

Astronomy is an observationally-driven science, so, although the bulk of this course will focus on building a physical model for the stars and their properties, we will begin with what we learn from observations. With the naked eye under optimal conditions, one can distinguish  $\sim 6,000$  individual stars from Earth, but the bulk of the Milky Way Galaxy appears as an indistinguishable blur. In 1610 Galileo published the first telescopic observations showing that this blur in fact consists of numerous individual stars (Figure 1).

While these early observations are of course important, in order to study stars systematically we must be able to make quantitative measurements of their properties. Only quantitative measurements can form the nucleus of a theoretical understanding and against which model predictions can be tested. In this first class, we will focus on how we obtain quantitative information about stars and their properties.

## I. Luminosity

The most basic stellar property we can think of measuring is its luminosity – its total light output.

### A. Apparent brightness and the magnitude system

The first step to measuring stars' luminosity is measuring the flux of light we receive from them. The Greek astronomer Hipparchus invented a numerical scale for describing stars' brightnesses. He described the brightest stars as being of first magnitude, the next brightest of second, etc., down to sixth magnitude for the faintest objects he could discern. In the 1800s, Pogson formalized this system, and unfortunately we are still stuck with a variant of this system today.

I say unfortunate because the magnitude system has several undesirable features. First, higher magnitudes corresponds to dimmer objects. Second, since it was calibrated off human senses, the system is, like human senses, logarithmic. Every five magnitudes corresponds to a change of a factor of 100 in brightness. In this class we will not make any further use of the magnitude system, and will instead discuss only fluxes from stars, which can be measured directly with enough accuracy for our purposes.

While fluxes are a first step, however, they don't tell us much about the stars themselves. That is because we cannot easily distinguish between stars that are bright but close and stars that are dim but far. The flux depends on the star's intrinsic luminosity and distance:

$$F = \frac{L}{4\pi r^2}. \tag{1}$$

From the standpoint of building a theory for how stars work, the quantity we're really interested in is luminosity. In order to get that, we need to be able to measure distances.

In terms of the magnitude system, the flux is described as an apparent magnitude. We are interested instead in the absolute magnitude, which is defined as the brightness that a star would have if we saw it from a fixed distance.

### B. Parallax and distances

The oldest method, and still the only really direct one, for measuring the distance to a star is parallax. Parallax relies on the apparent motion of a distant object



Figure 1: A drawing of the Pleiades (a constellation) from Galileo's *Sidereal Messenger*. The stars that are shown in outline are visible with the naked eye, while the rest are only visible using a telescope. Source: [https://en.wikipedia.org/wiki/Sidereus\\_Nuncius](https://en.wikipedia.org/wiki/Sidereus_Nuncius).

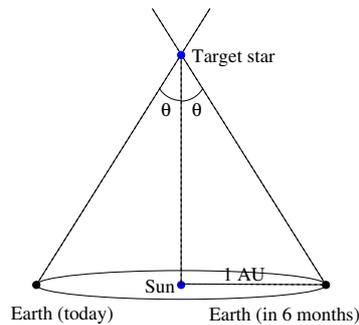


Figure 2: Measuring distances by parallax, in the simplest case of a star whose direction from the Sun is perpendicular to the plane of the Earth's orbit.

relative to a much more distant background as we look at it from different angles. The geometric idea is extremely simple, and illustrated in [Figure 2](#): we measure the position of the target star today, then we measure it again in 6 months, when the Earth is on the opposite side of its orbit.

We then measure the change in the apparent position of the star, relative to some very distant background objects that don't appear to move appreciably. The change is described in terms of the parallax angle  $\theta$ . For a measured change  $2\theta$ , the distance  $r$  to the target star is simply given by

$$r = \frac{1 \text{ AU}}{\tan \theta} \simeq \frac{1 \text{ AU}}{\theta} \quad (2)$$

where the distance between the Earth and the Sun is  $1 \text{ AU} = 1.5 \times 10^{13} \text{ cm}$ , and in the second step we used the small angle formula to say  $\tan \theta \simeq \theta$ , since in practice  $\theta$  is always small.

