpc.

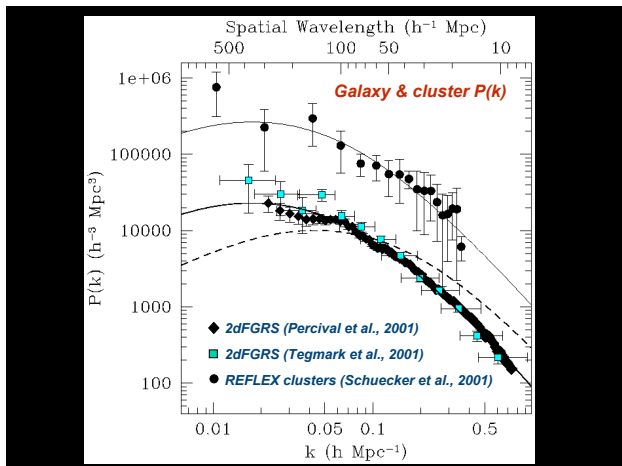


Evolution of Structure

- The **goal** is to derive the evolution of the mass density field, represented by the dimensionless density perturbation:
 $\delta(x) = \rho(x)/\langle\rho\rangle - 1$
- The **framework** is the growth of structures from 'initial' density fluctuations by gravitational instability.
- Up to the decoupling of matter and radiation, the evolution of the density perturbations is complex and depends on the interactions of the matter and radiation fields - '**CMB physics**'
- After decoupling, the linear growth of fluctuations is simple and depends only on the cosmology and the fluctuations in the density at the surface of last scattering - '**large-scale structure in the linear regime**'.
- As the density perturbations grow the evolution becomes non-linear and complex structures like galaxies and clusters form - '**non-linear structure formation**'. In this regime additional complications emerge, like gas dynamics and star formation.

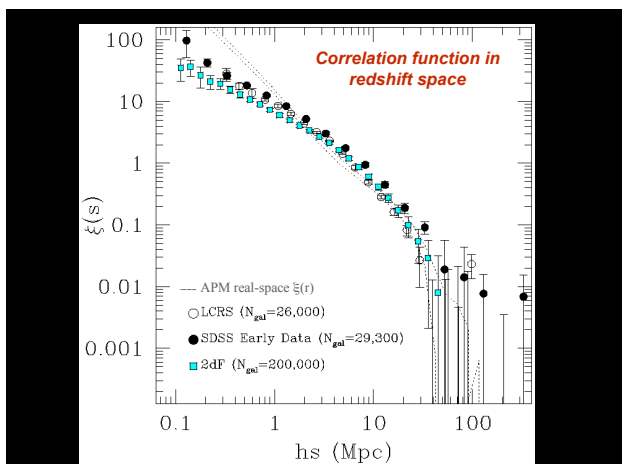
The power spectrum

- It is helpful to express the density distribution $\delta(r)$ in the Fourier domain:
 $\delta(k) = V^{-1} \int \delta(r) e^{ik \cdot r} d^3r$
- The power spectrum (PS) is the mean squared amplitude of each Fourier mode:
 $P(k) = \langle |\delta(k)|^2 \rangle$
 - Note $P(k)$ not $P(k)$ because of the (assumed) isotropy of the distribution (i.e. scales matter but directions don't).
 - $P(k)$ gives the power in fluctuations with a scale $r=2\pi/k$, so that $k=(1.0, 0.1, 0.01) \text{ Mpc}^{-1}$ correspond to $r=(6, 60, 600) \text{ Mpc}$.
- The PS can be written in dimensionless form as the variance per unit $\ln k$:
 $\Delta^2(k) = d\langle |\delta(k)|^2 \rangle / d\ln k = (V/2\pi)^3 4\pi k^3 P(k)$
 - e.g. $\Delta^2(k)=1$ means the modes in the logarithmic bin around wavenumber k have rms density fluctuations of order unity.



