Iteration and Problem Solving Strategies

How to solve anything!

How to work out really complicated motion

- Break it up into little tiny steps.
- Use an approximate method for each step.
- Add them all up.

Vertical spring-mass

- Time t with $t+\Delta t$
- Position x with $x+v\Delta t$
- Velocity v with $v+\left(g-\frac{k}{m}x\right)\Delta t$

Let's go

- Start off with t=0, x=0, v=0
- Apply our equations:
  - New value of t is t + Δt = 0+0.1 = 0.1
  - New value of x is x + v Δt = 0+0×0.1 = 0
  - New value of v is

$$v+\left(g-\frac{k}{m}x\right)\Delta t=0+\left(9.8-\frac{5}{0.1}0\right)\times 0.1=0.98$$

So after 0.1 seconds…

- According to our method, the position hasn't changed (still zero) but the velocity has increased to 0.98 m/s.
- Now do this again, using these new numbers as the starting parameters..

Second iteration

- Start off with t=0.1, x=0, v=0.98
- Apply our equations:
  - New value of t is t + Δt = 0.1+0.1 = 0.2
  - New value of x is x + v Δt = 0+0.98$\times$0.1 = 0.098
  - New value of v is

$$v+\left(g-\frac{k}{m}x\right)\Delta t=0.98+\left(9.8-\frac{5}{0.1}0\right)\times 0.1=1.96$$

Third iteration

- Start off with t=0.2, x=0.098, v=1.96
- Apply our equations:
  - New value of t is t + Δt = 0.2+0.1 = 0.3
  - New value of x is x + v Δt = 0.098+1.96×0.1 = 0.294
  - New value of v is

$$v+\left(g-\frac{k}{m}x\right)\Delta t=1.96+\left(9.8-\frac{5}{0.1}0.098\right)\times 0.1=2.45$$

Fourth iteration

- Start off with t=0.3, x=0.294, v=2.45
- Apply our equations:
  - New value of t is t + Δt = 0.3+0.1 = 0.4
  - New value of x is x + v Δt = 0.294+2.45$\times$0.1 = 0.539
  - New value of v is

$$v+\left(g-\frac{k}{m}x\right)\Delta t = 2.45+\left(9.8-\frac{5}{0.1}0.294\right)\times 0.1 = 1.96$$