The importance of this method of distance measurement is illustrated by the fact that in astronomy the most common unit of distance measurement is the parsec (pc), which is defined as the distance away that an object must have in order to produce a parallax shift of 1 second of arc, which is  $4.85 \times 10^{-6}$  radians:

$$1 \text{ pc} = \frac{1 \text{ AU}}{4.85 \times 10^{-6} \text{ rad}} = 3.09 \times 10^{18} \text{ cm} = 3.26 \text{ ly} \quad (3)$$

The nice thing about this definition is that the distance in parsecs is just one over the parallax shift in arcseconds.

Although this technique has been understood since antiquity, our ability to actually use it depends on being able to measure very small angular shifts. The nearest star to us is Proxima Centauri, which has a parallax of  $0.88''$  – to put this in perspective, this corresponds to the size of a quarter at a distance of half a kilometer. As a result

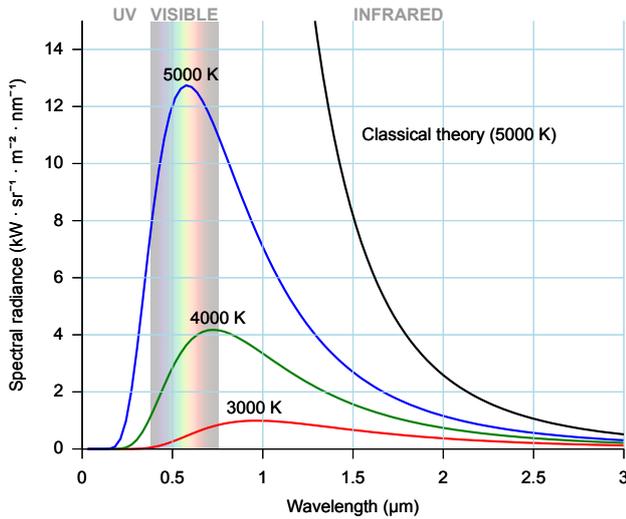


Figure 3: The Planck function, plotted for three different temperatures (red, green, and blue curves). Source: [https://en.wikipedia.org/wiki/Planck's\\_law](https://en.wikipedia.org/wiki/Planck's_law).

of this difficulty, the first successful use of parallax to measure the distance to a star outside the solar system was not until 1838, when Friedrich Bessel measured the distance to 61 Cygni.

In the 1980s and 90s, the Hipparcos satellite made parallax measurements that gave distances accurate to  $\sim 10\%$  for about 20,000 nearby stars – up to about 500 pc distance for the brightest stars. The Gaia satellite, launched in 2013, will eventually provide distances at comparable accuracy for more than 1 billion stars, out to tens of kpc, with the exact distance limit depending in the brightness of the target star. Gaia released its first batch of data late last year, which gave distances for about 2 million stars. For these stars, simply measuring the flux provides the absolute luminosity. These luminosities form a crucial data set against which we can test theories of stellar structure.

## II. Temperatures

Luminosity is one of two basic direct observables quantities for stars. The other is the star's surface temperature. To understand how we can measure the surface temperature of a star, we need to digress a bit into the thermodynamics of light. Since this topic is covered in classes on quantum mechanics and statistical mechanics, I will assert rather than prove the statements below.

To first approximation, we can think of a star as a blackbody, meaning an object that absorbs all light that falls on it. Blackbodies have the property that the spectrum of light they emit depends only on their temperature. The intensity of light that a blackbody of temperature  $T$  emits at wavelength  $\lambda$  is given by the Planck function

$$B(\lambda, T) = \frac{2hc^2}{\lambda^5} \left( \frac{1}{e^{hc/\lambda k_B T} - 1} \right), \quad (4)$$

where  $h = 6.63 \times 10^{-27}$  erg s is Planck's constant,  $c = 3.0 \times 10^{10}$  cm s $^{-1}$  is the speed of light, and  $k_B = 1.38 \times 10^{-16}$  erg K $^{-1}$  is Boltzmann's constant. Figure 3 shows this function.

It is straightforward to show, as you will on your homework, that this function reaches a maximum at a wavelength

$$\lambda_{\max} = 0.20 \frac{hc}{k_B T} = \frac{0.29 \text{ cm}}{T}, \quad (5)$$

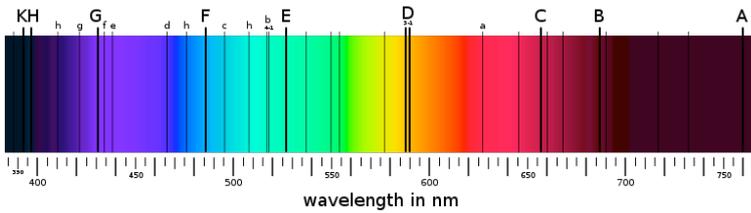


Figure 4: A spectrum of the Sun; the visible dark lines are the Fraunhofer lines. Source: [https://en.wikipedia.org/wiki/Fraunhofer\\_lines](https://en.wikipedia.org/wiki/Fraunhofer_lines).

