Iteration and Problem Solving Strategies
How to solve anything!
How to work out really complicated motion

- Break it up into little tiny steps.
- Use an approximate method for each step.
- Add them all up.
Vertical spring-mass

- Time $t$ with $t + \Delta t$
- Position $x$ with $x + v \Delta t$
- Velocity $v$ with $v + \left( g - \frac{k}{m} x \right) \Delta t$
Let’s go

• Start off with $t=0$, $x=0$, $v=0$
• Apply our equations:
  – New value of $t$ is $t + \Delta t = 0 + 0.1 = 0.1$
  – New value of $x$ is $x + v \Delta t = 0 + 0 \times 0.1 = 0$
  – New value of $v$ is

\[
v + \left( g - \frac{k}{m} x \right)\Delta t = 0 + \left( 9.8 - \frac{5}{0.1} \right) \times 0.1 = 0.98\]
So after 0.1 seconds…

• According to our method, the position hasn’t changed (still zero) but the velocity has increased to 0.98 m/s.

• Now do this again, using these new numbers as the starting parameters.
Second iteration

• Start off with $t=0.1, x=0, v=0.98$

• Apply our equations:
  – New value of $t$ is $t + \Delta t = 0.1 + 0.1 = 0.2$
  – New value of $x$ is $x + v \Delta t = 0 + 0.98 \times 0.1 = 0.098$
  – New value of $v$ is

\[
v + \left( g - \frac{k}{m} x \right) \Delta t = 0.98 + \left( 9.8 - \frac{5}{0.1} \right) \times 0.1 = 1.96
\]
Third iteration

• Start off with $t=0.2$, $x=0.098$, $v=1.96$
• Apply our equations:
  – New value of $t$ is $t + \Delta t = 0.2 + 0.1 = 0.3$
  – New value of $x$ is $x + v \Delta t = 0.098 + 1.96 \times 0.1 = 0.294$
  – New value of $v$ is
    \[
    v + \left( g - \frac{k}{m} x \right) \Delta t = 1.96 + \left( 9.8 - \frac{5}{0.1} 0.098 \right) \times 0.1 = 2.45
    \]
Fourth iteration

- Start off with $t=0.3$, $x=0.294$, $v=2.45$
- Apply our equations:
  - New value of $t$ is $t + \Delta t = 0.3 + 0.1 = 0.4$
  - New value of $x$ is $x + v \Delta t = 0.294 + 2.45 \times 0.1 = 0.539$
  - New value of $v$ is

$$v + \left( g - \frac{k}{m} x \right) \Delta t = 2.45 + \left( 9.8 - \frac{5}{0.1} \times 0.294 \right) \times 0.1 = 1.96$$
And so on…

• Do the calculations for each step, and then use the results as the input for the next step.

• That’s what iteration means!

• What results do we get?
Results for first few iterations (steps)

<table>
<thead>
<tr>
<th>t</th>
<th>x</th>
<th>v</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.1</td>
<td>0</td>
<td>0.98</td>
</tr>
<tr>
<td>0.2</td>
<td>0.098</td>
<td>1.96</td>
</tr>
<tr>
<td>0.3</td>
<td>0.294</td>
<td>2.45</td>
</tr>
<tr>
<td>0.4</td>
<td>0.539</td>
<td>1.96</td>
</tr>
<tr>
<td>0.5</td>
<td>0.735</td>
<td>0.245</td>
</tr>
</tbody>
</table>
A graph of the first twenty iterations...
Good and bad

• If you remember - the correct solution is an oscillation.
• Our iteration has correctly produced an oscillation.
• But it has the amplitude steadily increasing - which is wrong.
• Springs don’t do that!
Our time step was too big.

- The approximation (that the speed and velocity are approximately constant within each time-step) wasn’t good enough.
- If we make our time-step smaller… (say 0.01 sec)…
- We have to do a lot more steps…
But it gets better...
And if we make our time step smaller still - say 0.001 sec...
Really rather good…

- But I needed to do 2000 steps (iterations) to get the last plot.
- Which would have been very tedious and error-prone had I not used a computer…
- Luckily we have computers and doing those 2000 steps took less than 0.1 sec…
But it’s painful

- So do it by computer!
- Example python program
So even this crude approximation…

• It pretty good with small timesteps.
• And with the speed of modern computers, small timesteps are not much of a problem.
• Using a better (more complicated) approximation to the motion in each timestep will mean that you can get away with bigger timesteps.
• But each timestep needs more calculations to evaluate - so overall you may not be better off.
Let’s try an example

• A spaceship near a black hole...
What forces apply?

