DARK MATTER IN GALAXIES

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These lectures present a brief overview of what we know about dark matter in galaxies. I will stress some of the current problems

1 Introduction

We believe that galaxies formed through a hierarchy of merging. The merging elements were a mixture of baryonic and dark matter. The dark matter settled into a partially virialized spheroidal halo, while the baryons (in disk galaxies) settled into a rotating disk and bulge.

What can we learn about the properties of dark halos ? Do the properties of dark halos predicted by simulations correspond to what is inferred from observational studies ?

These lectures will primarily be about dark matter in disk galaxies. Disk galaxies are flat systems, supported against gravity by their rotation, and they are the simplest galaxies for studying the properties of the dark halos.

2 Rotation of Spirals

Most spirals do not rotate like rigid bodies. They show a wide range of rotation curve morphology, depending on the radial distribution of stars. The extremes range from almost solid body rotation, as seen for some lower luminosity disks, to rotation curves in which the rotational velocity is almost constant with radius throughout the galaxy which is more typical of the brighter disks like the Milky Way. See Figure 1 for some extreme examples.

What keeps the disk in equilibrium (this is an important question to ask for any stellar system) ? Most of the kinetic energy is in the rotation. In the radial direction, gravity provides the radial acceleration needed for the approximately circular motion of gas and stars in the disk. In the vertical direction, gravity is balanced by the vertical pressure gradient associated with the random vertical motions of the disk stars.

For the gas in a disk galaxy, the radial potential gradient provides the acceleration for the circular motion.

$$\frac{V^2}{R} = -\frac{\partial \Phi}{\partial R} \simeq \frac{GM(R)}{R^2}$$

where $V(R)$ and $\Phi(R)$ are the rotational velocity and potential at radius R in the plane of the disk, and $M(R)$ is the enclosed mass within radius R. As shown in Figure 1, the shape of $V(R)$ can be anything from solid body to $V \simeq$ constant (flat). For the larger spirals like our Galaxy, $V(R)$ is usually close to flat, so the enclosed mass increases linearly with R, at least out to the maximum extent of the rotation curve.

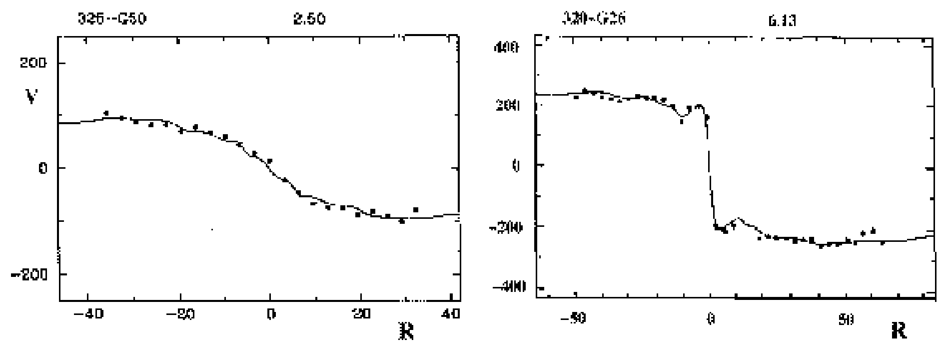

Figure 1. Optical rotation curves for two spiral galaxies from Buchhorn (1991), showing the wide variety of rotation curve morphology seen among spiral galaxies.The units of V and R are km s^{-1} and arcsec respectively. The points show the rotation data. See the text for explanation of the curves.

$M(R) \propto R$ is not what we would expect for a gravitating system of stars. We would expect $M(R)$ to tend to some asymptotic mass M for large R. Is $M(R) \propto R$ evidence for a dark halo ? Not necessarily. It depends on how far the observed rotation curve extends. Most spirals have a light distribution that is roughly exponential: $I(R) \propto \exp(-R/h)$ where the scale length h is about 4 kpc for a large galaxy like the Milky Way. Rotation curves measured optically from the spectra of ionized gas typically extend to about $r = 3h$.

Now assume that the surface *density* distribution of stars in our disk galaxy is proportional to the optical surface *brightness* distribution. Can this surface density distribution, with its associated gravitational potential $\Phi(R)$, explain the observed rotation curve $V(R)$? The answer to this question is yes and no.

The answer is yes for optical rotation curves extending out to about 3 radial scale lengths. In Figure 1, the points are the observed rotational velocities and the curve is the expected curve derived from the surface density distribution, assuming that mass follows light. Despite the very different shapes of the rotation curves, the light distribution can explain the observed optical rotation curves out to about 3 scale lengths. The only scaling is in the velocity coordinate, through the adopted mass to light ratio M/L.