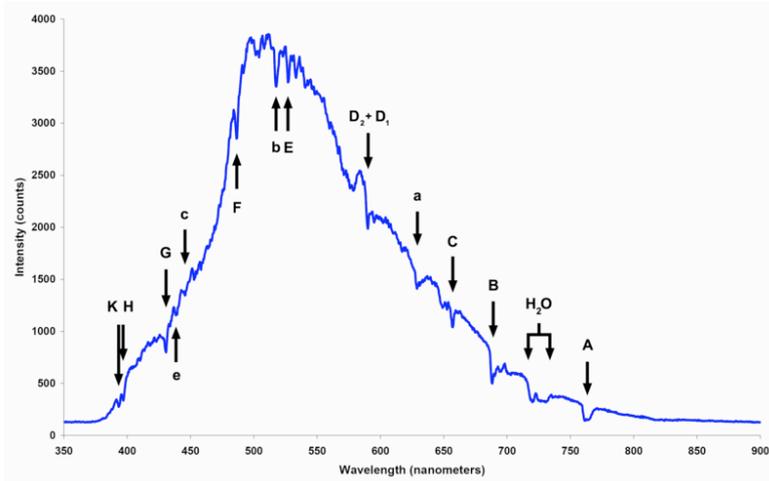


Figure 5: A plot of the intensity of light versus wavelength corresponding to the spectrum shown in Figure 4. The visible dips are the Fraunhofer lines. Source: [https://en.wikipedia.org/wiki/Fraunhofer\\_lines](https://en.wikipedia.org/wiki/Fraunhofer_lines).

where in the last step the temperature is measured in K. This implies that, if we measure the wavelength at which the emission from a star peaks, we immediately learn the star's surface temperature. Even if we don't measure the full spectrum, just measuring the colour of a star by measuring its flux through a set of different-coloured filters provides a good estimate of its surface temperature.

The total light output by a blackbody of surface area  $A$  is

$$L = A\sigma T^4 = 4\pi R^2\sigma T^4, \quad (6)$$

where the second step is for a sphere of radius  $R$ . This means that we measure  $L$  and  $T$  for a star, we immediately get an estimate of its radius. Unfortunately this is only an estimate, because stars aren't really blackbodies – they don't have well-defined solid surfaces, and they have diffuse atmospheres the process the light that passes through them on the way to us. As a result, the spectrum doesn't look exactly like our blackbody function, and the radius isn't exactly what we infer from  $L$  and  $T$ .

### III. Spectra and spectral classification

Temperature is only the most crude description possible of stars' colours. We can get much more information by considering stars' full spectra. That is because a real stellar spectrum is not just a simple continuous function like a blackbody. Instead, there are all sorts of spiky features. These were first studied by the German physicist Fraunhofer in 1814 in observations of the Sun, and for the Sun they are known as Fraunhofer lines in his honour. They are called lines because when you look at the light spread through a prism, they appear as dark lines superimposed on the bright background.

Each of these lines is associated with a certain element or molecule – they are caused by absorption of the star's light by atoms or molecules in the stellar atmosphere at its surface. As you will have learned in quantum mechanics, every element or molecule has certain energy levels that it can be in. The dark lines correspond to wavelengths of light where the energy of photons at that wavelength matches the difference in energy between

two energy levels in some atom or molecule in the stellar atmosphere. Those photons are strongly absorbed by those atoms or molecules, leading to a drop in the light we see coming out of the star at those wavelengths.

Although this is not the case for the Sun, in some stars there are strong emission lines as well as absorption lines. Emission lines are like absorption lines in reverse: they are upward spikes in the spectrum, where there is much more light at a given frequency than you would get from a blackbody. Emission lines appear when there is an excess of a certain species of atoms and molecules in the stellar atmosphere that are in excited quantum states. As these excited states decay, they emit extra light at certain wavelengths. We can figure out what lines are caused by which atoms and molecules using laboratory experiments on Earth, and as a result tens of thousands of spectral lines that appear in stars have been definitively assigned to the species that produces them.

