

## RADIATION FEEDBACK AND FRAGMENTATION IN MASSIVE PROTOSTELLAR CORES

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Received 2006 January 26; accepted 2006 March 1; published 2006 March 20

### ABSTRACT

Star formation generally proceeds inside-out, with overdense regions inside protostellar cores collapsing rapidly, and progressively less dense regions following later. Consequently, a small protostar will form early in the evolution of a core, and collapsing material will fall to the protostellar surface and radiate away its gravitational potential energy. The resulting accretion luminosity will heat the core and may substantially affect the process of fragmentation. This is of particular interest for massive cores that, at their initial temperatures, have masses much greater than a thermal Jeans mass and thus might be expected to fragment into many stars during collapse. Here I show that accretion luminosity can heat the inner parts of a core to above 100 K very early in the star formation process, and that this in turn strongly suppresses fragmentation. This has implications for a number of outstanding problems in star formation, including the mechanism of massive star formation, the origin of the stellar initial mass function and its relationship to the core mass function, the demographics of massive binaries, and the equation of state in star-forming gas.

*Subject headings:* binaries: general — equation of state — ISM: clouds — methods: numerical — radiative transfer — stars: formation

### 1. INTRODUCTION

In the last few years, millimeter and submillimeter observations have reached resolutions sufficient to identify dense molecular condensations with masses  $\sim 100 M_{\odot}$  and sizes  $\sim 0.1$  pc in regions of massive star formation (a recent review is given in Garay 2006). These objects are cold, supersonically turbulent, and centrally condensed. The condensations have a mass distribution that resembles the stellar initial mass function (IMF) (Beuther & Schilke 2004; Reid & Wilson 2005), which naturally leads to the question whether they could collapse to form massive stars, making them the high-mass analogs of the cores from which low-mass stars form.

One objection to this scenario is the possibility of fragmentation. Since the thermal Jeans mass in a cold ( $T \sim 10$  K), dense ( $n_{\text{H}} \sim 10^6 \text{ cm}^{-3}$ ) gas is  $\sim 1 M_{\odot}$ , why should a core of mass  $\sim 100 M_{\odot}$  collapse to form a single massive object (or a small multiple system), rather than many smaller objects? Bate & Bonnell (2005) argue based on simulations that denser regions produce smaller fragments, since the Jeans mass decreases with density at fixed temperature (but see Martel et al. 2006). Dobbs et al. (2005) simulate the collapse of a massive turbulent core and find that it fragments to as many as 20 and as few as two objects, depending on the assumed equation of state. If fragmentation of massive cores to many objects were common, there would be no direct mapping between the core and stellar mass distributions; their agreement would simply be a coincidence. If, on the other hand, massive cores typically collapse to one or a few objects, then one can explain the IMF in terms of the core mass function. (Fragmentation to two or three objects does not pose a problem in mapping from core to star masses, because the massive star IMF is uncorrected for multiplicity.)

Almost all published work on fragmentation of massive cores treats the gas as either isothermal or barotropic. The barotropic approximation is based on calculations showing that radiative cooling keeps gas isothermal until it reaches a density of  $\sim 10^{-14} \text{ g cm}^{-3}$ , at which point it makes a transition to adiabatic

(Larson 1969; Masunaga et al. 1998; Masunaga & Inutsuka 2000). However, this approach assumes that gas is heated solely by compression. While this is probably reasonable far from point sources of radiation, it is likely to fail once a protostar forms, since heating of the gas by irradiation from the luminous, accreting central object vastly exceeds heating due to gas compression. Indeed, in detailed one-dimensional radiation-hydrodynamic calculations of the evolution of a  $1 M_{\odot}$  core, Masunaga & Inutsuka (2000) find that accretion luminosity can heat gas to over 100 K out to hundreds of AU. Matzner & Levin (2005) find that heating due to accretion luminosity is sufficient to prevent fragmentation of low-mass protostellar disks into brown dwarfs.