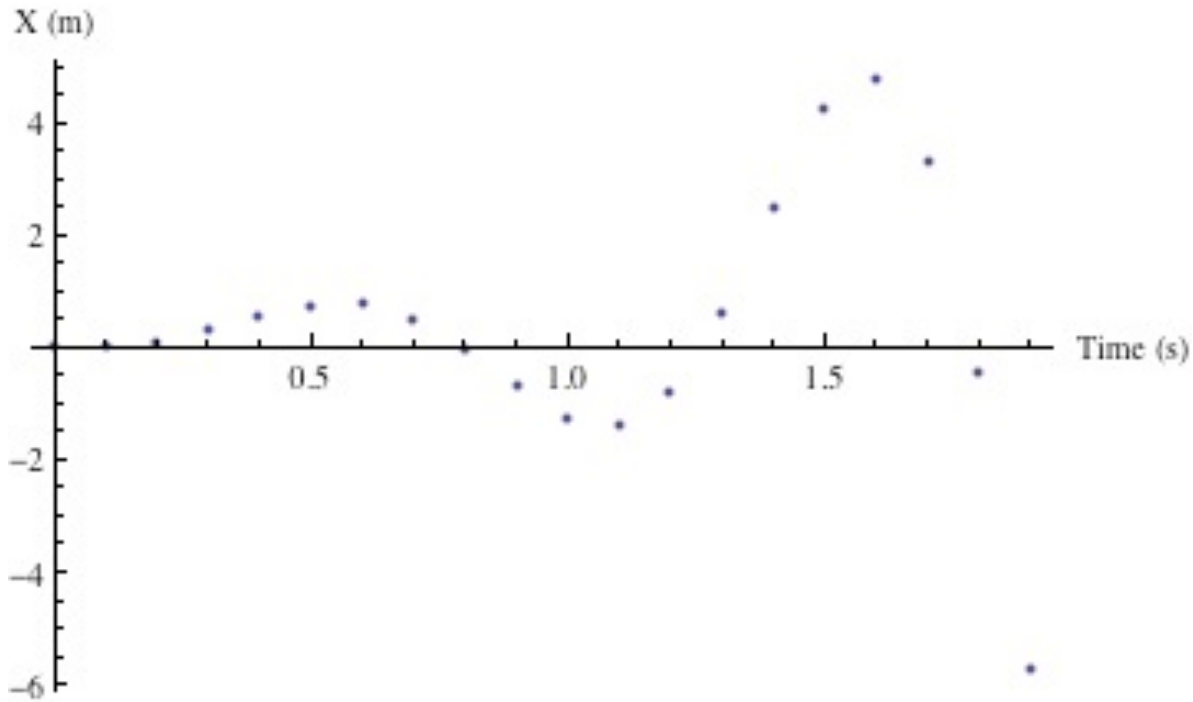
And so on…

- Do the calculations for each step, and then use the results as the input for the next step.
- That's what iteration means!
- What results do we get?

Results for first few iterations (steps)

t	**x**	**v**
0	*0*	*0*
0.1	*0*	*0.98*
0.2	*0.098*	*1.96*
0.3	*0.294*	*2.45*
0.4	*0.539*	*1.96*
0.5	*0.735*	*0.245*

