Supplementary Discussion

In this Supplementary Discussion, we give more detailed derivations of several of the results in our letter.

First, we describe in detail our method of calculating the temperature structure of our clouds and determining when they are dominated by internal illumination. As discussed in the main text, we consider a cloud of mass M, column density Σ, radius $R = \sqrt{M/(\pi\Sigma)}$, and density profile $\rho \propto r^{-k_\rho}$. The gas in the cloud is mixed with dust, which provides an opacity per unit mass $\kappa = \delta\kappa_0(\lambda_0/\lambda)^\beta$, where λ is the radiation wavelength, $\lambda_0 = 100$ μm, $\kappa_0 = 0.54$ cm^2 g^{-1}, $\beta = 2$, and the choice $\delta = 1$ corresponds to typical properties in cold gas in the Milky Way, where ice mantles on dust grains double the opacity compared to that in the diffuse interstellar medium[17]. For convenience we define $T_0 = hc/(k_B\lambda_0) = 144$ K. There is a source of luminosity L located at the cloud center, and we define $\eta = L/M$.

To compute the cloud temperature, we first estimate the dust temperature using the method of ref. [13], who show that the dust temperature profile in a cloud of this sort takes an approximately powerlaw form:

$$T = T_{\rm ch}\left(\frac{r}{R_{\rm ch}}\right)^{-k_{\rm T}}. \qquad (1)$$

The characteristic emission radius $R_{\rm ch}$ and emission temperature $T_{\rm ch}$ are defined by two conditions: the total luminosity must be $L = 4\pi\tilde{L}R_{\rm ch}^2\sigma_{\rm SB}T_{\rm ch}^4$, where $\tilde{L}$ is a constant of order unity to be determined, and the optical depth from the cloud edge to $R_{\rm ch}$ must be unity for radiation whose wavelength is $\lambda = hc/(k_\mathrm{B} T_\mathrm{ch})$. Together these conditions imply

$$\tilde{R} \equiv \frac{R}{R_\mathrm{ch}} = \left\{ \left(\frac{\eta}{4\sigma_\mathrm{SB}\tilde{L}} \right)^{\beta} \Sigma^{4+\beta} \left[\frac{(3-k_\rho)\kappa_0}{4(k_\rho - 1)T_0^\beta} \right]^4 \right\}^{-\alpha} \quad (2)$$

$$T_\mathrm{ch} = \left\{ \left(\frac{\eta}{4\sigma_\mathrm{SB}\tilde{L}} \right)^{k_\rho - 1} \Sigma^{k_\rho - 3} \left[\frac{4(k_\rho - 1)T_0^\beta}{(3-k_\rho)\kappa_0} \right]^2 \right\}^{\alpha}, \quad (3)$$

where $\alpha \equiv 1/[2\beta + 4(k_\rho - 1)]$. The approximation

$$\tilde{L} \approx 1.6 \tilde{R}^{0.1} \quad (4)$$

reproduces numerical solutions of the transfer equation very accurately, so we adopt it. The temperature structure within the cloud takes an approximately powerlaw form, $T = T_\mathrm{ch}(r/R)^{-k_\mathrm{T}}$, with k_T well-fit by the approximation

$$k_\mathrm{T} \approx \frac{0.48 k_\rho^{0.05}}{\tilde{R}^{0.02 k_\rho^{1.09}}} + \frac{0.1 k_\rho^{5.5}}{\tilde{R}^{0.7 k_\rho^{1.9}}}. \quad (5)$$

Note that the approximations in ref. [13] break down and the equations become singular as $k_\rho \to 1$, but numerical solutions of the transfer equation show that the temperature profiles for $k_\rho = 1$ and $k_\rho = 1.1$ are nearly identical. Thus, we handle the case $k_\rho < 1.1$ by approximating it with $k_\rho = 1.1$. Also note that equation (4) is a slightly different approximation than that given in ref. [13]. This approximation provides a somewhat better fit than the approximation given there (S. Chakrabarti, private communication, 2007).