Massive star-forming regions have high densities that produce accretion rates that are orders of magnitude larger than in low-mass regions, and they are optically thick even in the mid-infrared. Thus, heating by accretion luminosity within them is likely to have even more profound effects. In this Letter, I test the assumption that massive cores can be described with a barotropic equation of state and explore how a more realistic treatment of gas thermodynamics changes the picture of fragmentation in these objects. In § 2, I develop a simple analytic model for the temperature distribution in massive cores, and in § 3, I use the results to estimate how fragmentation is likely to be affected. Finally, in § 4, I discuss some of the broader implications of my findings.

### 2. THE TEMPERATURE STRUCTURE IN MASSIVE CORES

Since the focus of this Letter is fragmentation, I concentrate on cores at early times when the gas mass greatly exceeds the stellar mass and accretion is the dominant source of luminosity. To make the calculations definite, I use the turbulent-core model proposed by McKee & Tan (2003, hereafter MT03), in which the density distribution within a core is a power law,  $\rho \propto r^{-k_p}$ , and both MT03 and I adopt  $k_p = 1.5$  as a fiducial value. In this model a choice of core mass  $m_c$  and mean column density  $\Sigma_c$  specifies the outer radius of the core,  $r_o = [m_c / (\pi \Sigma_c)]^{1/2}$ , and hence the density profile. Requiring hydrostatic balance gives the effective sound speed versus radius,  $\sigma = \sigma_o (r/r_o)^{1-k_p/2}$  with

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$\sigma_o = [2(k_p - 1)]^{-1/2}(Gm_c/r_o)^{1/2}$ . Within the core is an embedded protostar of mass  $m_*$ . This model neglects turbulent structure within a core, the effects of which I discuss in more detail in § 3.

One can estimate the accretion rate in a massive core from equation (41) of MT03:

$$\dot{m}_* = 4.6 \times 10^{-4} \left( \frac{m_c}{60 M_\odot} \right)^{3/4} \Sigma_c^{3/4} \left( 2 \frac{m_*}{m_c} \right)^{1/2} M_\odot \text{ yr}^{-1}. \quad (1)$$

(This calculation assumes that half the mass reaching the star is blown away in an outflow.) For a protostar in the mass range  $0.01$ – $1 M_\odot$ , this gives a typical accretion rate of roughly  $10^{-5}$  to  $10^{-4} M_\odot \text{ yr}^{-1}$  for a core with  $m_c = 50 M_\odot$  and  $\Sigma_c = 1.0 \text{ g cm}^{-2}$ . Though I compute  $\dot{m}_*$  from MT03, one can obtain similar values by simple order-of-magnitude arguments. For example, Beuther et al. (2005) observe a velocity dispersion  $\sigma = 2.0 \text{ km s}^{-1}$  in IRDC 18223-3 Main, giving an expected accretion rate  $\dot{m}_* \sim \sigma^3/G \approx 10^{-3} M_\odot \text{ yr}^{-1}$ . Reid & Wilson (2005) find cores with typical values  $m_c \sim 100 M_\odot$  and  $r_o \sim 0.25 \text{ pc}$ . At the time when the forming protostar has mass  $m_*$ , the accretion rate should be  $\dot{m}_* \sim m_*/t_{\text{ff}}(m_*) \approx 10^{-5} M_\odot \text{ yr}^{-1}$ , where  $t_{\text{ff}}(m_*)$  is the free-fall time at the edge of the central region in the core that contains  $m_*$  of gas and the evaluation assumes  $k_p = 1.5$ . Simulations obtain comparable values. Dobbs et al. (2005) find accretion rates onto single objects of  $10^{-5}$  to  $10^{-4} M_\odot \text{ yr}^{-1}$  for protostars of mass  $m_* < 1 M_\odot$  in a core with  $m_c = 30 M_\odot$  and  $\Sigma_c = 0.6 \text{ g cm}^{-2}$ .

