Assignment 3 Semester 1, 2009

1. Write a fortran program to compute the difference in distance modulus between the two nearby galaxies the Large Magellanic Cloud (LMC)and the Small Magellanic Cloud (SMC). This will be done by fitting period-luminosity relations in the form $\mathrm{V} = a \times \log \mathrm{P} + b$ to Cepheid variables in these two galaxies. Then, assuming the distance to the LMC is 51 kpc, calculate the distance to the SMC.

A template fortran program that you can complete for the present task can be downloaded from the WebCT Assignment 3 page. You will also find there the Cepheid data files and a plot of the period luminosity relations. Hand in a printout of your fortran source code and a printout of the output file produced by the code (which will give the distance modulus difference and the SMC distance).

8 Marks

2. Write some brief notes on the following three classes of variable stars: RR Lyraes; Classical Cepheids; and Long-period Variables (Miras and Semi-Regular Variables). For each class, your notes could mention: stellar mass range; luminosities and/or period luminosity laws; mode(s) of pulsation; where found (old populations, young populations, star clusters,...); evolutionary state; astronomical "uses"; and connection with, or revelations about, mass loss.

9 Marks

3. A low mass asymptotic giant branch star consists of a compact degenerate core, which is essentially a white dwarf around which hydrogen and helium burn in shells, and an extended envelope. The compact core has a mass of about 0.6 $\mathrm{M}_\odot$ and the envelope mass is about one or two tenths of a solar mass. The radius of the nuclear burning shells is $\sim$0.1 $\mathrm{R}_\odot$ while the radius of the extended envelope is $\sim$200 $\mathrm{R}_\odot$.

Such a star may be approximated as a central point source of mass M and luminosity L and an envelope whose mass is so small that it does not contribute to the gravitational acceleration felt by mass elements in the envelope. The envelope can be assumed to consist of a perfect gas and to have a radius $r = R$ outside which the pressure is zero.

(a) Let $x = r/R$ and assume the density ρ in the envelope is given by $\rho = \rho_o x^{-2}$. Show that the pressure in the envelope of such a star is given by

$$P = \frac{GM\rho_o}{3R}(\frac{1}{x^3} - 1).$$

2 Marks

(b) Use this expression for the pressure to show that when Γ_1 is constant, the adiabatic wave equation for the star can be written as

$$x\frac{d}{dx}[(x - x^4)\frac{d\xi}{dx}] + \frac{3}{\Gamma_1}[\frac{R^3}{GM}\omega^2 x^3 - (3\Gamma_1 - 4)]\xi = 0.$$

2 Marks

(c) Show that near x = 0, the solution to this equation is of the form

$$\xi = Ax^\alpha,$$

where A is a constant and

$$\alpha^2 = \frac{3}{\Gamma_1}(3\Gamma_1 - 4).$$

3 Marks

...over

(d) For $0 \leq x \leq 1$, try a power series of the form

$$\xi = \sum_{n=0}^{\infty} a_n x^{\alpha+n}$$

as a solution to the adiabatic wave equation in (b). Show that this leads to a recurrence relation between the coefficients

$$\frac{a_{n+3}}{a_n} = \frac{(n+\alpha)(n+\alpha+3) - J}{(n+\alpha+3)^2 - \alpha^2}, \qquad n = 0, 3, 6, ...$$

$$\text{where} \quad J = \frac{3R^3\omega^2}{\Gamma_1 GM}.$$

Hence, derive a formula for the eigenvalues ω^2.

6 Marks

(e) Derive and sketch the eigenfunctions for the fundamental mode and first overtone mode, with each eigenfunction normalized to 1.0 at the surface. Assume $\Gamma_1 = \frac{5}{3}$ for your plot.

3 Marks