The answer is no for galaxies with 21 cm neutral hydrogen (HI) rotation curves that extend out to $R \gg 3h$. Figure 2 shows a decomposition of the rotation curve of the spiral NGC 3198, adopting the maximum value for the M/L ratio for the stellar disk that is consistent with the observed rotation curve (*i.e.* the adopted M/L ratio cannot be so high that the calculated rotation curve is higher anywhere than the observed rotation curve. In this galaxy the HI rotation curve extends to about $11h$. With the maximum possible M/L ratio for the stars, the expected $V(R)$ from the stars and gas falls well below the observed rotation curve in the outer region of the galaxy. This kind of shortfall is seen for almost all spirals with rotation curves that extend out to many scale lengths. We conclude that the luminous matter dominates the radial potential gradient $\partial\Phi/\partial R$ for $R \lesssim 3h$ but beyond this radius the dark halo becomes progressively more important.

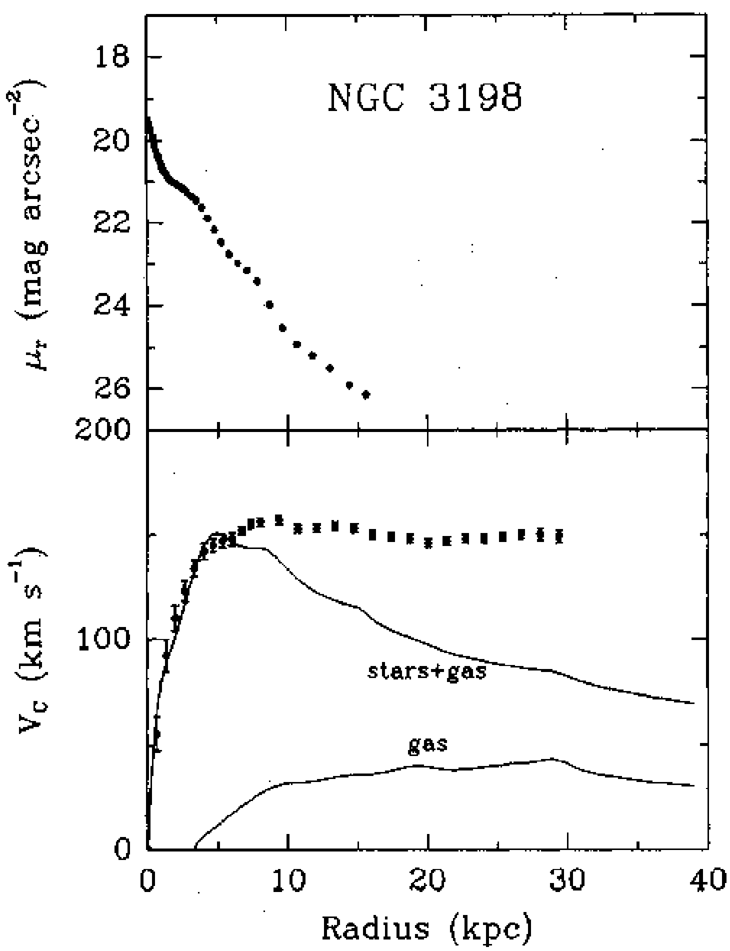

Figure 2. The *upper panel* shows the surface brightness distribution of the spiral galaxies NGC 1398, from Begeman(1989). The *lower panel* shows the large discrepancy between the HI rotation curve (points) and the expected contribution to the rotation curve from the stars plus gas, adopting the maximum disk hypothesis as explained in the text (§3).

Typically, out to the radius where the HI data ends, the ratio of dark to luminous mass is 3 to 5. Values of 10 to 20 are found in a few examples.

For the decomposition of NGC 3198 described above, the stellar M/L ratio was taken to be as large as possible without leading to a hollow dark halo. This kind of decomposition is known as a maximum disk (or minimum halo) decomposition. Many galaxies have been analysed in this way. The decomposition usually works out as for NGC 3198, with comparable peak circular velocity contributions from disk and dark halo. This is believed to be at least partly due to the adiabatic compression of the dark halo by the baryons as they dissipate and condense to form the disk.

3 The Maximum Disk Question

The inferred stellar M/L ratios from maximum disk decompositions are usually consistent with those expected from synthetic stellar populations, at least for the brighter spirals like the Milky Way. Nevertheless, some people still do not believe that the maximum disk approach is correct. They argue that the dark halo is probably more significant gravitationally than the maximum disk / minimum halo hypothesis would indicate; this is equivalent to adopting a smaller stellar M/L ratio for the disk. One reason for this belief comes from the Milky Way itself: the apparent surface density of the galactic disk and halo near the sun is only about 50 $M_\odot$ pc^{-2}, which may be too low to be consistent with a maximum disk (Kuijken & Gilmore 1989).