To compute the accretion luminosity, one must know the protostellar radius. Initially collapsing gas forms a pressure-supported “first core” a few AU in size, but this becomes unstable and collapses to stellar sizes once its mass reaches roughly  $0.05 M_\odot$  (Masunaga et al. 1998; Masunaga & Inutsuka 2000). Thereafter one can compute the protostellar radius using the simple evolution model of MT03, which agrees well with the more detailed numerical calculations of Palla & Stahler (1992). The accretion luminosity is  $L_{\text{acc}} = f_{\text{acc}} m_* \dot{m}_*/r_*$ , where  $f_{\text{acc}} \approx 0.5$  is the fraction of the accretion energy that goes into radiation rather than driving a wind. For typical massive star-forming regions, this gives  $L_{\text{acc}} \sim 10$ – $100 L_\odot$ .

Given a central luminosity and a density structure, one could numerically determine the temperature structure (see, e.g., Ivezić & Elitzur 1997; Whitney et al. 2003). However, for simplicity I instead use the analytic approximation given by Chakrabarti & McKee (2005), which agrees well with numerical calculations. I will not review the full Chakrabarti-McKee formalism, but the central idea is that the majority of the energy radiated away at a given frequency comes from a thin radial shell whose position is determined by a competition between opacity, which suppresses emission from small radii, and decreasing temperature, which reduces emission from large radii. Once one knows the characteristic emission radius versus frequency, one can estimate the temperature profile, which is roughly a power law  $T = T_{\text{ch}}(r/R_{\text{ch}})^{-k_T}$ . For a given model, I compute  $R_{\text{ch}}$ ,  $T_{\text{ch}}$ , and  $k_T$  from Chakrabarti & McKee’s equations (6), (7), and (41), using the opacity law of Weingartner & Draine (2001). Since I am concerned with early times when the protostar has accreted a negligible fraction of the cloud, I assume that the core density profile is unchanged from its initial MT03 form.

I plot the temperature profile for a selection of parameters in Figure 1. The gas is clearly quite warm, but the true temperature is likely even hotter. The Chakrabarti & McKee (2005)

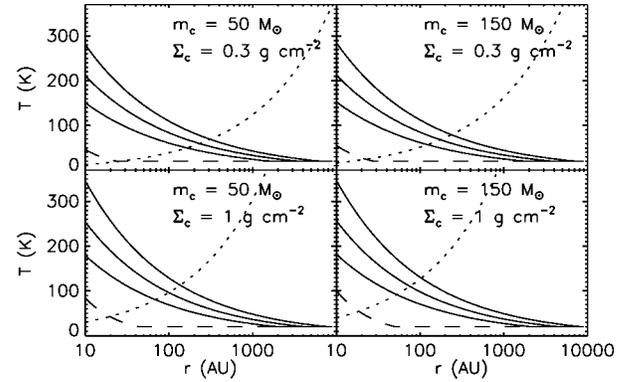


FIG. 1.—Temperature vs. radius for models with the values of  $m_c$  and  $\Sigma_c$  indicated in the panels. Temperatures shown are for protostellar masses of  $m_* = 0.8 M_\odot$ ,  $m_* = 0.2 M_\odot$ , and  $m_* = 0.05 M_\odot$  (solid lines, top to bottom) and for the Dobbs et al. (2005) barotropic equation of state (dashed lines). I also show the effective temperature set by the turbulent velocity dispersion,  $T_{\text{turb}} = 2.33 m_* \sigma^2 / k_B$  (dotted lines).

approximation underestimates the temperature at very small radii, I have neglected compressional heating, and I have assumed that all the energy that goes into driving a wind escapes from the core rather than being radiated away by shocks within it. Figure 1 also shows the temperature set by the barotropic equation of state used by Dobbs et al. (2005),  $T = 20 \text{ K}$  for  $\rho < 10^{-14} \text{ g cm}^{-3}$ ,  $T = 431 \text{ K}$  for  $\rho > 10^{-12} \text{ g cm}^{-3}$ , and  $T \propto \rho^{2/3}$  in between, which is similar to those used in other simulations (e.g., Bonnell et al. 2003, 2004; Bonnell & Bate 2005; Bate & Bonnell 2005). As the figure shows, the barotropic approximation severely underestimates the temperature. One can understand this intuitively: the barotropic approximation says that at some critical density, gas becomes adiabatic and ceases radiating away the energy it gains through compression. However, since potential energy varies as  $r^{-1}$ , the vast majority of the energy released by a collapse comes out in the final plunge onto the stellar surface and is then radiatively transferred to the rest of the gas. Even when the mass accreted onto a star is a small fraction of the core mass, the energy released by its fall to very small radii can dominate the total gravitational potential energy released. By including only the energy released by the fall from  $r_o$  to  $\sim 10 \text{ AU}$ , roughly the numerical resolution of most simulations, and not the energy released from  $\sim 10 \text{ AU}$  to  $\sim 1 R_\odot$ , the barotropic approximation misses the vast majority of the energy of collapse.

