ATA 2009 Dynamics of our Galaxy: ASSIGNMENT

Due September 18, 2009 at lecture

Please hand this work in on time: otherwise, I may not be able to grade it. Remember: this assignment represents 30% of your assessment for this module.

1. Give a simple order of magnitude argument to show why the relaxation time for 2-body encounters in a stellar system $T_R \sim \sigma^3/m^2 n$, where σ is the velocity dispersion, m the typical stellar mass and n the number density of stars. (10 marks)

2. (From B&T 99, question 2.3). The density distribution

$$\rho(r) = \frac{M}{4\pi r_J^3} \frac{r_J^4}{r^2(r+r_J)^2}$$

(where M and r_J are constants) for a spherical galaxy is called a Jaffe model (after Walter Jaffe of Leiden University, who devised it).

- Verify that the total mass is M.
- Show that the potential is

$$\Phi = \frac{GM}{r_J} \ln(\frac{r}{r+r_J})$$

  by (i) using Poisson's equation, and (ii) deriving the potential from the density distribution.

- Show that the circular speed v_c is approximately constant for $r \ll r_J$ and falls off like $v_c \sim r^{-1/2}$ for $r \gg r_J$.
- Comment on Jaffe models as models for real galaxies.

(10 marks for whole question)

3.a) In a spherical galaxy with potential

$$\Phi(r) = V^2 \ln(r)$$

with $V = 200$ km s^{-1}, a star at $r = 10$ kpc has $v_t = 100$ km s^{-1} , $v_r = 50$ km s^{-1}, where v_t and v_r are the tangential and radial components of the stellar velocity. What are the maximum and minimum values of r for the star's orbit ? (Hint: energy and angular momentum are conserved).

3.b) Integrate this orbit numerically until it reaches its third apogalacticon (maximum r).

- plot the orbit in its own plane
- plot $r(t)$

- what is the angular distance between successive apogalactica
- check that the apo- and perigalactic distances agree with your estimates in **3.a)**.

3.c) In this same potential, sketch the following in the (E, J) plane, where $E = v^2/2 + \Phi$, $J = rv_t$ are the energy and angular momentum of a star: *(i)* loci of circular orbits, *(ii)* a few loci of constant apogalactica, *(iii)* a few loci of constant perigalactica.

Say we allow no orbits with $E > E_\circ$, where $E_\circ$ is some constant. What is the ratio of the apogalactic radii for circular and radial orbits with $E = E_\circ$? Mark these two orbits on your (E, J) plane. What do you conclude about the properties of orbits in the outer parts of stellar systems with a sharp energy cutoff ?

3.d) The *infall time* is the time taken for a particle initially at rest at radius r to fall in to the center of the galaxy. What is the ratio of the infall time to the rotation period at radius r ? (Hint: you can do this analytically). (40 marks for whole question).

4. A spherical galaxy has a density distribution

$$\rho(r) = \rho_o[1 + r^2/a^2]^{-1}.$$

- Show that the enclosed mass $M(r) \propto r^3$ for $r \ll a$ (*i.e.* in the core) and $M(r) \propto r$ for $r \gg a$ (*i.e.* in the outer halo).
- Now take a population of massless test particles in the potential of this galaxy. Assume that this population is spherical, non-rotating, isothermal and isotropic, with velocity dispersion σ in each velocity component. What is the radial density distribution of this test particle population ? (Hint: use Jeans' equation)
- At large r, this density distribution simplifies and its form depends on a dimensionless number. Give a physical interpretation of this dimensionless number. What is the condition that the density distributions of the test particle population and the galaxy itself have similar forms at large r ?

(20 marks for whole question)

5) This is an essay question. Write a short essay (about 500 words) on the goals and methods of chemical tagging in galactic archaeology. The paper "Panoramic Spectroscopy" on the website, and the background slides at the end of the ATA2009.ppt file may be useful. (20 marks)