Stellar spectra show certain characteristic patterns, which lead astronomers to do what they always do: when confronted with something you don't understand, classify it! The modern spectral classification system, formally codified by Annie Jump Cannon in 1901, recognises 7 classes for stars: O, B, A, F, G, K, M. This unfortunate nomenclature is a historical accident, but it has led to a useful mnemonic: **Oh Be A Fine Girl/guy, Kiss Me**. Each of these classes is subdivided into ten sub-classes from 0 – 9 – a B0 star is very close to an A9, an A0 is close to an F9, etc. Examples are shown in [Figure 6](#).

In the 1920s, [Cecilia Payne-Gaposchkin](#) showed that these spectra correlate with surface temperature, so the spectral classes correspond to different ranges of surface temperature. O is the hottest, and M is the coolest. Today we know that both surface temperature and spectrum are mostly determined by stellar mass, as we'll discuss later in the course. Thus the spectral classes correspond to different stellar masses – O stars are the most massive, while M stars are the least massive. O stars are also the largest. The Sun is a G star.

In modern times observations have gotten better, and we can now see objects too dim and cool to be stars. These are called brown dwarfs, and two new spectral types have been added to cover them. These are called L and T, leading to the extended mnemonic **Oh Be A Fine Girl/guy, Kiss Me Like That**, which proves one thing – astronomers have way too much time on their hands.

Cecilia Payne-Gaposchkin's work in the 1920s did not just demonstrate that spectral classes correlated with surface temperatures. More importantly, she was able to use the spectra of stars to deduce their compositions. We will recreate her calculation in our tutorial.<sup>1</sup>

#### IV. The HR Diagram

Given the ability to measure stars' temperatures / spectral classes and luminosities, the natural thing to do is to plot them against each other. When one does this, the result is remarkable. Such a plot was first made by two astronomers, Ejnar Hertzsprung in

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<sup>1</sup>For those interested in history of science, it is worth learning more about Payne-Gaposchkin. She was not just an incredible scientist, but a pioneer for women in science. She completed the work for a degree in astronomy at Cambridge, but was not awarded a degree because Cambridge at the time did not grant degrees to women. Realising that she could have a future in science only outside the UK, she moved to the United States in 1923, and was the first person to be awarded a PhD in astronomy from Radcliffe, Harvard's sister college. Her PhD thesis was called by Otto Struve, one of the leading astronomers of the day, "undoubtedly the most brilliant PhD thesis ever written in astronomy". In it, she showed for the first time that the Sun is mostly made of hydrogen. She went on to become the first female full professor at Harvard.

