Magnetorotational instability in weakly ionised accretion discs

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Abstract

We present a linear analysis of the vertical structure and growth of the MRI in weakly ionised, stratified accretion discs, such as protostellar and quiescent dwarf novae systems.

Because of the low conductivity, the magnetic coupling is weak and depends critically on density, temperature, ionisation rate and charged species. These factors vary radially and vertically within the disc.

The method includes the effects of the magnetic coupling, the conductivity regime of the fluid and the strength of the magnetic field, which is initially vertical. The conductivity is treated as a tensor and assumed constant with height.

Methodology

System of equations

Conservation of mass

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0$$

Conservation of Momentum

$$\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} \cdot \nabla)\mathbf{V} - 2\Omega V_\varphi \hat{\mathbf{r}} + \tfrac{1}{2}\Omega V_r \hat{\boldsymbol{\varphi}} - \frac{V_k^2}{r}\hat{\mathbf{r}} + \frac{C_s^2}{\rho}\nabla\rho + \nabla\Phi - \frac{\mathbf{J}\times\mathbf{B}}{c\rho} = 0$$

Induction Equation

$$\frac{\partial \mathbf{B}}{\partial t} + \nabla\times(\mathbf{v}\times\mathbf{B}) - c\nabla\times\mathbf{E} - \tfrac{3}{2}\Omega B_r \hat{\boldsymbol{\varphi}} = 0$$

Ampere's Law

$$\mathbf{J} = \frac{c}{4\pi}\nabla\times\mathbf{B}$$

Ohm's Law

$$\mathbf{J} = \boldsymbol{\sigma}\cdot\mathbf{E}$$

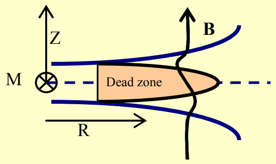
Perturbations

$$q = q_o + \delta q(z)e^{i\omega t}$$

This system is linearised about an initial state where the fluid is in keplerian rotation and the magnetic field is vertical.

Parameters

χ_o	Strength of the coupling between the magnetic field and the disc at the midplane
v_A/c_s	Ratio of Alfven to sound speed at the midplane -Strength of the magnetic field.
σ_1/σ_2	Ratio of conductivity terms perpendicular to the field -Conductivity regime of the fluid.

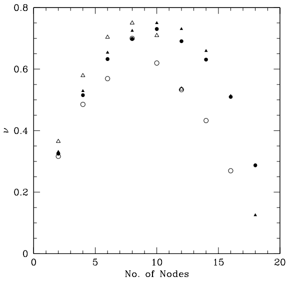


The linearised system is solved as a two-point boundary value problem by integrating the equations vertically from the midplane to the surface.

Main Findings

(1) Unstable modes

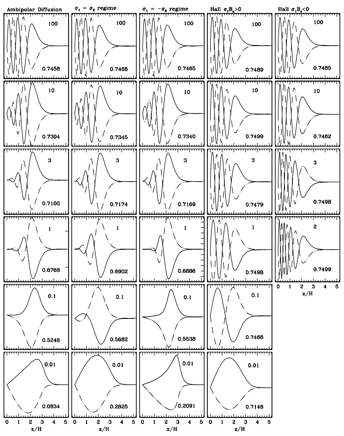
Stratification restricts the unstable frequencies to a discrete subset of the continuous range derived from a local analysis [4].



Growth Rate versus No. of Nodes of the MRI for different conductivity regimes and coupling at the midplane. Filled symbols correspond to $\chi_o=10$ and open ones to $\chi_o=1$. Circles show ambipolar diffusion and triangles the Hall limit ($\sigma_1 B_z>0$).

(2) Conductivity Regime

Ambipolar diffusion determines the structure and growth of the most unstable MRI perturbations when magnetic coupling is strong. Under a weak coupling, the Hall effect causes perturbations to grow faster and peak at a different height.



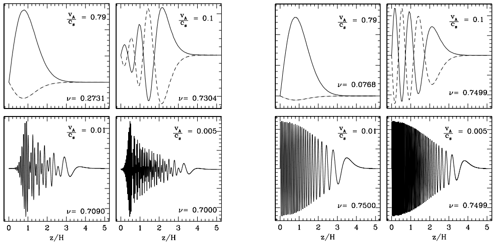
Structure of the most unstable MRI perturbations for different values of the coupling (χ_o) in all conductivity regimes. The coupling is shown at the top right corner of each panel and the growth rate is indicated at the bottom right corner.