The correlation function

- The autocorrelation function of the density field (often just called 'the correlation function', CF) is:
 $\xi(r) = \langle \delta(x)\delta(x+r) \rangle$
- The CF and the PS are a Fourier transform pair:
 $\xi(r) = V/(2\pi)^3 \int |\delta_k|^2 \exp(-ik \cdot r) d^3k$
 $= (2\pi^2)^{-1} \int P(k) [(\sin kr)/kr] k^2 dk$
 - Because $P(k)$ and $\xi(r)$ are a Fourier pair, they contain precisely the same information about the density field.
- When applied to galaxies rather than the density field, $\xi(r)$ is often referred to as the 'two-point correlation function', as it gives the excess probability (over the mean) of finding two galaxies in volumes dV separated by r :
 $dP = \bar{\rho}_g^2 [1 + \xi(r)] d^2V$
 - By isotropy, only separation r matters, and not the vector r .
 - Can thus think of $\xi(r)$ as the mean over-density of galaxies at distance r from a random galaxy.



Gaussian fields

- A Gaussian density field has the property that the joint probability distribution of the density at any number of points is a multivariate Gaussian.
- Superposing many Fourier density modes with random phases results, by the central limit theorem, in a Gaussian density field.
- A Gaussian field is fully characterized by its mean and variance (as a function of scale).
- Hence $\langle\rho\rangle$ and $P(k)$ provide a complete statistical description of the density field if it is Gaussian.
- Most simple inflationary cosmological models predict that Fourier density modes with different wavenumbers are independent (i.e. have random phases), and hence that the initial density field will be Gaussian.
- Linear amplification of a Gaussian field leaves it Gaussian, so the large-scale galaxy distribution should be Gaussian.

The initial power spectrum

- Unless some physical process imposes a scale, the initial PS should be scale-free, i.e. a power-law, $P(k) \propto k^n$.
- The index n determines the balance between large- and small-scale power, with rms fluctuations on a mass scale M given by:

$$\delta_{\text{rms}} \propto M^{-(n+3)/6}$$
- The 'natural' initial power spectrum is the power-law with $n=1$ (called the Zel'dovich, or Harrison-Zel'dovich, spectrum).
- The $P(k) \propto k^{-1}$ spectrum is referred to as the scale-invariant spectrum, since it gives variations in the gravitational potential that are the same on all scales.
- Since potential governs the curvature, this means that space-time has the same amount of curvature variation on all scales (i.e. the metric is a fractal).
- In fact, inflationary models predict that the initial PS of the density fluctuations will be approximately scale-invariant.

Growth of linear perturbations

- The (non-relativistic) equations governing fluid motion under gravity can be linearized to give the following equation governing the growth of linear density perturbations:

$$\ddot{\delta} + \frac{\dot{a}}{a} \dot{\delta} = \delta \left(4\pi G \rho_0 - \frac{c_s^2 k^2}{a^2} \right)$$

- This has growing solutions for on large scales (small k) and oscillating solutions for small scales (large k); the cross-over scale between the two is the Jeans length,

$$\lambda_J = c_s \sqrt{\frac{1}{G\rho}}$$

where c_s is the sound speed, $c_s^2 = \partial p / \partial \rho$.

- For $\lambda < \lambda_J$, sound waves cross an object on the same time-scale as the gravitational collapse, so pressure can counter gravity.
- In an expanding universe, λ_J varies with time; perturbations on some scales swap between growing and oscillating solutions.

Bias: light vs mass

- Gravitational instability theory applies to the mass distribution but we observe the galaxy distribution - are these 1-to-1?
- A bias factor b parameterises our ignorance: $\delta_g = b \delta_m$, i.e. fractional variations in the galaxy density are proportional to fractional variations in the mass density (with ratio b).
 - What might produce a bias? Do galaxies form only at the peaks of the mass field, due to a star-formation threshold?
 - Variation of bias with scale. This is plausible at small scales (many potential mechanisms), but not at large scales.
 - Observed variation with galaxy type. The ratio E:Sp is 10:1 in clusters ($\delta_g \gg 1$) but 1:10 in field ($\delta_g \ll 1$).