A graph of the first twenty interations…



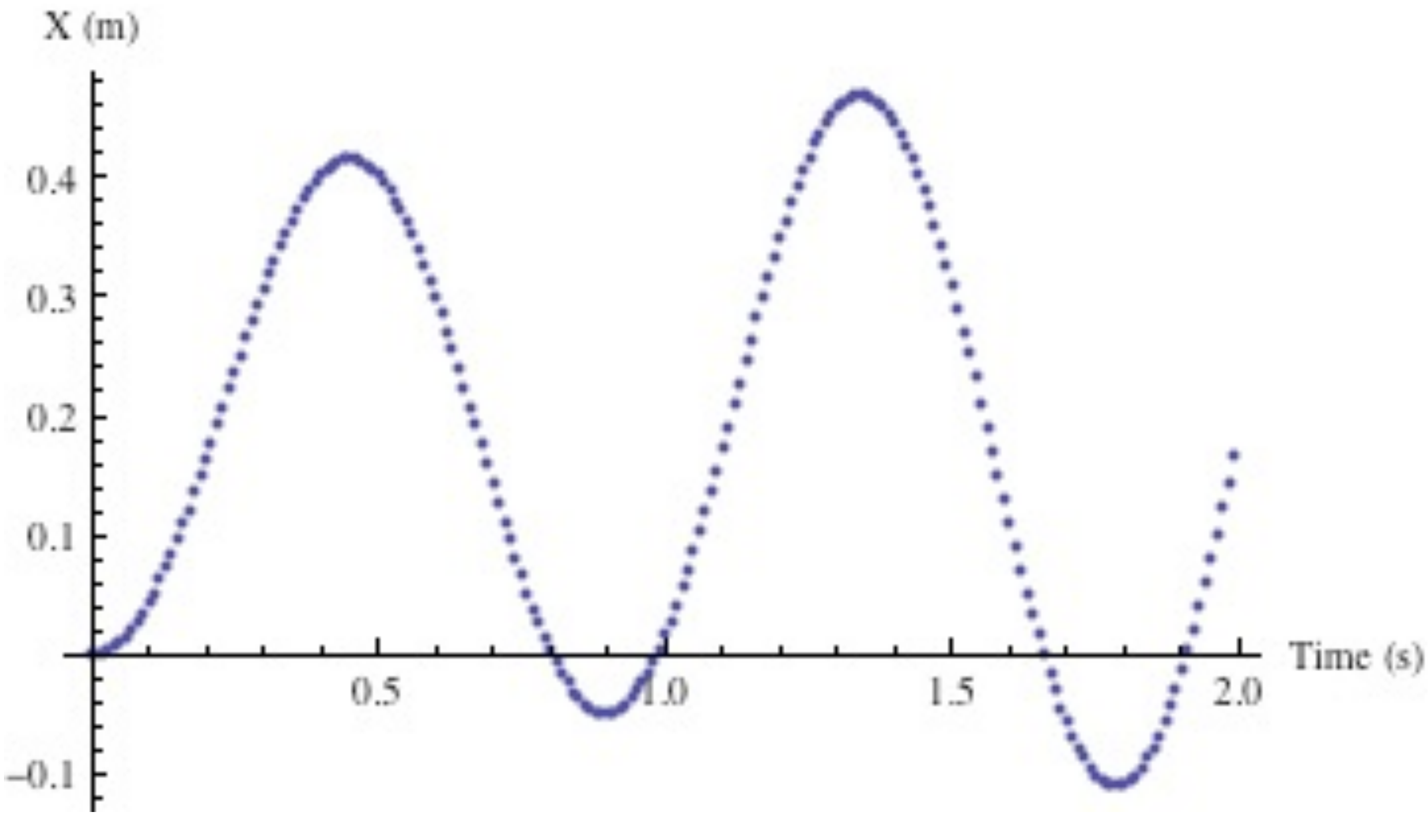
Good and bad

- If you remember - the correct solution is an oscillation.
- Our iteration has correctly produced an oscillation.
- But it has the amplitude steadily increasing - which is wrong.
- Springs don't do that!

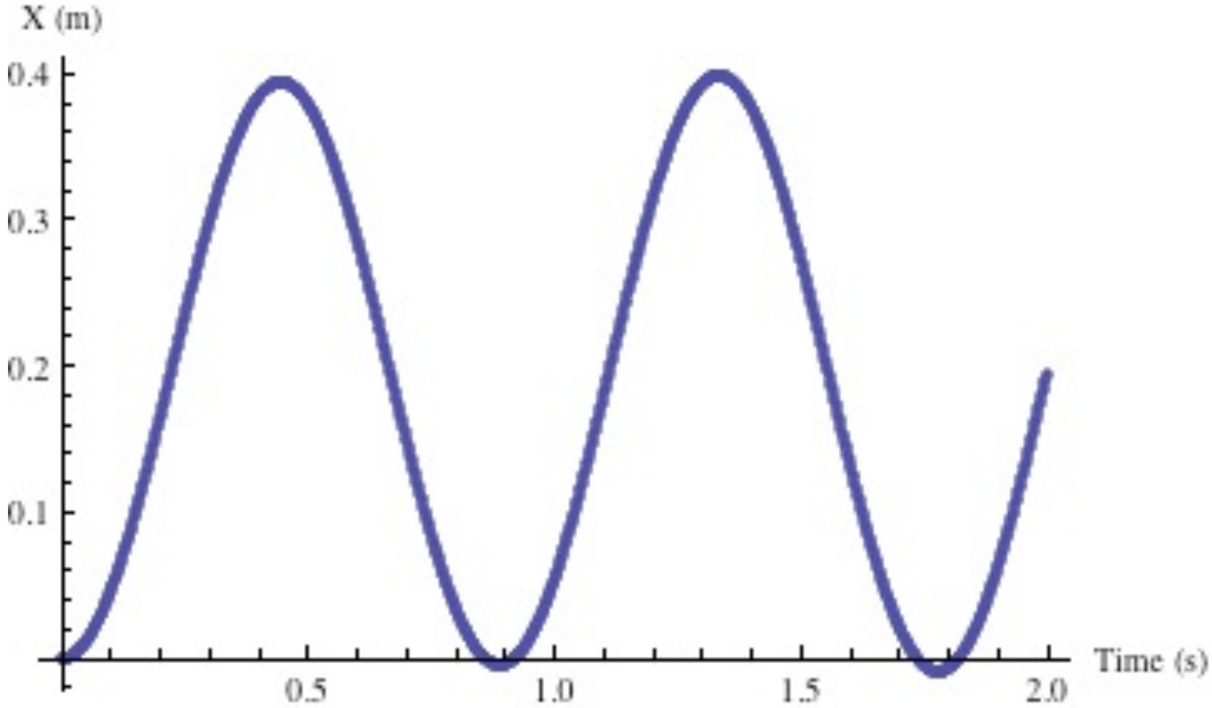
Our time step was too big.