References

[1] Balbus S. A., Hawley J. F., 1991, ApJ, 376, 214
[2] Gammie C. F., 1996, ApJ, 457, 355
[3] Igea J., Glassgold A. E., 1999, ApJ, 518, 848
[4] Wardle M., 1999, MNRAS, 307, 849

(3) Magnetic Field Strength

The weaker the magnetic field (B), the higher the wavenumber of the perturbations [1]. When B is very small the instability grows with very high wavenumbers.

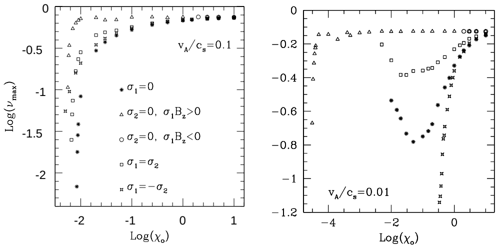


Structure and growth rate of the MRI for different choices of v_A/c_s. Left panel shows the ambipolar diffusion limit and right panel the Hall ($\sigma_1 B_z>0$) case. Solid lines show δB_r and dashed lines δB_ϕ. Pattern on lower left panels are caused by interference between two modes of similar growth rate.

(4) Parameter space

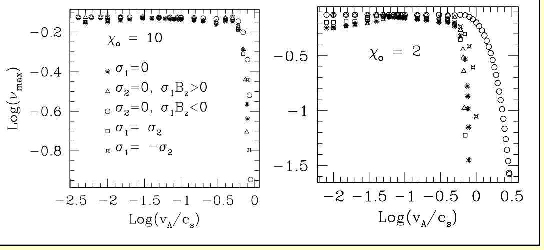
(a) **Magnetic Coupling**

Left panel: Ambipolar diffusion perturbations are damped when $\chi_o<v_A/c_s$. In the Hall case the limit is $\chi_o<v_A^2/c_s^2$ [4].
Right panel: For very weak fields, ν_{max} decreases with χ_o as per local results[4], until wavenumbers are small and global effects cause ν_{max} to increase.



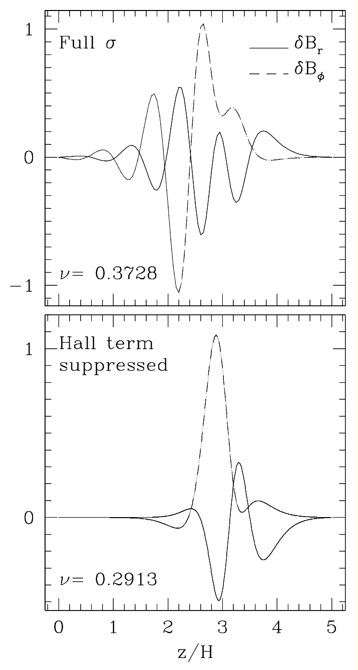
(b) **Magnetic field strength**

Left panel: Increasing the strength of the magnetic field has little effect on ν_{max} until $v_A/c_s \sim 1$, when it abruptly drops to zero.
Right panel: For weak coupling, Hall limit perturbations grow until $v_A/c_s \sim 2.9$. In both cases stratification is unimportant for weak fields and MRI perturbations grow at the local rates [4].



Implications

When the Hall regime dominates near the midplane and ambipolar diffusion is dominant closer to the surface, a thicker layer of the disc is unstable to MRI perturbations and modes grow faster than under the ambipolar diffusion approximation.
Including the Hall regime in the study of dynamical processes in low conductivity discs is essential to understand the true nature of accretion.



Structure and growth rate of the MRI for different configurations of the conductivity tensor and v_A/c_s = 0.01. Top panel shows the case where ambipolar diffusion dominates near the surface while the Hall regime is predominant close to the mid plane. Bottom panel shows the ambipolar diffusion approximation. Note the "dead zone" near the mid plane [2].

Future Work

-Evaluate a z-dependent conductivity tensor, including x-ray [3] and cosmic ray ionisation.
- Model structure and linear growth of MRI perturbations at different radial locations in low conductivity discs