Growth of perturbations

- What does it take for an object to Collapse in the Universe.
- We can estimate this by looking at the Gravitational Binding Energy of a spherical ball and comparing it to the thermal energy of the ball. When gravity dominates, the object can collapse. Scale where this happens is called the Jean's Length

$$\begin{aligned} \frac{GM^2}{R} &\approx \frac{M}{m} kT & E_{\text{pot}} &\approx kT & N_{\text{pot}} &= \frac{M}{m} \\ \frac{GM}{R} &\approx \frac{kT}{m} \\ \frac{G(\frac{4}{3}\pi\rho R^3)}{R} &\approx \frac{kT}{m} \\ R &\approx \sqrt{\frac{kT}{mG\rho}} = \lambda_J = c_s \sqrt{\frac{1}{G\rho}} \end{aligned}$$

Peculiar Velocity and Linear Growth

Peebles, (1976) demonstrated in the linear regime (i.e. acceleration Due to a mass concentration is constant – unaffected by the growth of the mass concentration) the following relationship holds.

$$\begin{aligned} \frac{v_{\text{pec}}(r)}{H_0 r} &= -\frac{1}{3} \Omega_M^{0.6} \delta(r) & \text{BIAS: } \delta_{\text{gal}}(r) &= b \delta(r) \\ \frac{v_{\text{pec}}(r)}{H_0 r} &= -\frac{1}{3} \frac{\Omega_M^{0.6}}{b} \delta_{\text{gal}}(r) \end{aligned}$$

So...We think $\Omega_M=0.3$,

between us and the Virgo Cluster the density of galaxies we see over the background is a factor of 2 in that sphere,

$H_0 = 70$ km/s

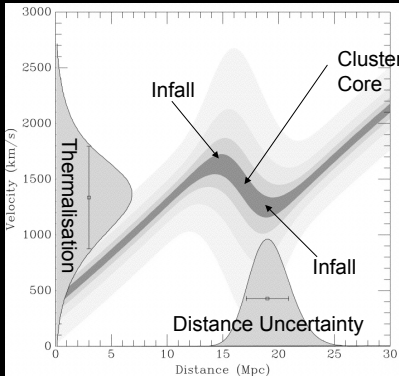
Distance to Virgo cluster is 16 Mpc...

$$\begin{aligned} v_{\text{pec}}(r) &= -\frac{H_0 r}{3} \frac{\Omega_M^{0.6}}{b} \delta_{\text{gal}}(r) \\ v_{\text{pec}}(r) &= -\frac{70 * 16 * 0.3^{0.6}}{3} * 2 = 362 \text{ km/s} \end{aligned}$$

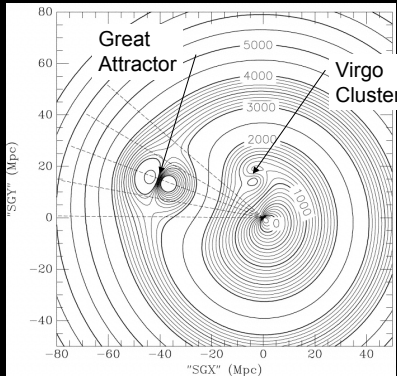
Non-Linear Growth

- Eventually structures grow and this causes their Mass to increase, and the linear regime to breakdown
- Galaxies start to interact with each other and thermalise (Called Virialisation)

Virgo Cluster as Measured with Surface Brightness Fluctuations

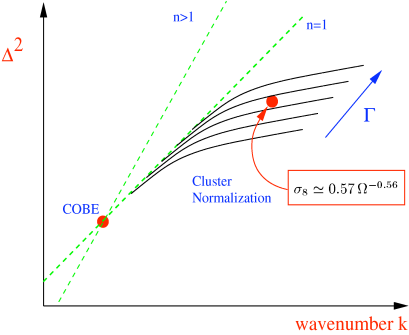


Map of velocities in nearby Universe-SBF



$$\Gamma_{eff}(k) = \Omega_0 h \left[\alpha_r + \frac{1 - \alpha_r}{1 + (0.43ks)^4} \right], \quad (30)$$

$$\alpha_r = 1 - 0.328 \ln(43.1\Omega_0 h^2) \frac{\Omega_b}{\Omega_0} + 0.38 \ln(22.3\Omega_0 h^2) \left(\frac{\Omega_b}{\Omega_0} \right)^2. \quad (31)$$



