In the Supplementary Discussion, we derive the some of the equations and numerical values used in the main text.

Recent simulations show that, over times comparable to a crossing time, the time-averaged accretion rate onto a point particle of mass m_* in a turbulent flow of three-dimensional Mach number $\mathcal{M}$ and outer length scale ℓ is well-fit by[9]

$$\phi_{\rm BH} = \frac{3}{\phi_{\rm u}} \left[\ln(2\phi_{\rm u}\mathcal{M}) - 1\right] \left(1 + 100\frac{r_{\rm B}}{\ell\mathcal{M}}\right)^{-0.68}, \tag{1}$$

where $\phi_{\rm u} \approx 0.95$ and

$$r_{\rm B} = \frac{Gm_*}{c_{\rm s}^2} \approx 0.11 \left(\frac{m_*}{0.1\ M_\odot}\right)\ {\rm pc} \tag{2}$$

is the Bondi radius of the accreting object, and we have assumed a typical sound speed of $c_{\rm s} \approx 0.2$ km s^{-1} for the region. The Mach number is therefore $\mathcal{M} = \sqrt{3}\sigma/(0.2\ {\rm km\ s^{-1}})$. The outer scale of the turbulence ℓ is roughly R for spherical clumps. For filamentary clumps, there are two candidate length scales. Since filamentary clumps will be unable to sustain turbulent motions on scales larger than R in two directions, we adopt $\ell = R$ for filamentary regions as well. With these choices, we can determine $\phi_{\rm BH}$ from the properties of a clump.

Under the approximations discussed in the main text, the rate of star-core captures is

$$\frac{1}{t_{\rm cap}} = \frac{n_{\rm co}R_{\rm co}^2}{8\pi^2\sigma^6} \int d\mathbf{v}_{\rm co} \int d\mathbf{v}_* v_0 \phi_{\rm r}^2 \exp\left(-\frac{v_{\rm co}^2 + v_*^2}{2\sigma^2}\right) H(v_{\rm cap} - v_0), \tag{3}$$

where $n_{\rm co}$ is the number density of cores within the clump, $\mathbf{v}_{\rm co}$ and $\mathbf{v}_*$ are the velocities of the core and star, $\mathbf{v}_0 \equiv \mathbf{v}_{\rm co} - \mathbf{v}_*$ is their relative velocity, $\phi_{\rm r}$ is a factor that depends on v_0 and accounts for the enhancement of the core-star cross section by gravitational focusing, $H(x)$ is the Heaviside step function, and $v_{\rm cap}$ is the critical velocity below which encounters lead to captures, and above which they do not. For cores of mass $M_{\rm co}$, assuming that a captured core will be accreted completely, the fractional mass gain per dynamical time of the clump is therefore

$$f_{\rm m-cap} = \frac{R}{q\sigma t_{\rm cap}}, \tag{4}$$

where we have defined $q \equiv m_*/M_{\rm co}$ as the star-core mass ratio.

The number density of cores is easy to estimate. If we let $\phi_{\rm co}$ be the fraction of the clump's mass that is in cores, then the number density of cores is

$$n_{\rm co} = \frac{\phi_{\rm co} M}{\pi R^2 M_{\rm co}} \left(\frac{3}{4R}, \frac{1}{L}\right) \tag{5}$$

for (spherical, filamentary) star-forming regions, and the fractional mass increase per crossing time becomes

$$f_{\rm m-cap} = \left(\frac{3}{4}, \frac{2}{\pi}\right) \frac{\phi_{\rm co}}{8\pi^3 q\sigma^7} \int d\mathbf{v}_{\rm co} \int d\mathbf{v}_* v_0 \phi_{\rm r}^2 \exp\left(-\frac{v_{\rm co}^2 + v_*^2}{2\sigma^2}\right) H(v_{\rm cap} - v_0), \tag{6}$$

where we have used our relation that cores and clumps have equal surface densities.