- The approximation (that the speed and velocity are approximately constant within each time-step) wasn't good enough.
- If we make our time-step smaller… (say 0.01 sec)…
- We have to do a lot more steps…

But it gets better…



And if we make our time step smaller still - say 0.001 sec…



Really rather good…

- But I needed to do 2000 steps (iterations) to get the last plot.
- Which would have been very tedious and error-prone had I not used a computer…
- Luckily we have computers and doing those 2000 steps took less than 0.1 sec…

But it's painful

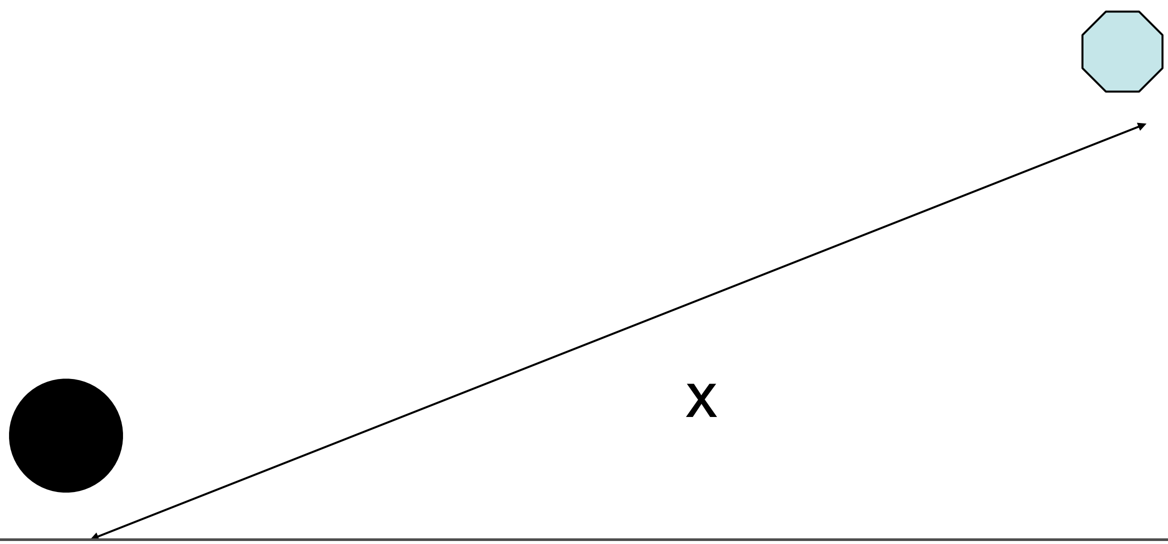
- So do it by computer!
- Example python program

So even this crude approximation…

- It pretty good with small timesteps.
- And with the speed of modern computers, small timesteps are not much of a problem.
- Using a better (more complicated) approximation to the motion in each timestep will mean that you can get away with bigger timesteps.
- But each timestep needs more calculations to evaluate - so overall you may not be better off.