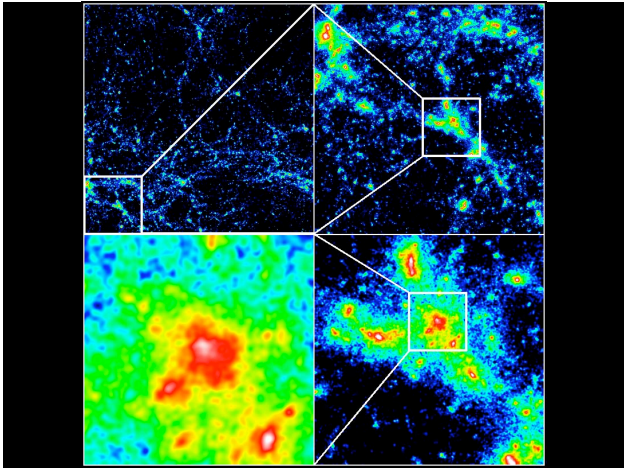
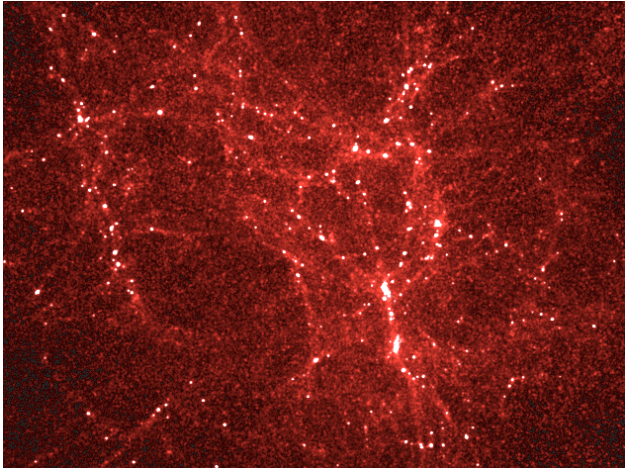
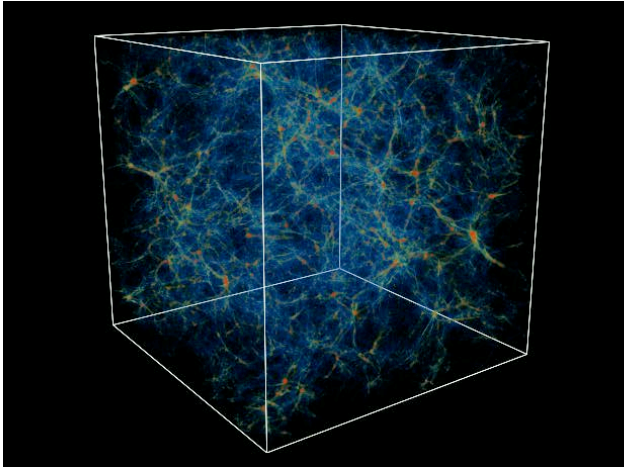
The CDM Power Spectrum

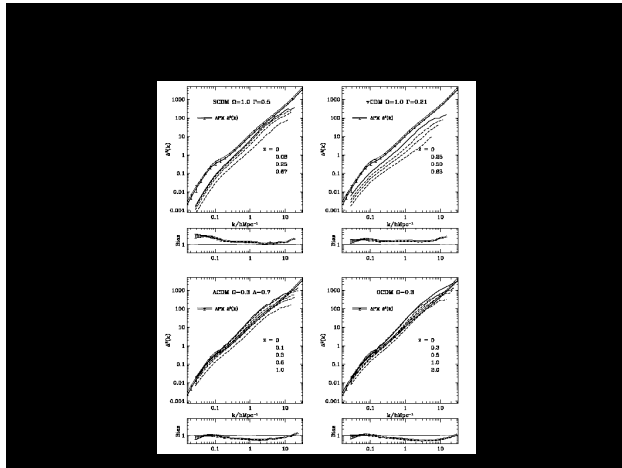
Fluctuations in the density grow as δ
(a) = a f(Ω [a])