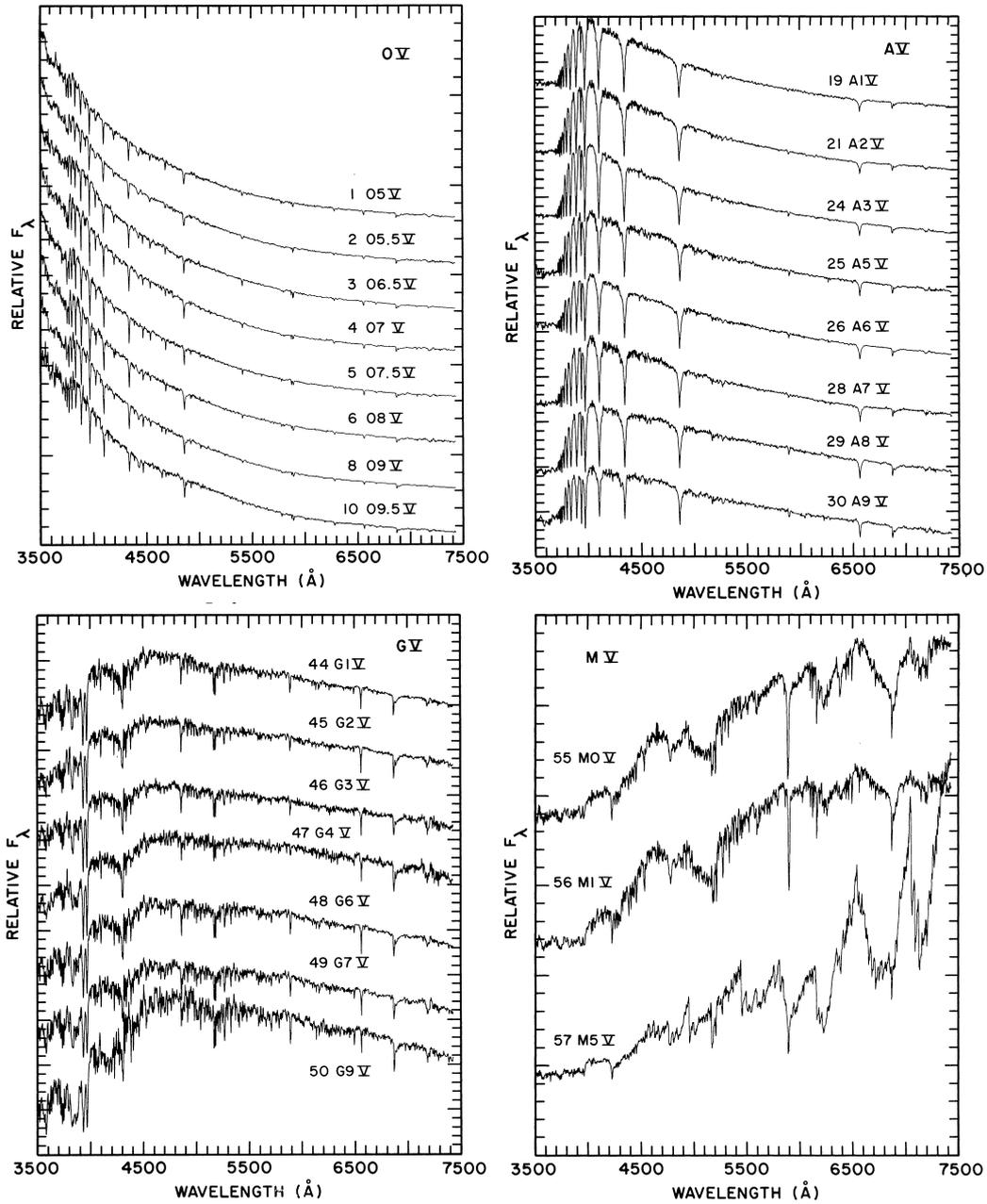


Figure 6: Sample spectra for stars of spectral type O, A, G, and M. Source: Jacoby, Hunter, & Christian, *The Astrophysical Journal Supplement*, 56, 257, 1984, <http://adsabs.harvard.edu/abs/1984ApJS...56..257J>.

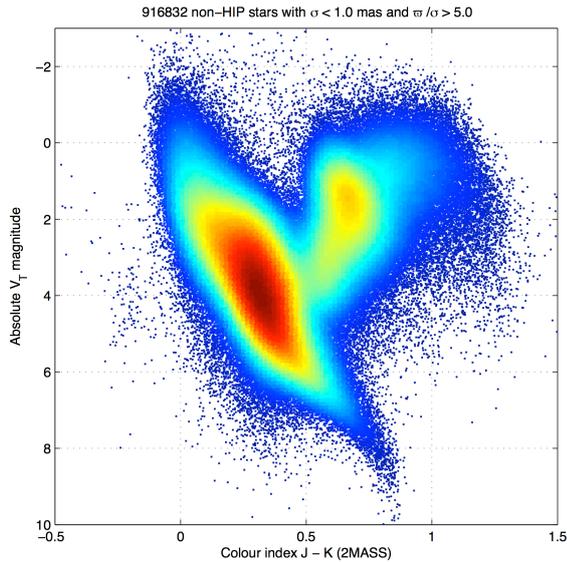


Figure 7: The HR diagram created using 1 million stars from the Gaia catalog; colour indicates the number of stars that fall into a particular pixel. The horizontal axis shows the difference in brightness between two filters, known as J and K, while the vertical axis shows the absolute luminosity measured in a V filter. Source: <http://sci.esa.int/gaia/56388-gaia-first-hertzsprung-russell-diagram/>.

Denmark and Henry Norris Russell in US, from 1911 - 1913. This plot is known as the Hertzsprung-Russell diagram, or HR diagram, in their honour.