Let’s try an example

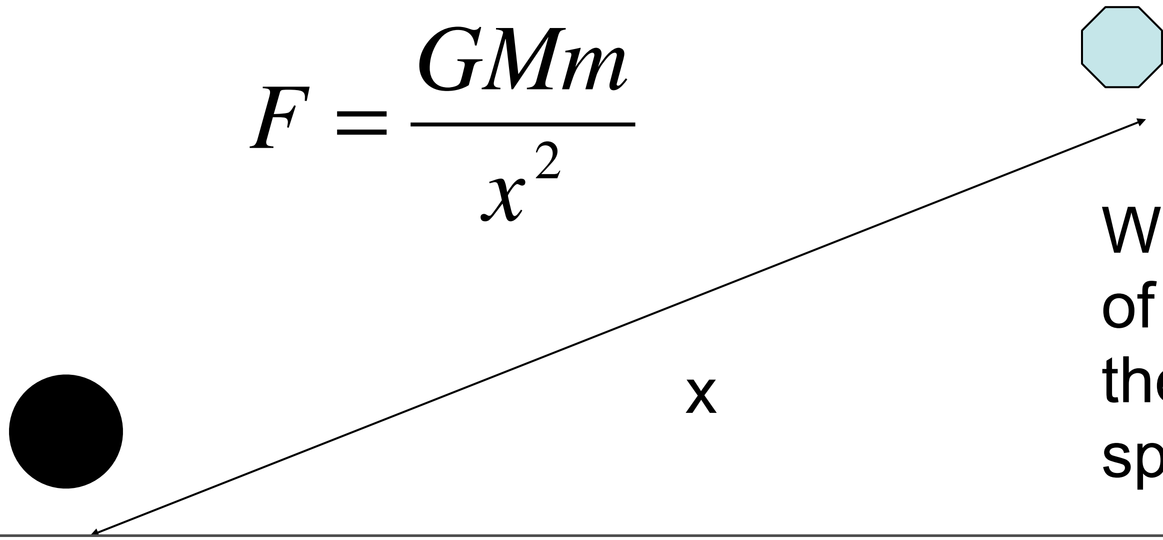
- A spaceship near a black hole…



What forces apply?

- In this case there is only one - the gravitational force.
- Being in space there is no friction or drag, so…

$$F = \frac{GMm}{x^2}$$



Where M is the mass of the black hole and m the mass of the spaceship.

What variables will we track?

- Time, position (x) and velocity (v) as before.
- For one-dimensional problems it will always be these.
- In 3D, you will need to track vector position and vector velocity.

Iteration equations

- For time: t + Δt (as before)
- For position: x+ v Δt (as before)
- But what about for velocity?

$$F = \frac{GMm}{x^2}$$

Write it down…

What will the velocity be at the end of a time-step?

x increases away from the black hole. Velocity is (as always) rate of change of x.

Clicker Question

- What is the new velocity?

The answer…

- Gravity works to decrease the (outwards) velocity

$$v - \frac{GM}{x^2}\Delta t$$

Let's chose some values

- Mass of the black hole = 10^{31} kg
- Starting distance = 1,000,000 km
- Starting speed = 2000 km/s away

(You've been blasting away from it as hard as you could - but now your fuel has run out… Is your speed great enough to escape?)

Python simulation

And then VPython simulation

Summary

- Divide up your problem into little tiny steps.
- Write down an approximate set of equations for each step
- Plug numbers into these formulae over and over again - taking the output from one step as the input to the next.

Chaos

- You can get extremely complicated results from this...
- TIny changes in the starting positions can cause huge changes in the outcomes.
- This is the hallmark of "Chaos"

Computer Lab

- You will get to practice iteration in the computer lab.
- This is one of four rotations - check in which week you are doing it.
- Venue is different - BOZO112

Contact Forces

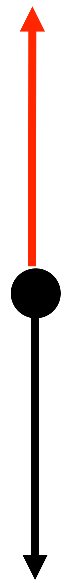
Whenever one object touches another…

Peculiarly tricky

- Because they can point in different directions
- Because there is no simple formula to work them out

Is there really a force when you sit on something?

- Newton's laws say there must be…



But how can a chair push?

- A force is normally thought of as a “push” or “pull”
- But you don’t normally think of chairs, walls, the floor pushing?