Scale of break in power spectrum relates to baryons suppressing growth of CDM

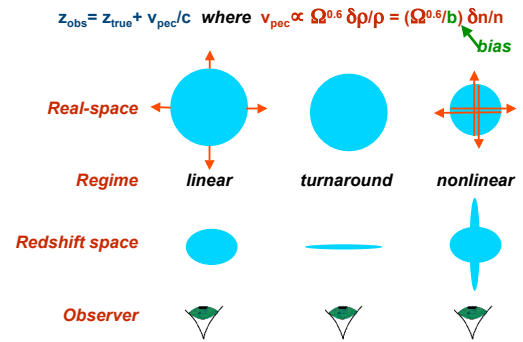
In practice, get shape parameter $\Gamma \approx \Omega_m h$

Γ is a shorthand way of fitting the actual power spectra coming out of Nbody models



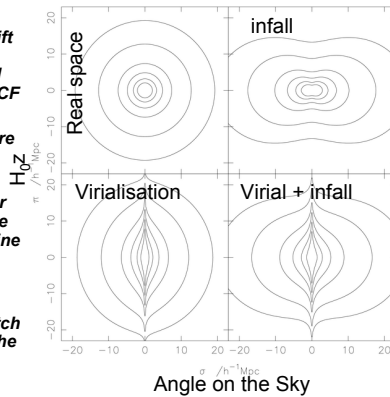


Redshift-space distortions



Redshift-space distortions

- Because of peculiar velocities, the redshift space Correlation Function is distorted w.r.t. the real-space CF
- In real space the contours of the CF are circular.
- Coherent infall on large scales (in linear regime) squashes the contours along the line of sight.
- Rapid motions in collapsed structures on small scales stretch the contours along the line of sight.



Some Relevant questions in Large Scale Structure

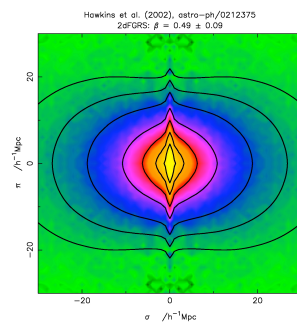
- What is the shape of the power spectrum? what is the value of $\Gamma = \Omega h$?
- Mass and bias:
 - what is the value of $\beta \approx \Omega^{0.6}/b$?
 - can we obtain Ω and b independently of each other?
 - what are the relative biases of different galaxy populations?
- Can we check the gravitational instability paradigm?
- Were the initial density fluctuations random-phase (Gaussian)?

Measuring β from $P(k)$

- z -space distortions produce 'Fingers of God' on small scales and compression along the line of sight on large scales.
- Or can measure the degree of distortion of ξ_s in σ - π plane from ratio of quadrupole to monopole:
- $P_2^s(k) = 4/3\beta + 4/7\beta^2$
 $P_0^s(k) = 1 + 2/3\beta + 1/5\beta^2$