#### A. The Main Sequence and Red Giant Branch

The HR diagram is an extremely simple plot. We simply find a bunch of stars, measure their luminosities and surface temperatures, and make a scatter plot of one against the other. Since surface temperatures and absolute luminosities are often expensive to measure in practice, more often we plot close proxies to them. In place of luminosity we put total luminosity (or magnitude) as seen in some particular range of wavelengths, and in place of surface temperature we plot the ratio of the brightness seen through two different filters – this is a proxy for colour, and thus for surface temperature. For this reason, we sometimes also refer to these diagrams as colour-magnitude diagrams, or CMDs for short.

An important point to make is that it is only possible to make an HR diagram for stars whose distances are known, since otherwise we don't have a way of measuring their luminosities. This is why the first HR diagram was not made until the early 20th century; there needed to be measurements of distances to enough stars to make it possible. The largest collection of stars ever placed on an HR diagram today comes from the Gaia catalog, using 1 million stars whose distances have been measured by parallax. This diagram is shown in [Figure 7](#).

A note on interpreting [Figure 7](#): the  $y$  axis in this plot is absolute visual magnitude, where visual here means as observed through a filter that approximates the sensitivity of the human eye. Note that, since the magnitude scale is backward and higher magnitude corresponds to lower luminosity, the scale is reversed – magnitude decreases upward, so that brighter stars are near the top, as you would intuitively expect.

On the  $x$  axis is J magnitude minus K magnitude; J is a bluer filter (i.e., closer to the visible part of the spectrum) than K. Since magnitudes are a logarithmic scale,  $J - K$  is a measure of the ratio of the star's luminosity in the two bands. Since magnitudes go in the opposite of the sensible direction (bigger numbers are dimmer), a high value of  $J - K$  corresponds to a small ratio of bluer to redder light.

A low value of  $J - K$  is the opposite. Thus moving to the right on this diagram corresponds to getting redder, and moving left corresponds to getting bluer. The value of  $J - K$  is our proxy for temperature.

The first thing to notice about the diagram is that the stars do not fall anything like randomly on it. The great majority of them fall along a single fat line from bright and blue to dim and red, which we call the main sequence. The Sun sits right in the middle of it. The main sequence extends from stars that are bright and blue to stars that are dim and red. It covers an enormous range of absolute luminosities – recall that 5 magnitudes is a factor of 100 in brightness, so this diagram spans something like a factor of  $10^5$  in total brightness.

We can also see a second prominent population, extending like a branch off the main sequence, that is bright and red. For reasons we will see in a moment, this means they must have very large radii, and so they are called red giants.

It's worth stopping for a moment to realise that the HR diagram is rather surprising. Why should it be that stars do not occupy the full range of luminosities and temperatures continuously, and instead seem to cluster into distinct groups? Along the main sequence, why should stars fall on a single curve such that luminosity tells you colour, and vice-versa. Explaining the existence of these distinct groups where stars live, their shape in the HR diagram, and their relationship to one another, is the single big theoretical problem from stellar physics. Our goal at the end of this class is to understand why the HR diagram looks like this, and what it means.

## B. The White Dwarf Sequence

The Gaia sample mostly contains main sequence stars and red giants, because these are the most common in a sample that is limited by brightness. However, there are other types of stars that can be added too, if we use a more extended catalog that includes some dimmer objects. [Figure 8](#) shows an HR diagram containing about 20,000 stars from Hipparcos, plus about 1,000 stars from the Gliese catalog chosen to give a wider range of stellar types.

As before, we see that the most prominent feature is the main sequence, and the second most prominent is a branch extending out of it consistent of bright, red stars. However, we can also see some other populations start to emerge. First, there is a collection of stars that run from medium colour to blue, but that are very dim – a factor of  $\sim 1000$  dimmer than the Sun. These too fall along a rough line. As we will see in a moment, the combination of high surface temperature and low luminosity implies that these stars must have very small radii. For this reason, we call them dwarfs. Because these stars have fairly high surface temperatures, their colours are whitish-blue. Thus, these stars are called white dwarfs.

One can also get glimpses of other types of stars in other parts of the diagram, which don't fall on either the main sequence, the red giant branch, or the white dwarf sequence. Almost all of these stars are very bright, and lie above the main sequence. These other types of stars must be very rare, since even a catalog containing 20,000 stars includes only a handful of them. We'll discuss these more exotic types of stars when we get to stellar evolution.