To validate my use of the equilibrium temperature profile, I also estimate the time required to reach radiative equilibrium. If  $E_{\text{diff}}$  is the difference in gas thermal energy between a core whose temperature is as shown in Figure 1 and a core with a constant temperature  $20 \text{ K}$ , the time to reach radiative equilibrium is roughly  $t_{\text{rad}} = E_{\text{diff}}/L_{\text{acc}}$ . This estimate neglects radiative cooling, but since the cooling rate varies as  $T^4$  in optically thick gas, cooling is negligible compared with heating until the gas is very close to equilibrium. For the parameter range  $m_* = 0.05$ – $1.0 M_\odot$ ,  $m_c = 50$ – $150 M_\odot$ , and  $\Sigma_c = 0.3$ – $1.0 \text{ g cm}^{-2}$ ,  $t_{\text{rad}} < 10 \text{ yr}$ . This is shorter than the free-fall time everywhere except in the central few tenths of an AU, so the assumption of radiative equilibrium is justified.

### 3. FRAGMENTATION IN MASSIVE CORES

The high temperatures produced by accretion luminosity will change how fragmentation proceeds. A cold centrally condensed core will undergo global collapse, but local inhom-

genities produced by turbulence will also be gravitationally amplified, producing a collapsing cluster of small stars, as seen in Dobbs et al. (2005), rather than monolithic collapse to a single star. Heating prevents the growth of small perturbations, thereby both reducing the number of perturbations that grow and increasing the mean mass of those that can. To study this effect, I compute the Jeans length  $\lambda_j = [\pi c_s^2 / (G\rho)]^{1/2}$  as a function of radius  $r$  in my model cores. (I take the mean particle mass to be  $2.33m_H$ .) At radii where  $\lambda_j > 2r$ , no perturbations that fit within the sphere of radius  $r$  are Jeans unstable and no fragmentation can occur. The radius where  $\lambda_j = 2r$  defines a length scale  $r_{\text{frag}}$  and mass scale  $m_{\text{frag}}$  below which there can be no fragmentation. I compute the Jeans length using the thermal sound speed rather than the effective sound speed set by turbulence because both simulations and theory show that the typical fragment mass in a turbulent medium is the thermal Jeans mass rather than the “turbulent” Jeans mass (Padoan & Nordlund 2002; Mac Low & Klessen 2004; references therein). Physically, this occurs because fragment formation in a turbulent medium tends to occur at stagnation points that are relatively nonturbulent.

Figure 2 shows  $r_{\text{frag}}$  and  $m_{\text{frag}}$  for a range of parameter choices. For a barotropic equation of state, there is no heating, so the mass scale is constant. For  $m_c = 30 M_\odot$  and  $\Sigma = 0.6 \text{ g cm}^{-2}$ , the parameters used by Dobbs et al. (2005),  $m_{\text{frag}} = 0.3 M_\odot$ , which roughly agrees with the typical fragment mass in the simulations. However, when one includes radiation, the resulting high temperatures suppress fragmentation to objects of mass  $\lesssim 1 M_\odot$  over length scales  $\gtrsim 1000$  AU from the moment that the first core collapses and a protostar appears. The barotropic approximation underestimates fragment masses by factors as large as 10, and it switches the problem from the regime where  $m_{\text{frag}} > m_*$  (which favors monolithic collapse) to  $m_{\text{frag}} < m_*$  (which favors fragmentation).