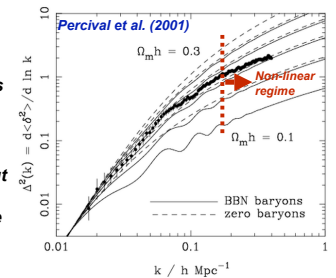
$$(\Omega_M)^6 = b * 0.49$$

$$\Omega_M = .30 * b^{5/3}$$



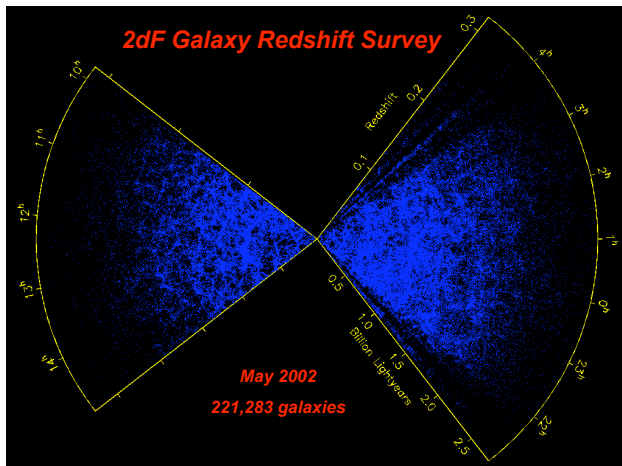
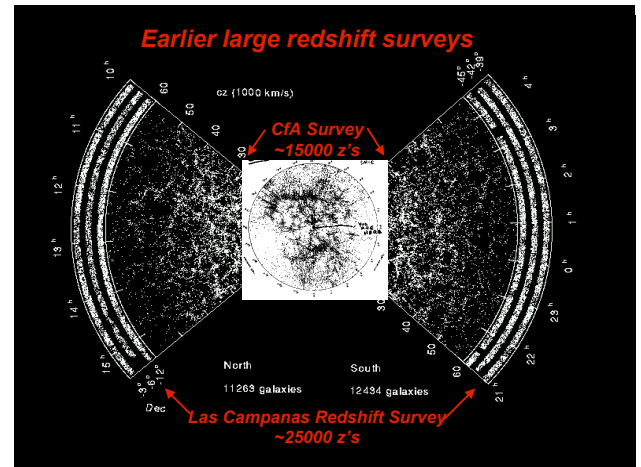
Large scales - $P(k)$

- $P(k)$ is preferred to $\xi(r)$ on large scales: it is more robust to compute, the covariance between scales is simpler, and the error analysis is easier.
- Fits to $P(k)$ give $\Gamma \approx 0.2$, implying $\Omega \approx 0.3$ if $h \approx 0.7$, but the turnover in $P(k)$ around $200 h^{-1} \text{ Mpc}$ (the horizon scale at matter-radiation equality) is not well determined.

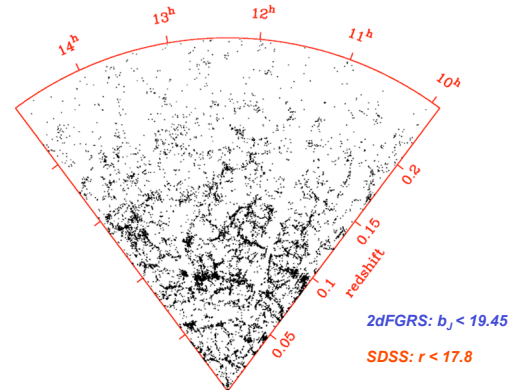


Major new Large Scale Structure Surveys

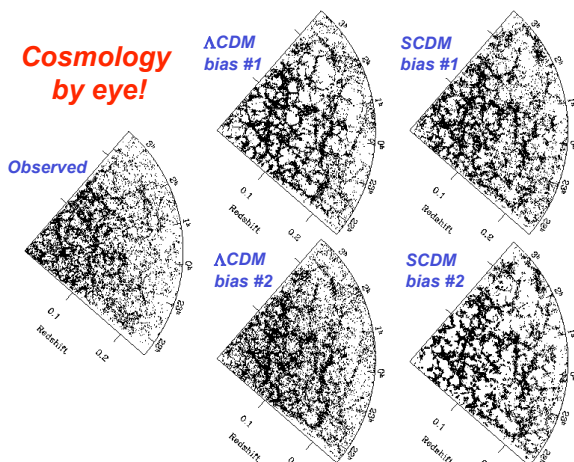
- **Massive surveys at low z** (10^5 - 10^6 galaxies $\langle z \rangle \approx 0.1$):
 - **2dF Galaxy Redshift Survey** and Sloan Digital Sky Survey
 - **high-precision Cosmology**: measure $P(k)$ on large scales and β from z -space distortions to give Ω_M and b .
 - **low- z galaxy population**: Φ and ξ as joint functions of luminosity, type, local density and star-formation rate
- **Massive surveys at high redshift** ($\langle z \rangle \approx 0.5$ - 1.0 or higher):
 - **VIMOS and DEIMOS surveys** (and others)
 - evolution of the galaxy population
 - evolution of the large-scale structure
- **Mass and motions survey (6dF Galaxy Survey)**:
 - **NIR-selected z -survey** of local universe, together with...
 - measurements of σ for 15000 E/S0 galaxies...
 - \Rightarrow masses and distances from Fundamental Plane
 - \Rightarrow density/velocity field to 15000 km/s ($150 h^{-1}$ Mpc)