If the dust and gas temperatures are equal, it is straightforward to solve equation (1) in the main text, so we proceed by assuming that they are equal and then check that assumption. Equations (2) - (4) constitute three equations in the unknowns R_ch, T_ch, and $\tilde{L}$, which we may solve for specified values of Σ, η, and k_ρ. Together with equation (5), this fully specifies the dust temperature profile in the cloud. To solve equation (1) in the main text for $\eta_{\rm halt}$, we simply fix δ and Σ, and iterate to obtain the value of η that satisfies the equation.

Our second calculation in the Supplementary Discussion is a check of the assumption that the dust and gas temperatures are equal using both a simple analytic estimate and using a detailed numerical calculation of dust-gas energy exchange. We perform this check for the threshold column densities $\Sigma_{\rm th}$ that we obtain under the assumption that the dust and gas temperatures are equal. In the vicinity of an illuminating source that heats the dust, the gas temperature will be determined by a competition between the dominant heating process, collisions with warm dust grains, and the dominant cooling process, molecular cooling. The former will be least effective and the latter most effective at the low density edge of a cloud, so we focus our attention at cloud edges, where the temperatures are near $T_{\rm b}$ and the densities are lowest. The volumetric gas heating rate by dust collisions is approximately[15]

$$\Gamma_{\rm gd} = 9.0 \times 10^{-34} n_{\rm H}^2 T_{\rm g}^{0.5} \sigma_{\rm d0} \left[1 - 0.8 \exp\left(-\frac{75\ {\rm K}}{T_{\rm g}}\right)\right] (T_{\rm d} - T_{\rm g})\ {\rm erg\ cm^{-3}\ s^{-1}}, \tag{6}$$

where $n_{\rm H}$ is the number density of hydrogen nuclei, $T_{\rm g}$ is the gas temperature, $T_{\rm d}$ is the dust temperature, and $\sigma_{\rm d0}$ is the average dust cross section per baryon, normalized to the fiducial Milky Way value, 6.09×10^{-22} cm^{-2}. We make the simple assumption that the dust-grain cross section varies with metallicity in the same way as the total dust opacity, so we adopt $\sigma_{\rm d0} = \delta$.

The dominant dust cooling process is molecular line emission, with CO dominating at lower densities and optical depths and other species taking over at higher densities. The exact cooling rate is a very complex function of density, temperature, and optical depth,

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but numerical radiative transfer and molecular excitation calculations[14] show that, for a gas temperature near $T_g = 10$ K, the total cooling rate has a roughly constant value of

$$\Lambda_{\rm mol} \approx 10^{-27} \delta n_{\rm H} \text{ erg cm}^{-3} \text{ s}^{-1} \tag{7}$$

for $n_{\rm H}$ in the range $10^3 - 10^7$ cm^{-3}. We have again taken the molecular cooling rate to be simply proportional to the metallicity. Adopting this cooling rate, and solving for the gas energy balance by setting $\Gamma_{\rm gd} = \Lambda_{\rm mol}$, we find

$$T_{\rm d} - T_{\rm g} \approx \frac{3.5 \times 10^5}{n_{\rm H}} \text{ K}. \tag{8}$$

Thus, the gas and dust temperature will be identical to within 1 K for densities $n_{\rm H} \gtrsim 3.5 \times 10^5$ cm^{-3}, roughly independent of δ.

We can now compare this to the minimum densities found in gas clouds at the critical column density. For a cloud of mass M and column density Σ, the number density at the cloud edge is

$$n_{\rm H} = \frac{3 - k_\rho}{4\mu} \sqrt{\frac{\pi \Sigma^3}{M}}, \tag{9}$$

where $\mu = 2.34 \times 10^{-24}$ g is the mean mass per H nucleus for a gas of the standard cosmic abundance. For stars in the mass range $10 - 200$ $M_\odot$, for the threshold column densities $\Sigma_{\rm th}$ shown in Figure 3 of the main text, the minimum values of $n_{\rm H}$ are 7.9×10^5, 3.4×10^5, and 3.8×10^6 cm^{-3} for the cases $\delta = 1$ and $T_{\rm b} = 10$ K, $\delta = 0.25$ and $T_{\rm b} = 10$ K, and $\delta = 0.25$ and $T_{\rm b} = 15$ K, respectively. These values suggest that the cloud volume densities are high enough for our threshold cases that our assumption of dust-gas temperature equality is well-justified, although the case $\delta = 0.25$, $T_{\rm b} = 10$ K is perhaps marginal.