The maximum disk question is important for us here, because inferences about the properties of dark halos from rotation curves depend so much on the correctness of the maximum disk interpretation. For example, if the maximum disk decompositions are correct, the contribution to $V(R)$ from the halo is approximately solid-body in the inner parts of the galaxy, so the dark halos have approximately uniform density cores which are much larger than the scale length of the disk. In contrast, the halos that form in cosmological simulations have *steeply cusped* inner halos with density distributions $\rho \sim r^{-1}$ or even steeper near the center.

Optical rotation curves favor the maximum disk interpretation. In the inner regions of the disks of larger spirals, the rotation curves are well fit by assuming that mass follows light. For example, Buchhorn (1991) analysed about 500 galaxies with I-band surface brightness distributions and a wide range of optical rotation curve morphologies spanning the extremes shown in Figure 1. He was able to match the observed and expected rotation curves well for about 97% of his sample, with realistic M/L ratios. The implication is that either the stellar disk dominates the gravitational field in the inner parts of the disk, or the potential gradient of the halo faithfully mimics the potential gradient of the disk in almost every spiral.

3.1 Other support for the maximum disk interpretation

Athanassoula et al (1987) used the dynamical theory of spiral structure to give a dynamical constraint on the stellar M/L ratio for the disk. From the number of spiral arms observed in each of their galaxies, they argue that most of the disks are indeed close to maximum.

Bell & de Jong (2001) and Perez (2003) compared the M/L ratio from synthetic stellar populations with those derived dynamically from maximum disk rotation curves. They find good agreement when they use a stellar mass function like that for the solar neighborhood.

Debattista & Sellwood (1998) showed that a dense halo (as in a submaximal disk decomposition) would rapidly slow down the rotation rate of the bars in barred spiral galaxies. In a low density halo (as in a maximum disk system), the bar rotation stays high. See Athanassoula (2002) for a more detailed study of the interaction of bars and dark halos. Evidence from gas flows in barred galaxies (eg Weiner et al 2001; Perez & Fux 2004) indicates that bars do rotate rapidly, with corotation just beyond the end of the bar.

I conclude that the maximum disk picture is probably correct, at least for galaxies of normal surface brightness. (We will discuss low surface brightness galaxies later).

4 Modelling the Dark Halo

Our goal is to estimate the typical parameters for dark halos (eg their density, scale length, velocity dispersion, shape) to compare with the properties of halos from cosmological simulations. Since about 1985, observers have used model dark halos with constant density cores to interpret rotation curves. Commonly used models include the nonsingular isothermal sphere, which has a well defined core radius and central density; its density falls off as $\rho \sim r^{-2}$ at large r so $V(r) \sim$ constant as often observed.

A simple analytical form is the pseudo-isothermal sphere

$$\rho = \frac{\rho_\circ}{1 + (r/r_c)^2}$$

which again has a well defined core radius and central density and $\rho \sim r^{-2}$ at large r. Using this model for the dark halos of large galaxies like the Milky Way, we find that $\rho_\circ \sim 0.01\ M_\odot \mathrm{pc}^{-3}$ and $r_c \sim 10$ kpc. For comparison, the density of the galactic disk near the sun as about $0.1\ M_\odot \mathrm{pc}^{-3}$. We will see later that the values of $\rho_\circ$ and r_c depend strongly on the luminosity of the galaxy.

Why were these models with central cores used ? I think it was because (1) rotation curves of spirals do appear to have an inner solid-body component which indicates a core of roughly constant density, and (2) hot stellar systems like globular clusters had been successfully modelled by King models, which are modified non-singular isothermal spheres (with cores). On the other hand, CDM simulations consistently produce halos that are cusped at the center. This has been known since the 1980's and has been popularized by Navarro et al (1996) with their NFW density distribution which parameterizes the CDM halos:

$$\rho(r) = \rho_\circ \frac{r_s}{r} \frac{1}{[1 + (r/r_s)]^2}$$

These are cusped at the center, with $\rho(r) \sim r^{-1}$.

