1. **Accretion disc spectra [25 points].**
Consider a geometrically thin accretion disc around some object (e.g., a black hole) with temperature as a function of radius $T(r)$. The disc has inner and outer radii R_0 and R_1. The material in the disc is in thermal equilibrium, and consists of particles with absorption cross section σ_ν, with no significant scattering. The column density as a function of radius is $N(r)$. The disc is a distance D from the Earth, and is seen face-on.

   (a) Write down an expression for the intensity I_ν received Earth from a line of sight that passes through the disc at radius r. [5 points]

   (b) Express the specific flux F_ν received from the disc at the Earth in terms of the variables given, and give limiting expressions for discs that are very optically thin or very optically thick. You may assume that the disc is far from the Earth, so $r \ll D$. [10 points]

   (c) Consider an optically thick disc with a temperature distribution $T(r) = T_0(r/R_0)^{-q}$. Suppose that the range of radii in the disc is large enough that, over a significant part of the oberved spectrum, the values of R_0 and R_1 do not matter, i.e., we can approximate $R_0 \to 0$ and $R_1 \to \infty$. Show that the spectral index $\beta = d\log F_\nu/d\log\nu$ is determined solely by the powerlaw slope q, so that for an optically thick disc one can determine the shape of the radial temperature dependence just from the spectral index. Hint: make a change from r to $x = (h\nu/k_B T_0)^{1/q}(r/R_0)$ as your integration variable. [10 points]

2. **Scattering screens [25 points].**
Consider light from an astrophysical source that passes through a scattering screen of plasma on its way to Earth. The scattering is done by electrons in the plasma, which have number density n and frequency-independent scattering cross section σ, with negligible emission or absorption. The thickness of the plasma layer is L.

   (a) Using the Eddington approximation, calculate the angle-averaged intensity J_ν as a function of depth z into the plasma layer. You answer should contain two constants of integration to be determined in subsequent parts of the problem. [5 points]

   (b) Use the two-stream approximation to write down two boundary conditions on J_ν for the problem. Express the boundary conditions in terms of F_ν, the incoming radiation flux reaching the screen from the source. You will need to make some approximations to do this; be sure to explain what approximations you are making. [5 points]

   (c) Use your boundary conditions to solve for the constants of integration, and thereby obtain a full expression for $J_\nu(z)$. [5 points]

   (d) Using your result for $J_\nu(\tau)$, estimate what fraction of the incoming flux is transmitted through the screen, and what fraction is reflected back. Again, you will need to make some approximations, which you should justify. [5 points]

   (e) What fraction of the flux would be transmitted if σ were an absorption opacity rather than a scattering opacity? (You don't need the Eddington approximation for this, you can just write the answer down.) Does more flux get through if the screen is scattering or if it is absorbing? Give a physical explanation for your result in a few sentences. [5 points]

3. **Radiation pressure from plane waves [25 points]**.

In this problem we will use a simple model system to deduce the pressure exerted by a plane wave against an absorbing medium. Our simple model of an absorbing medium (inspired by Rybicki & Lightman) consists of point masses of charge q suspended in a medium that exerts a viscous force $\mathbf{F}_{\rm visc} = -\eta \mathbf{v}$ on masses moving at velocity $\mathbf{v}$. The point masses have negligible inertia, so when subjected to Lorentz forces $\mathbf{F}_{\rm Lorentz}$ by an incoming electromagnetic wave, they instantly reach terminal velocity, whereby $\mathbf{F}_{\rm visc} = \mathbf{F}_{\rm Lorentz}$. This in turn transmits the Lorentz force to the rest of the medium.

(a) Consider a plane wave moving in the $\hat{\mathbf{z}}$ direction with angular frequency ω and electric field magnitude E impinging on this medium. Show that, as long as the charges move with speed $v = |\mathbf{v}| \ll c$, the electric force exerted by the wave is much greater than the magnetic force. [5 points]

(b) Suppose that the wave is linearly polarised, with the electric field vector lying in the $\mathbf{x}$ direction. Neglecting the sub-dominant magnetic force, show that the charges in the medium perform simple harmonic motion in the $\mathbf{x}$ direction at angular frequency ω, and derive the amplitude of the oscillation. [5 points]

(c) Compute the time-averaged power P delivered by the wave to the medium for each oscillating charge. [5 points]

(d) Next compute the time-averaged magnetic force $\mathbf{F}_{\rm mag}$ exerted by the wave on each oscillating charge. [5 points]

(e) Use the results of the previous two parts to argue that the radiation pressure exerted by a plane wave on a completely absorbing surface normal to its direction of travel is $p_{\rm rad} = F/c$, where F is the energy flux of the plane wave. [5 points]