Assignment 1  Semester 1, 2008

1. A molecular cloud in the interstellar medium may be considered as spherical and of uniform density and fixed temperature as it undergoes expansion or contraction. Assume the opacity of material in the molecular cloud has a value $\kappa$. Show that the optical depth $\tau_J$ from the centre to the surface of a sphere of one Jean’s mass is given by $\tau_J = \text{constant} \times \kappa \rho^{1/2} T^{1/4}$, and derive the value of the constant ($\tau_J$ may be taken as $\kappa \rho R_J$, where $R_J$ is the Jean’s length).

2. As the molecular cloud collapses and $\rho$ increases
   (a) How does $\tau_J$ vary during the early stages of collapse when the cloud is optically thin ($\tau_J << 1$)? Assume $\kappa$ independent of $T$ and $\rho$.
   (b) At approximately what value of $\tau_J$ will the temperature of each collapsing cloud fragment start to rise significantly due to collapse? Why?
   (c) When the cloud is optically thick ($\tau_J >> 1$) and the collapse may be considered adiabatic ($P \propto \rho^\gamma$), how will $\tau_J$ vary with $\rho$ during continued collapse?
   (d) Describe the variation of $M_J$ with cloud density both before and after each Jean’s mass becomes optically thick. How will this affect cloud fragmentation?
   (e) In the stages of collapse of a gas cloud before it becomes optically thick, its temperature is determined by the balance between compressional heating and radiation of energy from the cloud. Energy is radiated by atoms and molecules heavier than H and He, as well as by H$_2$ (the atoms and molecules are collisionally excited). Before the first stars polluted the interstellar gas with heavy metals, only H$_2$ cooling was available, and this is not as efficient at cooling the gas as the heavy element are. Given this information, suggest how the masses of the first stars might compare to the masses of stars formed from gas clouds today.

3. A particular molecular cloud in the interstellar medium may be considered to be of uniform density and temperature and to be made up of hydrogen molecules (we neglect helium and heavier elements). The cloud has a total mass $M$ of $10^6$ M$_\odot$, and the molecular hydrogen has a temperature $T = 20$ K and a number density $n = 200$ cm$^{-3}$.
   (a) What is the Jean’s mass (in M$_\odot$) associated with the molecular cloud?
   (b) How many sub-condensations could begin to fragment out of the cloud at this time?
   (c) Question 1 shows that the optical depth from centre to surface of one Jean’s mass increases with density. Thus, during collapse/fragmentation, an amount of gas equivalent to one Jean’s mass will become more optically thick. When the optical depth $\tau_J = 1$, Question 2 indicates that no more fragmentation will occur. Assuming the cloud temperature remains at 20 K until $\tau_J$ reaches 1, what is the minimum mass star that will condense out of the cloud? The opacity, due mainly to dust, may be taken as 1 cm$^2$ gm$^{-1}$.

4. Use the equations of stellar structure to show that for a star in hydrostatic equilibrium, the gravitational potential energy

$$\Omega = -3 \int_0^V PdV.$$  

This is the Virial Theorem for a star. When the pressure $P$ within the star is supplied
by a perfect gas, show that

\[ \Omega + 2U = 0, \]

where \( U \) is the thermal energy of the star.

Hence, show that the total thermal and gravitational potential energy of a star is negative. Before collapse, the gas cloud from which the star formed will have had \( E_{\text{total}} \approx 0 \) (because \( T \) was very low, and \( R \) was very large). What happened to the missing energy?

5. Write a brief description of each the following:

- The Chandrasekhar Limiting Mass
- The Hayashi Track
- The Horizontal Branch
- The Asymptotic Giant Branch
- The first and second dredge-ups
- The third dredge-up
- Helium burning and its nuclear products
- Nuclear statistical equilibrium (the e-process) and its modification in the case of alpha-rich freeze-out
- The s- and r-processes
- Nuclear reactions in Novae
- Supernovae of Types Ia, Ib, Ic and II
- The Schwarzschild criterion for convection and convective overshoot

Some quantities you may need:

- Solar mass \( M_\odot = 1.989 \times 10^{33} \text{ g} \)
- Boltzmann constant \( k = 1.38 \times 10^{-16} \text{ erg deg}^{-1} \)
- Gravitational constant \( G = 6.668 \times 10^{-8} \text{ dyn cm}^2 \text{ g}^{-2} \)
- Mass of a hydrogen atom = \( 1.67 \times 10^{-24} \text{ g} \)