Imagine replacing the chair…

- With a spring…

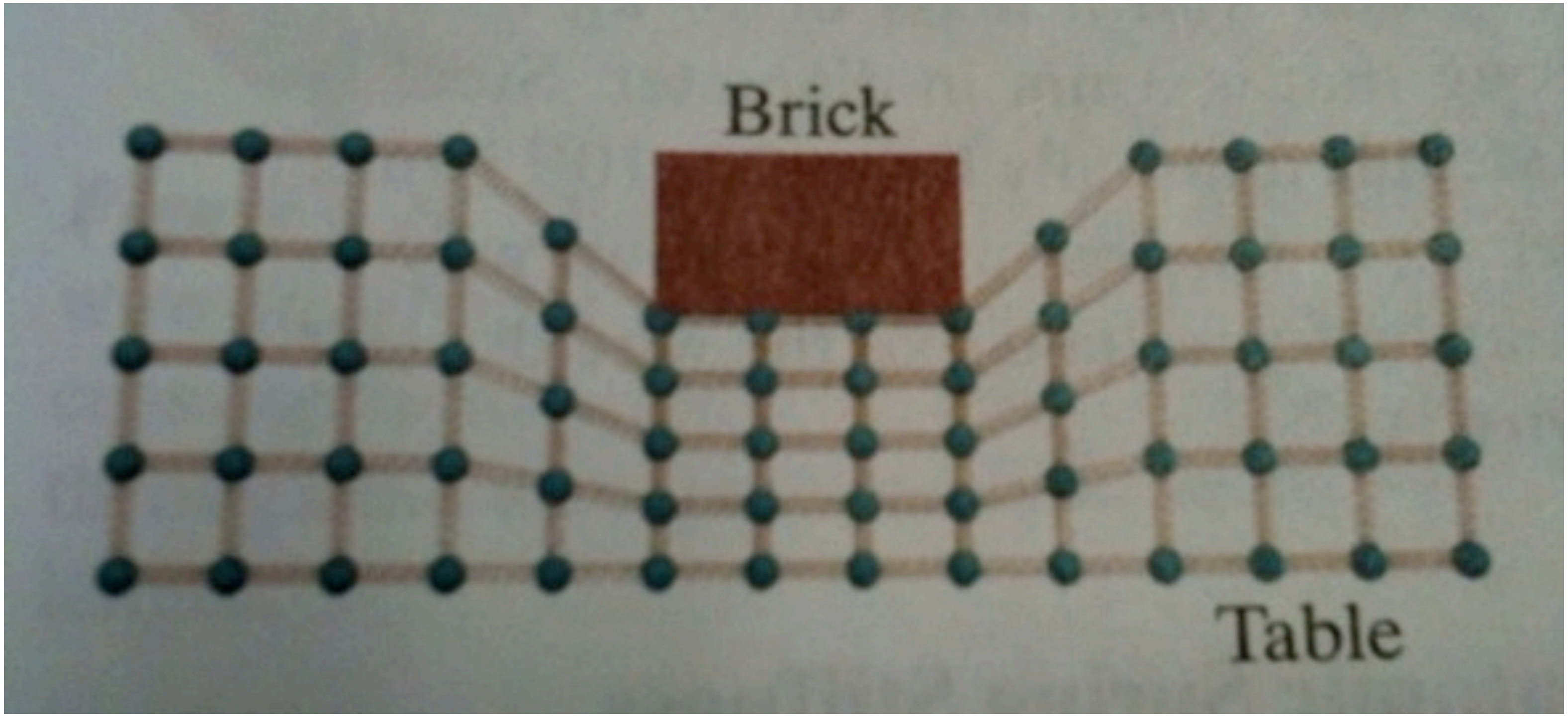
What would happen as I sat down?

- My weight would compress the spring.
- As I put more and more weight on it, the spring would compress more.
- And the more you compress a spring, the harder it pushes back.
- Eventually I would have compressed it so much that it would push back on me as much as my weight pushes down on it.

This is where contact forces come from

- At an atomic level, supposedly solid things (like chairs) are made of atoms stuck together by stretchy chemical bonds.
- These chemical bonds behave much like springs.
- When you apply a force to something, they bend and push back.

A brick on a table



Normal Force

- This is the explanation of normal force.
- Whenever you apply a force to a solid surface, it will push back with just enough force to stop you from sinking into that surface.
- Unless you push hard enough to break the solid surface.

How do you work it out?

- If you knew how much you were sinking into the surface, and the spring constant of the surface, you could use the spring equation. But you usually don't.
- Instead, work backwards from the motion. If an object is not sinking into a surface or leaping off it, the component of the forces perpendicular (normal) to the surface must add up to zero.
- The normal force is whatever you need to make this happen!