As illustrated in Figure 1, the velocity dispersion in the radiatively heated gas within a few hundred AU of the star can be larger than the velocity dispersion required to maintain hydrostatic balance. As a result, the gas may temporarily expand, until infall from the colder outer parts of the core raises the density. This could further suppress fragmentation. If the expansion is adiabatic with  $\gamma = 5/3$ , the Jeans length will increase as  $\lambda_j \propto \rho^{-1/6}$ . With radiative effects, the expansion effectively decreases  $\Sigma_c$  at constant  $m_c$ , the net effect of which is also to increase  $\lambda_j$ .

Finally, note that turbulence in massive cores likely produces multiple density peaks within the overall centrally condensed structure. The densest of these peaks (which is almost certainly close to the center) has the shortest free-fall time and will collapse first. At the accretion rate given by equation (1) for  $m_c = 50\text{--}150 M_\odot$  and  $\Sigma_c = 0.3\text{--}1.0 \text{ g cm}^{-2}$ ,  $0.05 M_\odot$  of gas will accumulate, collapse to stellar density, and begin irradiating its surroundings in 30–300 yr, which is vastly shorter than the mean density free-fall time of  $3 \times 10^4$  to  $1 \times 10^5$  yr. Unless there is an unreasonably synchronized collapse of several density peaks, all the gas in the cloud except the first  $0.05 M_\odot$  to collapse will be subject to radiative feedback. In a turbulent core, self-shielding may leave sufficiently overdense regions cooler than the spherical calculations indicate, so fragmentation can only be followed in detail by radiation-hydrodynamic simulations. Nonetheless, the spherical calculations definitively show that radiative heating will raise the Jeans length and mass over a large volume within the core well before the vast majority of the gas collapses, so that the number of fragments should be significantly fewer, and their mean mass

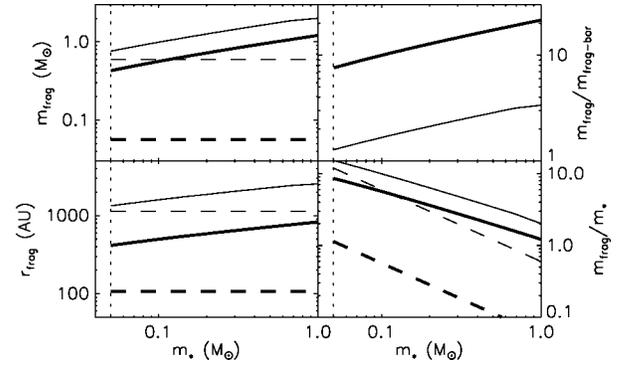


FIG. 2.—Mass  $m_{\text{frag}}$  and radius  $r_{\text{frag}}$  below which fragmentation is suppressed for  $(m_c, \Sigma_c)$  of  $(50 M_\odot, 0.3 \text{ g cm}^{-2})$  (thin lines) and  $(150 M_\odot, 1.0 \text{ g cm}^{-2})$  (thick lines). I plot  $m_{\text{frag}}$  (top left),  $r_{\text{frag}}$  (bottom left), the ratio of  $m_{\text{frag}}$  computed using radiative models to the value one infers using a barotropic equation of state (top right), and the ratio of  $m_{\text{frag}}$  to the protostellar mass  $m_*$  (bottom right). For all plots but the ratio of  $m_{\text{frag}}$  computed radiatively and barotropically, I show both radiative transfer models (solid lines) and barotropic models (dashed lines). The dotted vertical line at  $m_* = 0.05 M_\odot$  indicates the mass at which a protostar first forms.

much larger, than one would estimate by assuming an isothermal or barotropic equation of state.

#### 4. IMPLICATIONS AND CONCLUSIONS

##### 4.1. Massive Stars, Cores, and the IMF

The realization that radiative heating can suppress fragmentation argues that massive condensations really are cores, in that they are likely to collapse to produce single stars or small multiple systems, not clusters. This supports the idea that the massive stars form by accretion (MT03) and that the mass function of stars is determined at the stage of fragmentation into cores (Padoan & Nordlund 2002; Beuther et al. 2004; Reid & Wilson 2005). It removes the need to explain the common shape of the core and stellar mass functions as pure coincidence.