The last several years have seen a long controversy on whether the observed rotation curves imply cusped or cored dark halos. This continues to be illuminating. Galaxies of low surface brightness (LSB) are important in this debate. The disks of normal (or high surface brightness) spirals have a fairly well defined characteristic central surface brightness of about 21.5 B mag arcsec^{-2} (*e.g.* Freeman 1970). In the LSB galaxies, the disk surface brightness can be more than 10 times lower than in the normal spirals. These LSB disks are fairly clearly sub-maximal, and the rotation curve is believed to be dominated everywhere by the dark halo. So the rotation curves of these LSB galaxies potentially give a fairly direct estimate of the structure of the inner parts of the dark halo. The observational problem is to determine the shape of the rotation curve near the center of the galaxies. Near the center, a cored halo gives a solid body rotatio curve, while the rotation curve for a cusped halo rises very steeply.

Observationally, it is not easy to tell. HI rotation curves have limited spatial resolution, so the beam smearing can mask the effects of a possible cusp. Optical rotation curves, including the 2D optical rotation data with Fabry-Perot interferometers, have much better spatial resolution and favor a cored halo with a power law slope near zero (de Blok et al 2001). The recent HI study of the very nearby LSB galaxy NGC 6822, with 20 pc linear resolution (Weldrake et al 2003), also clearly favor a cored halo.

What is wrong: observations or theory ? Does it matter ? Yes: the density distribution of the dark halos provides a critical test of the nature of dark matter and of galaxy formation theory. For example, the proven presence of cusps can exclude some dark matter particles (eg Gondolo 2000). The halo density profiles can also provide some constraints on the fluctuation spectrum (eg Ma & Fry 2000).

Maybe CDM is wrong. For example, self-interacting dark matter can give a flat central $\rho(r)$ via heat transfer into the colder central regions. But further evolution can then lead to core collapse (as in globular clusters) and even steeper r^{-2} cusps (eg Burkert 2000; Dalcanton & Hogan 2001).

Alternatively, there are ways to convert CDM cusps into flat central cores, so that we do not see the cusps now. For example, bars are very common in disk galaxies: about 70% of disk galaxies show some kind of central bar structure. Many galaxies that do not appear to be barred from their optical images show clear central bars in near-infrared images which are dominated by older stars and are less affected by dust absorption. The bars are believed to come from gravitational instability of the disk. Weinberg & Katz (2002) showed that the angular momentum transfer and dynamical heating of the inner halo by the bar can remove a central cusp in about 1.5 Gyr.

This issue is far from settled. I think that the current belief is that the cusp structure may be flattened by the effect of blowout of baryons in early bursts of star formation as the halo is built up (eg Dekel *et al.* 2003). This idea has a couple of additional major attractions. Before discussing these, a short digression is needed on two important dynamical processes involved in hierarchical galaxy formation: dynamical friction and tidal disruption. The discussion follows Binney & Tremaine (1987).

4.1 Dynamical Friction

Dynamical friction is the frictional effect on a mass M moving through a sea of stars of mass m. Assume that the smaller masses m are uniformly distributed, and adopt the “Jeans Swindle” (i.e. ignore the potential of the uniform distribution