We further check this approximation by computing the gas temperature using a detailed gas-temperature calculation code[15,16]. The code takes as input fixed spherically-symmetric density and dust temperature profiles, and computes the resulting gas temperature profile. It includes grain-gas energy exchange, cosmic ray heating, and line cooling from a large number of species, including an approximate treatment of radiative trapping effects in the Sobolev approximation. In the code, we set the cosmic ray ionization rate to its Milky Way value, we set the grain-gas energy exchange rate to the value given by equation (6), and we scale the tabulated molecular cooling rate by δ under the assumption that the molecular cooling rate is simply proportional to metallicity. We also assume that our clouds are embedded in an environment where the column density is 10% of Σ_{th}.

Supplementary Figure 1 shows the cloud temperature profiles and the fractional difference between the dust and gas temperatures for the cases $\delta = 1$ and $T_{\mathrm{b}} = 10$ K, $\delta = 0.25$ and $T_{\mathrm{b}} = 10$ K, and $\delta = 0.25$ and $T_{\mathrm{b}} = 15$ K. The cases shown are for clouds with mass $M = 200$ $M_{\odot}$, which we approximate will produce 100 $M_{\odot}$ stars if they do not fragment[27], but different cloud masses do not produce qualitatively different results. As the plots show, the dust and gas temperatures agree to within a few percent even at the cloud edges. Thus, our assumption of dust-gas temperature equality is well-founded. In fact, the detailed calculation shows that our analytic approximation overestimates the grain-gas temperature difference. This is likely because in our analytic estimate we neglected cosmic ray heating, which becomes comparable to the assumed cooling rate Λ_{mol} when the temperature is near T_{b}, and because our analytic estimate of Λ_{mol} is based on calculations for somewhat lower column density regions than the ones we are considering, and the cooling rate is higher at lower column density due to decreased line opacity.

The third and final calculation we present in the Supplementary Discussion is a derivation of the estimated total rate of star formation in our centrally condensed cloud. We base our estimate on a fit to the star formation rate in simulations of driven turbulent motion in periodic boxes[22]

$$\mathrm{SFR}_{\mathrm{ff-uniform}} \approx 0.073\alpha_{\mathrm{vir}}^{-0.68}\mathcal{M}^{-0.32}, \tag{10}$$

where α_{vir} is the cloud virial ratio and $\mathcal{M}$ is its one-dimensional Mach number on scales comparable to the size of the entire cloud. The definition of the virial ratio is

$$\alpha_{\mathrm{vir}} = \frac{5\mathcal{M}^2 c_s^2 R}{GM}, \tag{11}$$

where c_s is the cloud's isothermal sound speed, which we compute at T_{b}. Following observations, we adopt $\alpha_{\mathrm{vir}} = 1.3$ as typical of star-forming clouds[9], and this implies that

$$\mathcal{M} = \frac{1}{c_s}\left(\frac{\pi\alpha_{\mathrm{vir}}^2 G^2 M\Sigma}{25}\right)^{1/4} = 6.2\left(\frac{M_2\Sigma_0}{T_{\mathrm{b},1}^2}\right)^{1/4}, \tag{12}$$