We next evaluate $v_{\rm cap}$. If a core and star approaching each other with velocity v_0 when they are far apart travel ballistically until they impact, the relative velocity when the star reaches the core surface will be

$$v_{\rm rel} = v_0 \sqrt{1 + (q+1)\frac{v_{\rm esc}^2}{v_0^2}}, \tag{7}$$

where $v_{\rm esc} \equiv (2GM_{\rm co}/R_{\rm co})^{1/2}$ is the escape velocity from the surface of the core, given by equation (4) of the main text. When the star is within the core, the accretion rate of mass onto the star is roughly

$$\dot{m}_* \approx \frac{3G^2 q^2 M_{\rm co}^3}{R_{\rm co}^3 v_{\rm rel}^3}, \tag{8}$$

where we assume that the star-core velocity is highly supersonic, and we have used the mean density and initial relative velocity to compute the accretion rate rather than attempting to follow the core's motion as it passes through the star. Since cores smaller than ~ 1 $M_\odot$ are supported by thermal pressure rather than turbulence, we do not include a factor $\phi_{\rm BH}$. The drag force on an object undergoing Bondi-Hoyle accretion in such a manner is[15] $F_d \approx \dot{m}_* v_{\rm rel}(\ln\Lambda - 1)$, where $\Lambda \approx R_{\rm co}/r_*$ is the ratio of the core size to the stellar size. A typical 0.1 $M_\odot$ protostar might have a radius of several 10^{11} cm, and a typical core might be tenths of a pc in size, so we take $\ln\Lambda \approx 14$. The distance the star travels through the core is of order $R_{\rm co}$, and the work done during that passage is therefore $F_d R_{\rm co}$. The star and core will become bound if this work exceeds the total kinetic energy of the system at infinity, which is, in the center of mass frame, $(1/2)M_{\rm co}v_0^2 q/(q+1)$. Re-arranging these expressions, we can derive a condition that the relative velocity must satisfy for a given encounter to lead to a capture:

$$v_0 < \left\{\frac{q+1}{2}\left[\sqrt{1+6\frac{q}{q+1}(\ln\Lambda - 1)} - 1\right]\right\}^{1/2} v_{\rm esc} \equiv u_{\rm cap}\sigma, . \tag{9}$$

where we have introduced the dimensionless velocity $u = v/\sigma$.

Finally, we estimate $\phi_{\rm r}$, which is defined such that the maximum star-core impact parameter that leads to a capture is $\phi_{\rm r} R_{\rm co}$. Again assuming that the star and core travel ballistically up to the point of impact, we can solve for the maximum impact paramter $\phi_{\rm r} R_{\rm co}$ that will bring the star and core into contact. Doing so, we find

$$\phi_{\rm r} = \sqrt{1 + (q+1)\frac{u_{\rm esc}^2}{u_0^2}}. \tag{10}$$

Now that we have estimated all the terms upon which the number of captures depends, we can solve for $f_{\rm m-cap}$. The integral in equation (6) can be evaluated using the substitution $\mathbf{v}_0 = \mathbf{v}_{\rm co} - \mathbf{v}_*$, $\mathbf{v}_{\rm cm} = \mathbf{v}_* + \mathbf{v}_0/2$, giving

$$f_{\rm m-cap} = \left(\frac{3}{4}, \frac{2}{\pi}\right) \phi_{\rm co} \frac{1}{\pi^{1/2} q} \left\{4 + (q+1)u_{\rm esc}^2 - \left[u_{\rm cap}^2 + 4 + (q+1)u_{\rm esc}^2\right] \exp\left(-\frac{u_{\rm cap}^2}{4}\right)\right\}. \quad (11)$$

If we adopt $M_{\rm co} = m_*$, the largest value of $M_{\rm co}$ consistent with competitive accretion, then we have $q = 1$. Evaluating (11) for this case gives equation (5) in the main text.