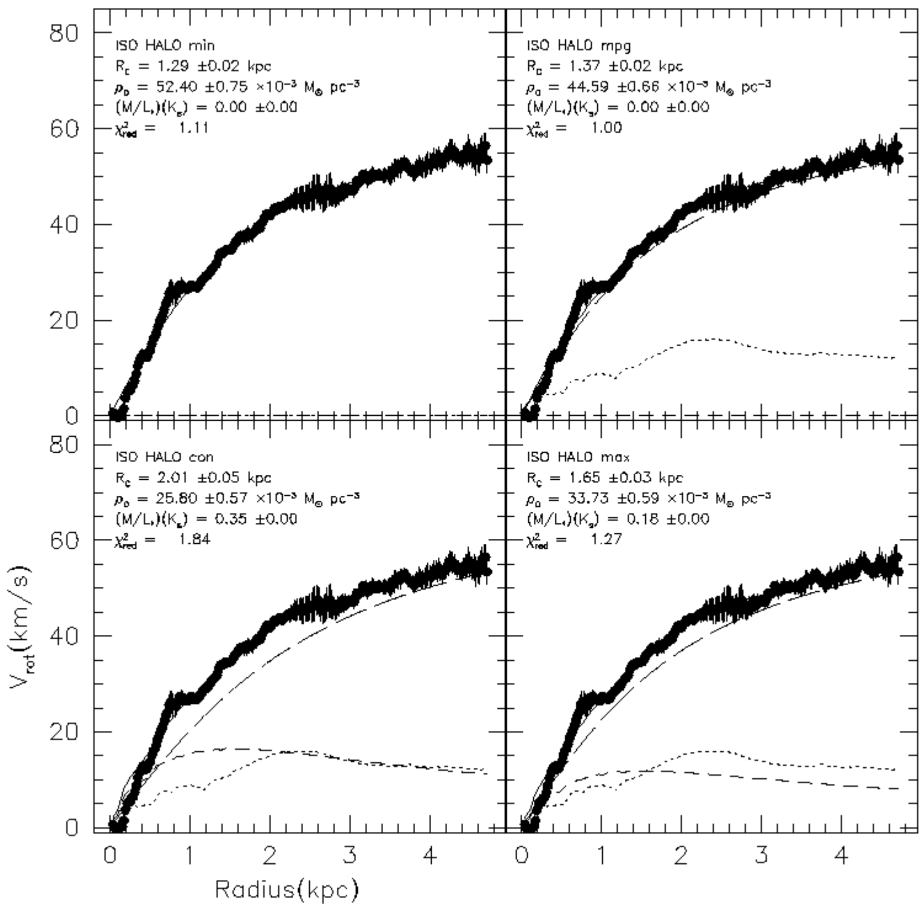

Figure 3. The rotation curve of the nearby LSB galaxy NGC 6822. The panels show fits of models with isothermal halos and different adopted stellar M/L ratios. Excellent fits are achieved with low M/L ratios, favoring the presence of a cored halo (Weldrake et al 2003).

of the m objects. Then the motion is determined only by the force of M and the *disturbances* that M produces on the distribution of the m objects.

M raises a response in the sea of smaller objects, and this response acts back on M itself. Summing the effects of the individual encounters of M and m, we see that M suffers a steady *deceleration* parallel to its velocity $\mathbf{v}$. If the velocity distribution of m is Maxwellian

$$f = \frac{n_\circ}{(2\pi\sigma^2)^{3/2}} \exp(-v^2/2\sigma^2)$$

then the drag is

$$\frac{d\mathbf{V}_M}{dt} = -\frac{4\pi \ln \Lambda G^2 \rho_m M}{V_M{}^3} \left(\text{erf}\chi - \frac{2\chi}{\sqrt{\pi}} \exp(-\chi^2) \right) \mathbf{V}_M$$

for $M \gg m$. $\chi = V_M/\sqrt{2}\sigma$ and $\Lambda =$ (maximum impact parameter) $\times$ (typical speed)$^2/GM$: $\Lambda \gg 1$.

So (i) the drag acceleration is $\propto \rho_m$ and $\propto M$ and (ii) the drag force $\propto M^2$. This comes about because stars deflected by M generate a downstream density enhancement: the enhancement $\propto M$, and the force back on $M \propto M^2$.

This estimate neglects the self-gravity of the density enhancement; i.e. it includes the attraction of m on M, but not m on m. The estimate seems to be fairly consistent with the results of N-body simulations, as long as the ratio of M to the total mass of the m objects $\lesssim$0.2 and the orbit of M is not confined to the core or to the exterior of the larger system. The estimate also neglects resonances between the orbit of M and the orbits of m objects within their system: such resonances enhance dynamical friction.

For example, consider the likely fate of the LMC, now located at about 60 kpc from the Galaxy. For circular orbits, the torque from dynamical friction due to the dark halo of our Galaxy gives a decay time

$$t_{fric} = \frac{10^{10}}{3} \left(\frac{r}{60 \text{ kpc}} \right) \left(\frac{V_c}{220 \text{ km s}^{-1}} \right) \left(\frac{2 \times 10^{10}}{M_\odot} \right) \quad \text{yr}$$

so if the galactic halo extends out beyond a radius of 60 kpc and the LMC orbit is approximately circular (both of which are true), then the LMC (and SMC) will sink into the Galaxy in a time less than the Hubble time.

4.2 Tidal Disruption

Consider a satellite of mass m in a circular orbit around a host of mass M at a distance D. The angular speed around the common center of mass is $\Omega^2 = G(m+M)/D^3$. In this rotating frame, we have the Jacobi integral $E_J = E - \mathbf{\Omega}.\mathbf{L} = \frac{1}{2}v^2 + \Phi_{eff}(\mathbf{r})$, where Φ_{eff} is the effective potential of the gravity plus the centrifugal force. The contours of Φ_{eff} have a saddle point between the two masses, where $\partial\Phi_{eff}/\partial x = 0$ (see Binney & Tremaine, Figure 7.8). Beyond this saddle point the contours open out. For $m \ll M$, the distance of the saddle point from the smaller mass is

$$r_J = \left(\frac{M}{3M} \right)^{1/3} D$$

This is a measure of the *tidal radius* of m. It is a rough estimate because

1. the zero velocity (ZV) surface is not spherical
2. orbits do not necessary escape because the ZV surface is open
3. the orbit of m is not usually circular
4. m often lies within M, so the point mass approximation is poor

but the main point here is that tidal removal of matter can occur at a radius from m such that $\rho_m(r_J) \sim \rho_M(D)$. For example, we expect an infalling satellite to remain intact in to a distance D from the larger galaxy, such that $\rho_M(D) \sim$ the mean density of the satellite.