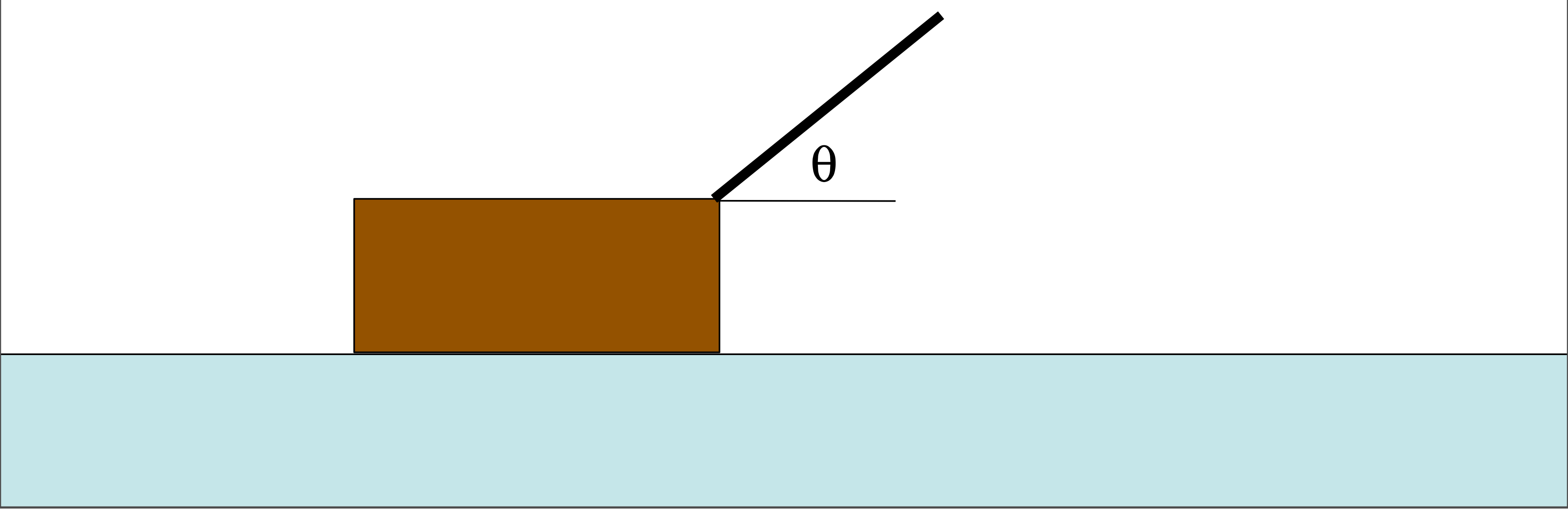
Perpendicular

- It’s called the “normal force” because it is perpendicular (“normal”) to the surface.
- How do you work it out?
- Usually by elimination. Work out all the other forces on some object.
- Add up (vector sum) these forces.
- Work out the component perpendicular to the surface.
- The normal force will be equal and opposite.

For example

You are dragging a box of mass M along the floor at a constant speed. You do it by pulling on a rope with force T.