Fine detail: 2-deg NGP slices (1-deg steps)



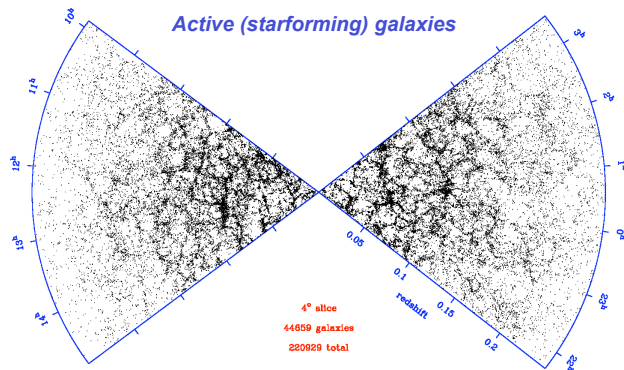
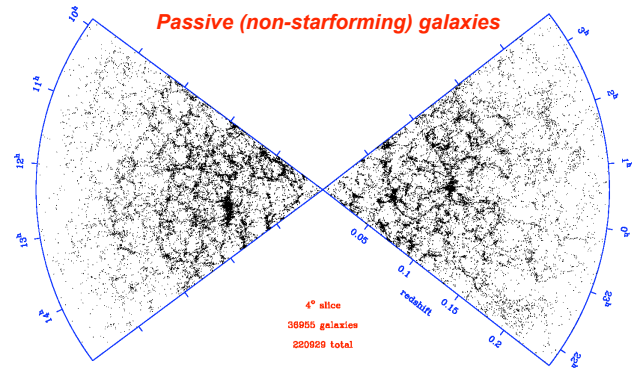
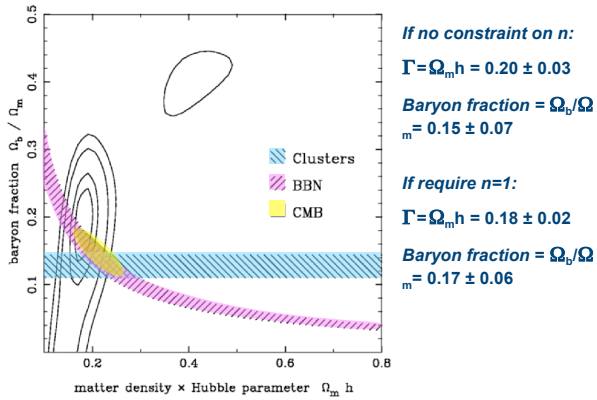
Cosmology by eye!



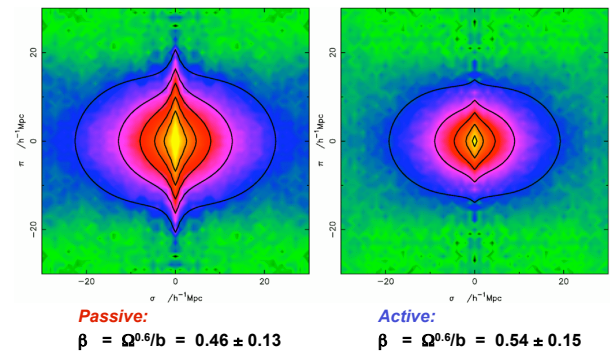
2dFGRS: LSS + Cosmology Highlights

- The most precise determination of the large-scale structure of the galaxy distribution on scales up to $600 h^{-1}$ Mpc.
- Unambiguous detection of coherent collapse on large scales, confirming structures grow via gravitational instability.
- Measurements of Ω_M (mean mass density) from the power spectrum and redshift-space distortions: $\Omega = 0.27 \pm 0.04$
- First measurement of galaxy bias parameter: $b = 1.00 \pm 0.09$
- An new upper limit on the neutrino fraction, $\Omega_\nu / \Omega < 0.13$, and a limit on the mass of all neutrino species, $m_\nu < 1.8$ eV.

Confidence Limits on Γ and Ω_b/Ω_m



Redshift-space distortions and galaxy type



The 2dF Galaxy Redshift Survey