To summarise the merger preliminaries:

- *Dynamical friction* (M in a sea of m). The drag force $\propto \rho_m M^2$, neglecting resonances and the selfgravity of the wake.
- Tidal disruption. Occurs when the mean density of the host within the satellite orbit $\sim$ the mean density of the satellite. Very dense satellites can survive accretion while low density satellites are broken up.

5 Galaxy Formation Problems

In simulations of galaxy formation (*e.g.* Moore *et al.* 1999), the virialized halos are quite lumpy, with much substructure, corresponding to many more satellites and dwarf galaxies than observed in the environment of the Milky Way. The simulations suggest that a galaxy like the Milky Way should have about 500 satellites with bound masses $> 10^6\ M_\odot$. These are not seen optically and probably not in HI. What is wrong ? Maybe there are a large number of baryon-depleted dark satellites, or there is some problem with the details of CDM (eg that the short wavelength end of the fluctuation spectrum needs modification).

The baryons also clump and, as they settle to the disk, the clumps suffer dynamical friction against the halo and so lose angular momentum. The resulting disks then have smaller angular momentum than those observed: they are therefore smaller in radius and spinning more rapidly than real galaxies. This remains one of the more serious problems in the current theory of galaxy formation (eg Abadi *et al.* 2003). We need to find ways to suppress the loss of angular momentum of the baryons to the dark halo.

One way to avoid this loss of angular momentum is by blowout of baryons early in the galaxy formation process. For example, Sommer-Larsen *et al.* (2003) made N-body + SPH simulations with a star formation prescription. Star formation begins early in the galaxy formation process. Small elements of the hierarchy (dwarf galaxies) form stars long before the whole system has virialized. The stellar winds and SN from the forming stars temporarily eject most of the baryons from the forming galaxy. The halo virializes and then the baryons settle smoothly to the disk. Because they settle smoothly, the loss of angular momentum via dynamical friction is much reduced.

The blowout process (§4) can also contribute to reducing the problem of too much substructure and to the cusp problem in another way (eg Dekel *et al.* 2003). Because the smaller elements of the hierachy grow first, they are denser (we will see observational evidence for this later). This means that they are less likely to be tidally disrupted as they settle to the inner parts of the halo via dynamical friction, so they can contribute to the high density cusp in the center of the virialized halo. Blowout of the baryon component of these dense small elements can contribute to unbinding them. Their chances of survival against the tidal field of the virializing halo are then reduced, so (1) the substructure problem (ie too many small elements) is reduced, and (2) the cusp problem is reduced.

6 How large are Dark Halos

Flat rotation curves imply that $M(r) \propto r$, like the isothermal sphere with $\rho(r) \sim r^{-2}$ at large r. This cannot go on forever: the halo mass would be infinite. Halos must have a finite extent, and their density distribution is probably steeper than $\rho(r) \sim r^{-2}$ at very large r. For example, the NFW halo with

$$\rho(r) = \rho_\circ \frac{r_s}{r} \frac{1}{[1 + r/r_s]^2}$$

has $\rho(r) \sim r^{-3}$ at large r.

Tracers of dark matter in the Milky Way (the rotation curve observed out to a radius of about 20 kpc, kinematics of stars and globular clusters in the stellar halo, and kinematics of satellites out to $R > 50$ kpc) all indicate that the enclosed mass rises linearly as in other galaxies, and is well approximated by $M(r) = r(\text{kpc}) \times 10^{10}$ $M_\odot$. This is what we would expect if the galactic rotation curve stays flat out to $r > 50$ kpc. This still does not tell us how far the dark halo extends. Other arguments are needed.