## C. The HR Diagram of Star Clusters

Before moving on, there is one final type of HR diagram that is worth examining. The

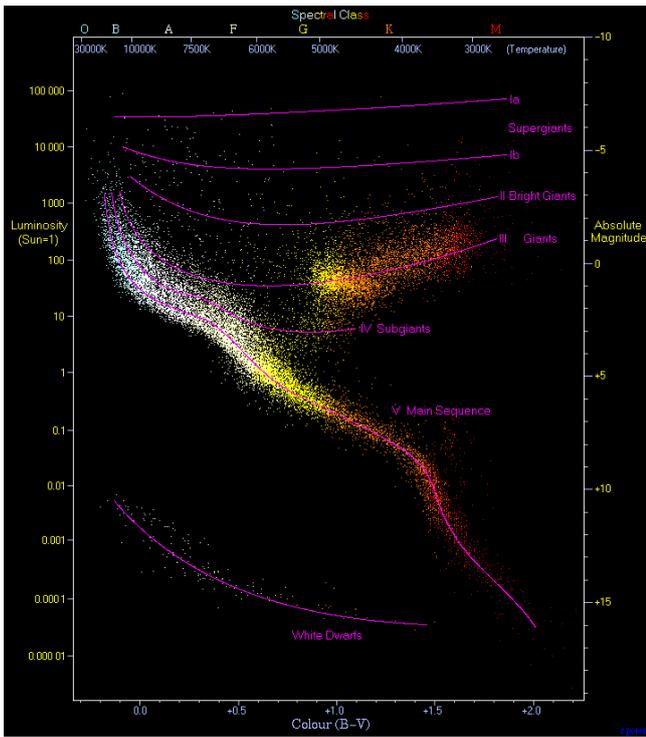


Figure 8: Extended HR diagram, containing 20,000 stars from Hipparcos plus about 1,000 stars from the Gliese catalog. This diagram uses B (blue) and V (visible) as its filters to define the colour, in contrast to Figure 7, but the sense of variation (red is right, blue is left) is the same. Source: <http://spiff.rit.edu/classes/phys440/lectures/lumclass/lumclass.html>.

two we have looked at thus far are HR diagrams for stars in the Solar neighbourhood, which contains a wide range of ages and elemental compositions. However, it is very instructive to instead look at the HR diagram for more homogenous populations. How do we pick a homogenous population? Fortunately, nature has provided for us. Some stars are found in clusters that are held together by the stars mutual gravity. These stars formed together, and are therefore nearly identical in age and chemical composition.

Figure 9 shows the HR diagram for an example star cluster, known as NGC 6397. The axis labels here correspond to the filters used on the Hubble telescope, but the idea is the same as before: the  $y$  axis shows the magnitude in a single filter using a reverse axis scale, so it measures luminosity, with up corresponding to higher luminosity. The  $x$  axis shows the ratio of the luminosities in two different colour filters, with the orientation chosen so that red is to the right and blue is to the left.

As in the previous HR diagrams we have examined, most stars fall along a single curve – the main sequence – and that there is a secondary curve at lower luminosity and bluer colour – the white dwarf sequence. In comparison to the other HR diagram, however, this main sequence is much narrower. Instead of a fat line we have a very thin one. In fact, it’s even thinner than it appears in the diagram – many of the points that lie off the main sequence turn out to be binary stars that are so close to one another that the telescope couldn’t separate them, and thus treated them as a single star.

The thinness of this main sequence suggests that the spread we saw in the solar neighbourhood main sequence must be due to factors that are absent in the star cluster: a spread in stellar ages and a spread in chemical composition. The fact that all the stars fall along a single thin line is compelling evidence that there is a single intrinsic property of stars that varies as we move along the main sequence, and is responsible for determining a star’s luminosity and temperature. The age and chemical composition can alter this slightly, causing the thin sequence to puff up a