In contrast, the suppression of fragmentation by radiative heating presents a serious problem for models of the type proposed by Bate & Bonnell (2005) and Bonnell & Bate (2005), in which ejection and competitive accretion determine the IMF and direct collisions are required in order to create massive stars (but see Krumholz et al. [2005] for a critique of these models on other grounds). In such models, all stars are born as brown dwarfs or very low mass stars in clusters  $\sim 1000$  AU in size. Most objects then accrete until being ejected by  $N$ -body interactions, while a few experience runaway collisions, leading to the formation of massive stars. However, in dense regions where the accretion luminosity is high, the formation of the first protostar within a core will inhibit the formation of subsequent ones around it. Rather than producing a large cluster of low-mass protostars or brown dwarfs as the competitive accretion and collision models demand, heating will produce a small number of larger protostars.

It is unclear whether these results present a problem to IMF models that depend on the structure of the equation of state for collapsing gas (Larson 2005). If the typical separation between the fragments in a protocluster is larger than  $\sim 1000$  AU, then, as Larson suggests, radiative heating will not substantially modify the typical fragment mass until enough stars have formed to heat the entire protocluster. On the other hand, if the typical interfragment spacing is small, then radiative feedback effects cannot be ignored.

#### 4.2. Binary Formation

Suppression of fragmentation by feedback also provides a natural explanation for the observation that when massive stars are in binaries, the binaries tend to consist of two massive stars rather than a high-mass star and a low-mass one (Pinsonneault & Stanek 2006). If binaries result from the fragmentation of massive cores, then one would expect the resulting stars to be massive, because once the first protostar forms, it will raise the Jeans mass and thereby favor the formation of massive stars thereafter. Although I have only considered heating due to accretion luminosity, it is likely to be even more significant once protostars become massive enough to start fusing deuterium and then hydrogen. This will greatly increase their luminosity and raise the Jeans mass even more. Testing whether this effect can quantitatively explain the observed properties of binaries will require radiation-hydrodynamic simulations. Radiation feedback may also be important for low-mass binaries. Lada (2006) recently pointed out that most low-mass stars are singletons rather than binaries. Barotropic simulations of low-mass star formation almost invariably produce multiple systems (Goodwin et al. 2004), and radiation feedback may provide a way of reducing the multiplicity.

#### 4.3. The Barotropic Approximation in Simulations

In some cases, the barotropic approximation produces results at odds with radiative transfer calculations even when there is no point source of radiation (Boss et al. 2000; Whitehouse &

Bate 2006). Feedback makes the problem far worse. Simulations on scales  $\gg 1000$  AU, such as those that model core formation within a larger molecular cloud or clump (e.g., Li et al. 2004; Tilley & Pudritz 2004), are probably safe. On the other hand, as Matzner & Levin (2005) point out for disk fragmentation, the barotropic approximation is likely to produce incorrect results in simulations that go to smaller scales (e.g., Bate et al. 2003; Goodwin et al. 2004). For this reason, simulations of fragmentation should be done with radiative transfer. However, in simulations that do use a barotropic equation of state, one can at least attempt to check the validity of the approximation *ex post facto* by selecting a few time slices and calculating the temperature distribution, including accretion luminosity, using a radiative transfer code (e.g., Whitney et al. 2003). If the temperature is significantly higher than indicated by the assumed equation of state in the simulation, then the results for fragmentation in the region whose temperature has been underestimated must be regarded with suspicion.

I thank I. Bonnell, R. Fisher, R. Klein, C. McKee, J. Stone, and the anonymous referee for helpful comments. This work was supported by NASA through Hubble Fellowship grant HSF-HF-01186 awarded by the Space Telescope Science Institute, which is operated by the Association of Universities for Research in Astronomy, Inc., for NASA under contract NAS 5-26555.

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