Energy
Energy Equations

- Energy $E$ is a scalar. The energy of an object is given by

$$E^2 = p^2 c^2 + m_0^2 c^4$$

where $p$ is the momentum, $m_0$ the rest mass and $c$ the speed of light.

There is also (potential) energy in fields such as gravitational or electric fields:

$$E = \frac{G m_1 m_2}{r} + \frac{1}{4 \pi \varepsilon_0} \frac{q_1 q_2}{r}$$

In a given collection of objects (a system), energy is conserved unless an external force $\mathbf{f}$ is applied to this system, in which case the change in energy of the system is:

$$\frac{dE}{dt} = \mathbf{v} \cdot \mathbf{F}$$

The dot is the vector dot product.

Tuesday, 22 February 2011
That’s actually all you need to know

• But now we’ll talk about what it actually means...
Energy

• The basic principle is very familiar - things don’t do stuff without energy.
• Energy can take many forms, and change from one to another, but it is always conserved and you can’t get it for nothing.
• This can sometimes let you solve seemingly impossible problems with the greatest of ease
Impossible without details

- You don’t get something for nothing
- Perpetual motion machines can’t exist.
- If you see something doing stuff, there must be an energy source hidden somewhere.
- If you don’t have enough energy, no matter what, you can’t do something.
For example

- “All this talk of space travel is utter bilge”, Professor Woolley, ANU, 1956
- Launch of Sputnik 1, 1957
- Argument - the energy liberated by a kilogram of TNT is less than the energy needed to lift one kilogram into space.
- So the most explosive materials known cannot lift even themselves into space.
What’s wrong with this?
Two things

• You can get much more energy per unit weight from things like petrol compared to explosives - the explosives have less energy, but liberate it faster.

• You can burn tones of fuel to get 1kg of payload into space - most of the fuel is burned low down.
Energy

• Throw a ball into the audience

• Let’s see what you know.

• Write down on a scrap of paper what is going on with energy while a ball is thrown across the classroom.
Less kinetic energy and more gravitational potential energy

Kinetic energy

Chemical energy in my muscles

Back to Kinetic energy, mostly in the ball but a bit in air currents

Heat energy in the ball and my hand. Maybe some sound energy?

Tuesday, 22 February 2011
Is this plausible?
On energy grounds alone...
He goes higher at the peak than at the start. Where did the energy for this come from?
Maths behind this?

• The chemical energy is really hard to measure (at least without dissecting you followed by a cell-by-cell chemical analysis)
• But you can work out how much energy you used using the law of conservation of energy.
• The kinetic energy in the ball must have come from your muscles.
• And your arm probably warmed up a bit from the exercise.
• Add this heat energy to the kinetic energy and that’s how much chemical energy you must have used.
Kinetic energy

- The true energy of a moving object is given by:

\[ E^2 = p^2 c^2 + m_0^2 c^4 \]

If you set the momentum \( p \) to zero (i.e. the ball isn’t moving), you get:

\[ E^2 = m_0^2 c^4 \]

Take the square root of this, and you get an equation you may recognize:

\[ E = m_0 c^2 \]
Relativity

- Relativity (covered in PHYS1201) tells us that matter and energy are the same thing.
- The true energy equation takes this into account - even a stationary ball has lots of energy (you multiply the mass in kg by the velocity of light squared…)
- But in most everyday situations you don’t need to worry about this - you can ignore this rest-mass energy and just look at the change in energy due to the motion.
- If the speed of an object is much less than the speed of light, you can approximate this using another familiar equation:

\[ KE = \frac{1}{2}mv^2 \]
Kinetic and potential energy

- So if you know how fast the ball was moving when it left my hand, you can work out the kinetic energy.
- But as it moves higher into the air, it will slow down.
- Energy has been lost by the ball, and gained by the gravitational field of the Earth. This is called Potential Energy.
Potential Energy

• The true equation for gravitational potential energy (well, almost - it does need some corrections for General Relativity which I won’t go into here) is:

\[ E = -\frac{GMm}{r} \]

where \( M \) in this case would be the mass of the Earth, and \( m \) the mass of the ball (or vice versa - it makes no difference). \( r \) is distance between the centre of the ball and the centre of the Earth.
Approximation

• You can use that full equation - $r$ might start off at 6400 km and go to 6400.005 km.

• But over this small range in $r$ (the distance to the centre of the Earth), you can use a simpler approximate form of the potential energy equation:

$$PE = mgh$$

Where $h$ is the height, $m$ the mass of the ball, and $g = 9.8 \text{ m s}^{-2}$. 

Tuesday, 22 February 2011
Straight up?

• If I threw it straight up, all the kinetic energy would turn into potential energy for a moment when it’s at the top of its arc.

• So you could work out how high it would go, using conservation of energy.

• The Kinetic energy when it leaves my hand must equal the potential energy at the top of its motion.
\[ \frac{1}{2}mv^2 = mgh \]

Cancel \( m \)

\[ \frac{1}{2}v^2 = gh \]

Divide both sides by \( g \), and write it down backwards

\[ h = \frac{1}{2} \frac{v^2}{g} \]
Other forms of energy

- Energy in a spring: 
  \[ E = \frac{1}{2}kD^2 \]
  where \( k \) is the spring constant and \( D \) the distance by which the spring is extended or compressed.

- Rotational Energy: 
  \[ E = \frac{1}{2}I\omega^2 \]
  where \( I \) is the moment of inertia and \( \omega \) is the angular velocity
General Procedure

• Pick states (like beginning and end)
• Write down all the various forms of energy at each state
• Set them equal to each other
• Solve for whatever it is you want to know.