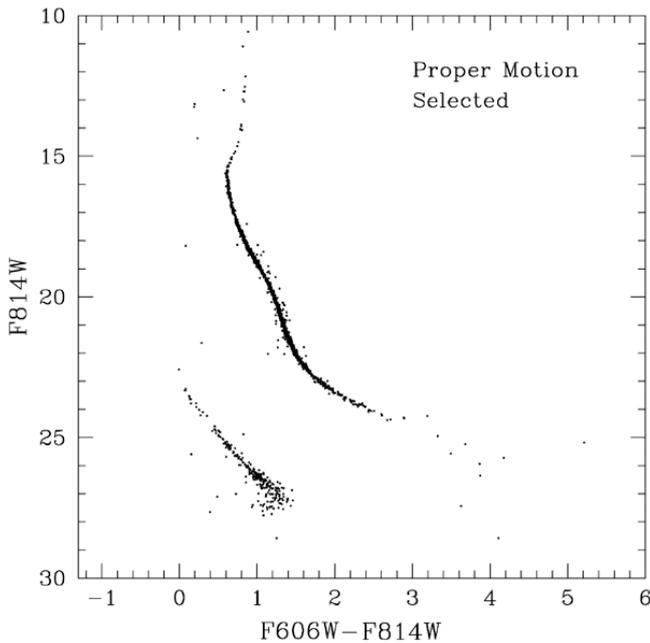


Figure 9: HR Diagram for the globular cluster NGC 6397, measured using two filters from the Hubble Space Telescope. Source: Richer et al., *The Astronomical Journal*, 135, 2141, 2008, <http://adsabs.harvard.edu/abs/2008AJ....135.2141R>.

little, but basically there’s one number that is going to determine everything about a star’s properties. The natural candidate, of course, is the star’s mass.

#### D. Stellar Radii in the HR Diagram

So far we have talked only about quantities we can observe directly – the observational HR diagram. However, from these two we can immediately deduce stars’ radii, at least to the extent that we are willing to approximate them as black bodies. Inverting [Equation 6](#), we have<sup>2</sup>

$$R = \sqrt{\frac{L}{4\pi\sigma T^4}} \quad (7)$$

$$= 1.33R_{\odot} \left(\frac{L}{L_{\odot}}\right)^{1/2} \left(\frac{T}{5000 \text{ K}}\right)^{-2}. \quad (8)$$

Just from this formula we can figure out what a line of constant radius will look like on the HR diagram. We’ve been plotting things on a logarithmic scale, so our HR diagrams have  $\log L$  on the  $y$  axis and  $\log T$  on the  $x$  axis. If we take the logarithm of this equation, we get

$$\log L = 4 \log T + \log(4\pi\sigma) + 2 \log R. \quad (9)$$

If  $R$  is constant, then in the  $(\log T, \log L)$  plane this is just a line with a slope of 4.

The main sequence in a plot of luminosity versus temperature does have a positive slope on the HR diagram<sup>3</sup>, but somewhat shallower than a slope of 4 in the  $(\log T, \log L)$  plane. This means that stars at the low temperature, low luminosity end have smaller radii than stars at the high luminosity, high temperature end. The range in radii is a factor of  $\sim 100$ .

<sup>2</sup>A quick notation note: anything with a subscript  $\odot$  refers to the Sun. Thus  $M_{\odot} = 1.99 \times 10^{33} \text{ g}$  is the mass of the Sun,  $L_{\odot} = 3.84 \times 10^{33} \text{ erg s}^{-1}$  is the luminosity of the Sun, and  $R_{\odot} = 6.96 \times 10^{10} \text{ cm}$  is the radius of the Sun. These are convenient units of measure for stars, and we’ll use them throughout the class.

<sup>3</sup>Note another annoying astronomy convention: we plot redder colours on the right, and thus lower temperatures on the right. Because the temperature axis is reversed, lines with positive slope go from upper left to lower right, the reverse of the normal convention.

This argument also immediately explains why red giants and white dwarfs have the names they do. Red giants are brighter than most main sequence stars, yet have redder colours. This is possible only if they have much larger radii than most main sequence stars – up to several hundred times the radius of the Sun. Conversely, white dwarfs have blue to white colours, indicating temperatures comparable to main sequence stars, but have much lower luminosities. This implies that they must have very small radii, typically about a percent of the radius of the Sun. To put these numbers in perspective,  $100R_{\odot} = 0.47 \text{ AU}$ , so a star with a radius just above  $200R_{\odot}$  would encompass the Earth's orbit. A radius of  $0.01 R_{\odot}$  corresponds to 10% more than the radius of the Earth. Thus the largest red giants would swallow the entire Solar System out to the orbit of the Earth, while the smallest white dwarfs are about the size of the Earth.