6.1 Timing arguments

M31 is now approaching the Galaxy at about 118 km s^{-1}. Its distance is about 750 kpc. Assuming that their initial separation was small, we can estimate a lower limit on the total mass of the Andromeda + Galaxy system such that they are now approaching at the observed velocity. The Galaxy's share of this mass is $(13 \pm 2) \times 10^{11}$ $M_\odot$. A similar argument for the Leo I dwarf at a distance of about 230 kpc gives $(12 \pm 2) \times 10^{11}$ $M_\odot$. Our relation for $M(r)$ for the galactic halo, derived for $r \sim 50$ kpc, then indicates that the dark halo extends out beyond a radius of 120 kpc, if the rotation curve remains flat, and possibly much more if the density distribution declines more rapidly at large radius. This radius is much larger than the extent of any directly measured rotation curves, so this "timing argument" gives a realistic lower limit to the total mass and radial extent of the galactic dark halo (Zaritsky 1999). This argument was originally due to Kahn & Woltjer (1959).

For our Galaxy, the luminous mass (disk + bulge) is about 6×10^{10} $M_\odot$. The luminosity is about 2×10^{10} L$_\odot$. The ratio of total dark mass to stellar mass is then at least $120/6 = 20$ and the total mass to light ratio is at least 60 in solar units.

Satellites of disk galaxies can also be used to estimate the total mass and extent of the dark halos. Individual galaxies have only a few observable satellites each, but we can make a super-galaxy by combining observations of many satellite systems and so get a measure of the mass of a typical dark halo. For example, Prada et al (2003) studied the kinematics of about 3000 satellites around about 1000 galaxies. With a careful treatment of interlopers, they find that the velocity dispersion of the super satellite system decreases slowly with radius. The halos typically extend out to about 300 kpc but their derived density distribution at large radius is steeper than the isothermal: $\rho(r) \sim r^{-3}$, like most cosmological models including the NFW halos. The total mass to light ratios are typically $100 - 150$, compared with the lower limit from the timing argument of 60 for our Galaxy. (Note that the Prada galaxies are bright systems, comparable to the Galaxy).

7 The Shapes of Dark Halos

What do we expect from simulations ? Dark halos from simulations are typically triaxial, with mean axial ratios 1 : 0.85 : 0.65 (eg Steinmetz & Muller 1995). What do we see ? The shapes of halos are difficult to measure, because the shape of the equipotentials (which affect the observed kinematics) is more spherical than the shape of the density distribution itself. Many different attempts have been made to measure the shapes of the dark halos. I will briefly review some of them.

7.1 Flaring of the HI layer in the Galaxy

The HI layer has an approximately isothermal velocity dispersion of about 8 km s^{-1}. In a spherical dark halo. the outer HI layer will then flare vertically more than if the dark halo is flattened. For our Galaxy, Olling & Merrifield (2000) use this flaring to estimate that the axial ratio of the dark halo is about 0.8.

7.2 Polar ring galaxies

Polar ring galaxies like NGC 4650A have matter rotating in two approximately orthogonal planes, so we can measure the potential gradient in these two planes. For example, in NGC 4650A, optical kinematics indicate that the dark halo has an axial ratio of about 0.3 to 0.4 (Sackett et al 1994). However an HI study of this system shows that the halo could be flattened to either of the two orbital planes (Arnaboldi & Combes 1996). We should also be aware that polar ring galaxies are unusual systems; it is possible that the survival of a well-developed polar ring may *require* a flattened and triaxial halo.

7.3 IC 2006

The elliptical galaxy IC 2006 is surrounded by a ring of HI at a radius of about 6.5 effective (ie half light) radii. The mass to blue light ratio at this radius is about 16, compared with the M/L ratio of about 5 in the inner regions. This is a good indication that IC 2006 has a dark halo like most galaxies. The kinematics of the HI ring show that the ring is almost perfectly circular (within 2%; Franx et al 1994), which suggests that the halo of this elliptical galaxy is very close to axisymmetric (i.e. two equal axes in the plane of the ring).

7.4 Carbon stars in the galactic halo

Ibata et al (2001) studied the kinematics of carbon stars in the galactic halo. At least half of them appear to be associated with the debris of the disrupting Sgr dwarf which extends in an almost polar great circle from a galactocentric radius of about 16 kpc to 60 kpc. The fact that the debris lies on a great circle suggests that the galactic halo does not exert a significant torque on the stream of debris. The distribution of carbon stars favors a nearly spherical galactic halo in the region $16 < R < 60$ kpc. Simulations of the precessing Sgr debris in potentials of different flattening show that an axial ratio as flat as 0.75 is very unlikely.

In summary, the evidence so far indicates that dark halos are fairly close to spherical.