• In this case there is only one - the gravitational force.
• Being in space there is no friction or drag, so...

\[ F = \frac{GMm}{x^2} \]

Where M is the mass of the black hole and m the mass of the spaceship.
What variables will we track?

- Time, position \((x)\) and velocity \((v)\) as before.
- For one-dimensional problems it will always be these.
- In 3D, you will need to track vector position and vector velocity.
Iteration equations

• For time: \( t + \Delta t \) (as before)
• For position: \( x + v \Delta t \) (as before)
• But what about for velocity?

\[ F = \frac{GMm}{x^2} \]

Write it down…

What will the velocity be at the end of a time-step?

\( x \) increases away from the black hole. Velocity is (as always) rate of change of \( x \).
Clicker Question

- What is the new velocity?
The answer...

- Gravity works to decrease the (outwards) velocity

\[ v - \frac{GM}{x^2} \Delta t \]
Let’s chose some values

• Mass of the black hole = $10^{31}$ kg
• Starting distance = 1,000,000 km
• Starting speed = 2000 km/s away

(You’ve been blasting away from it as hard as you could - but now your fuel has run out... Is your speed great enough to escape?)
Python simulation

And then VPython simulation
Summary

• Divide up your problem into little tiny steps.
• Write down an approximate set of equations for each step
• Plug numbers into these formulae over and over again - taking the output from one step as the input to the next.
Chaos

• You can get extremely complicated results from this...

• Tiny changes in the starting positions can cause huge changes in the outcomes.

• This is the hallmark of “Chaos”
Computer Lab

- You will get to practice iteration in the computer lab.
- This is one of four rotations - check in which week you are doing it.
- Venue is different - BOZO112
Contact Forces

Whenever one object touches another...
Peculiarly tricky

- Because they can point in different directions
- Because there is no simple formula to work them out
Is there really a force when you sit on something?

- Newton’s laws say there must be...

Gravitational force downwards, due to Earth

Must be an equal and opposite upward force or I’d be accelerating

Thursday, 10 March 2011
But how can a chair push?

- A force is normally thought of as a "push" or "pull"
- But you don’t normally think of chairs, walls, the floor pushing?
Imagine replacing the chair…

• With a spring…
What would happen as I sat down?

- My weight would compress the spring.
- As I put more and more weight on it, the spring would compress more.
- And the more you compress a spring, the harder it pushes back.
- Eventually I would have compressed it so much that it would push back on me as much as my weight pushes down on it.
This is where contact forces come from

- At an atomic level, supposedly solid things (like chairs) are made of atoms stuck together by stretchy chemical bonds.
- These chemical bonds behave much like springs.
- When you apply a force to something, they bend and push back.
A brick on a table
Normal Force

• This is the explanation of normal force.
• Whenever you apply a force to a solid surface, it will push back with just enough force to stop you from sinking into that surface.
• Unless you push hard enough to break the solid surface.
How do you work it out?

• If you knew how much you were sinking into the surface, and the spring constant of the surface, you could use the spring equation. But you usually don’t.

• Instead, work backwards from the motion. If an object is not sinking into a surface or leaping off it, the component of the forces perpendicular (normal) to the surface must add up to zero.

• The normal force is whatever you need to make this happen!
Perpendicular

- It’s called the “normal force” because it is perpendicular (“normal”) to the surface.
- How do you work it out?
- Usually by elimination. Work out all the other forces on some object.
- Add up (vector sum) these forces.
- Work out the component perpendicular to the surface.
- The normal force will be equal and opposite.
For example

You are dragging a box of mass $M$ along the floor at a constant speed. You do it by pulling on a rope with force $T$.

What is the normal force?
Draw a free-body diagram

• Show the box as a dot
• Show only the forces that act ON THE BOX
Now what’s the equation for the normal force?
Free-body diagram...

Forces perpendicular to the surface (vertical in this case) must balance. So -

\[ T \sin(\theta) + N = Mg \]
\[ N = Mg - T \sin(\theta) \]

(As it’s not accelerating, horizontal forces must balance too - so)

\[ F = T \cos(\theta) \]
Next time

- Friction - another contact force.
Key points

• Whenever objects touch, there is a contact force.

• The normal force is usually whatever is needed to stop the objects moving into each other or springing apart.