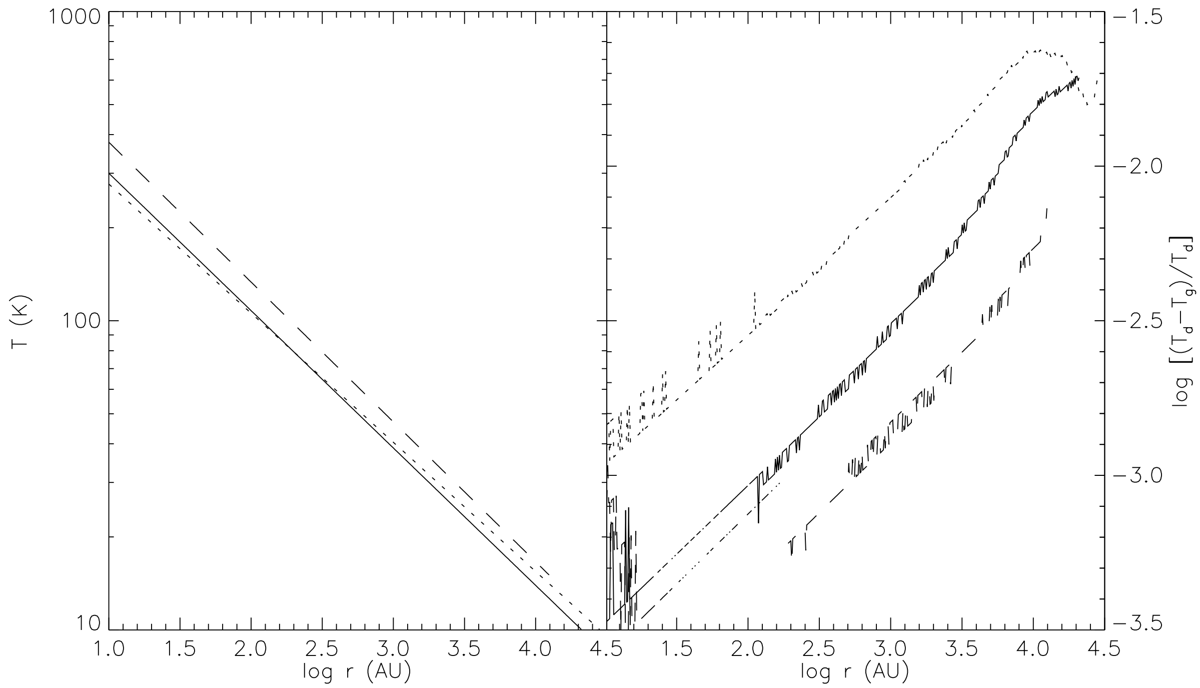
where $M_2 = M/(100\ M_\odot)$, $\Sigma_0 = \Sigma/(1\ \mathrm{g\ cm}^{-2})$, and $T_{\mathrm{b},1} = T_{\mathrm{b}}/(10\ \mathrm{K})$. In the numerical evaluation, we have taken the mean mass per particle of the gas to be $2.33m_{\mathrm{H}}$, appropriate for a gas of the standard cosmic abundance. Using this value of $\mathcal{M}$ in equation (10) gives an estimate of $\mathrm{SFR}_{\mathrm{ff}}$ on the outer scale of the cloud:

$$\mathrm{SFR}_{\mathrm{ff-out}} \approx 0.034\left(\frac{M_2\Sigma_0}{T_{\mathrm{b},1}^2}\right)^{-0.08}. \tag{13}$$

This would be the value of $\mathrm{SFR}_{\mathrm{ff}}$ if the cloud were uniform. For a centrally-concentrated cloud, if one assumes that equation (10) applies locally everywhere within the cloud, the star formation rate is enhanced over $\mathrm{SFR}_{\mathrm{ff-out}}$ by a factor of $(3-k_\rho)^{3/2}/[2.3(2-k_\rho)]$ (ref.

[20]). This enhancement occurs because the mass-averaged free-fall time is higher and the mass-averaged Mach number is lower for a centrally-concentrated cloud than for a uniform one. Applying this factor to $\mathrm{SFR}_{\mathrm{ff-uniform}}$ for $k_\rho = 1$, we arrive at our final estimate for $\mathrm{SFR}_{\mathrm{ff}}$ for the cloud:

$$\mathrm{SFR}_{\mathrm{ff}} \approx 0.041 \left(\frac{M_2 \Sigma_0}{T_{\mathrm{b},1}^2} \right)^{-0.08}. \tag{14}$$



Supplementary Figure 1 Dust temperature and dust-gas temperature difference in a gas cloud. The left panel shows the dust temperature as a function of radius computed for clouds of mass $M = 200\ M_\odot$ with critical column densities, for the cases $\delta = 1$ and $T_\mathrm{b} = 10$ K (*solid lines*), $\delta = 0.25$ and $T_\mathrm{b} = 10$ K (*dotted lines*), and $\delta = 0.25$ and $T_\mathrm{b} = 15$ K (*dashed lines*). We do not show the gas temperature profile separately because the curves so closely overlay one another as to be indistinguishable. In the right panel, we show the fractional difference in dust and gas temperatures $(T_\mathrm{d} - T_\mathrm{g})/T_\mathrm{d}$ as a function of radius.