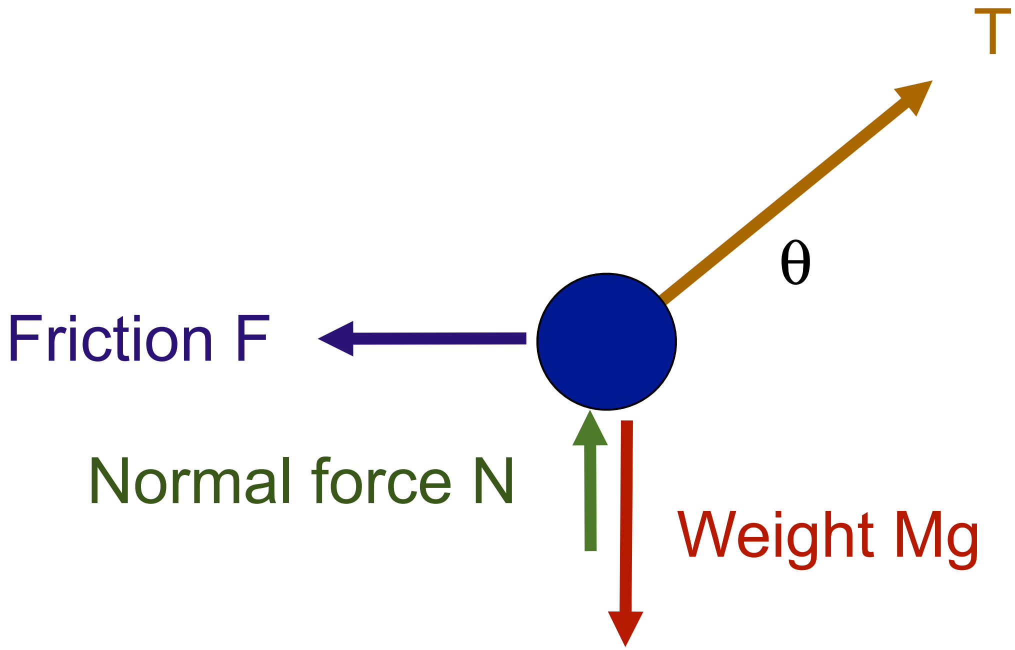
What is the normal force?



Draw a free-body diagram

- Show the box as a dot
- Show only the forces that act ON THE BOX

Free-body diagram…



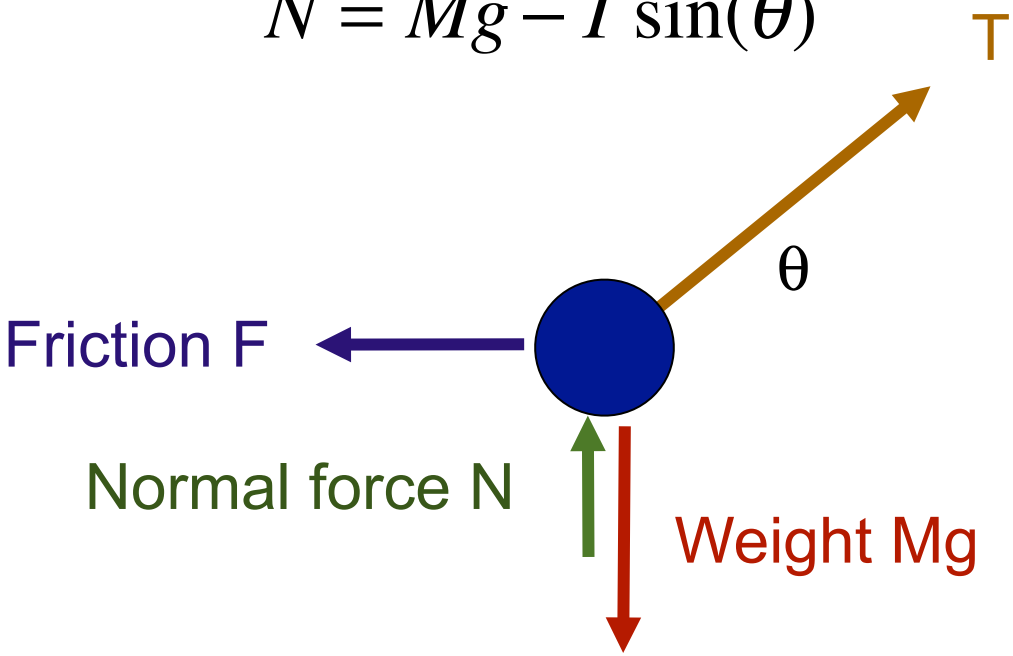
Now what's the equation for the normal force?

Free-body diagram…

Forces perpendicular to the surface (vertical in this case) must balance. So -

$$T\sin(\theta) + N = Mg$$

$$N = Mg - T\sin(\theta)$$



(as it's not accelerating, horizontal forces must balance too - so

$$F = T\cos(\theta)$$

Next time

- Friction - another contact force.

Key points

- Whenever objects touch, there is a contact force.
- The normal force is usually whatever is needed to stop the objects moving into each other or springing apart.