8 Rotation of Dark Halos

Halos are believed to acquire angular momentum through tidal interactions with other halos as they form. The dimensionless parameter $\lambda = J|E|^{\frac{1}{2}} M^{-\frac{5}{2}} G^{-1}$ where J is the angular momentum of a system and E and M are its binding energy and mass, is a measure of the ratio of (rotational velocity)/(virial velocity). For example, for a disk in centrifugal equilibrium, $\lambda \simeq 0.45$. Cosmological simulations give well-defined and similar distributions of λ, with a mean $\lambda \simeq 0.05$. So the simulated halos are relatively slowly rotating (eg Bullock et al 2001).

If baryons and dark matter are initially well mixed and have similar specific angular momentum J/M, and if the baryons conserve their angular momentum as they collapse to a disk in centrifugal equilibrium, then the radial collapse factor for the disk is $R_{\rm halo}/h_{\rm disk} = \sqrt{2}/\lambda \simeq 30$ (Fall 2002) where $R_{\rm halo}$ is the radius of the halo and $h_{\rm disk}$ is the exponential scalelength of the equilibrium disk. For example, for our Galaxy, the optical scale length of the disk is about 4 kpc, and the halo extends out to at least 120 kpc, consistent with the factor 30.

Galaxies with higher λ-values are initially closer to centrifugal equilibrium, so would typically form disks of lower surface brightness. This is supported by the observation that the distribution of surface brightness has a similar shape to the distribution of λ from the simulations (eg Bullock et al 2001).

So far we have discussed the angular momentum of dark halos in general terms. The shape or figure of a rotating body may be axisymmetric or triaxial. If it is triaxial *and* the triaxial figure itself is rotating, then the torque of the rotating figure may be important for galactic dynamics. For example, Bekki & Freeman (2002) argued that the figure rotation of a triaxial dark halo could be important for stirring up spiral structure in the outer regions of galaxies where self-gravity appears to be too low to sustain spiral structure. NGC 2915 is an example of a galaxy with HI spiral structure extending far beyond the optical galaxy (Meurer et al 1996). For some other spectacular examples, see www.nfra.nl/∼oosterlo.

9 Dwarf Spheroidal Galaxies

These are faint satellites of our Galaxy (seen also around M31). Their absolute magnitudes are as low as $M_V = -8$. They have very low surface brightnesses and masses that are typically about $10^7\ M_\odot$. Radial velocities of individual stars in several of these dSph galaxies show that their M/L ratios can be very high. Some of the faintest dSph galaxies have $M/L \sim 100$. Figure 4 shows M/L values for the Local Group dSph galaxies. Figure 5 shows the radial variation of the velocity dispersion in the Fornax galaxy, which is the largest of the Galactic dSph galaxies; the velocity dispersion is approximately constant with radius, and the inferred M/L ratio is about 10, significantly higher than the value of about 2 expected for an old metal-poor population.

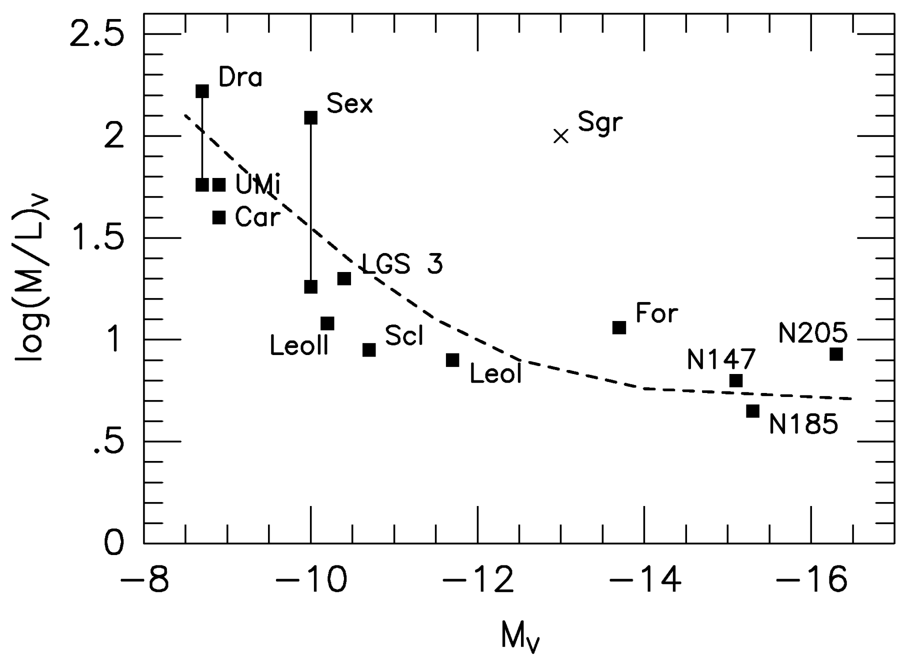

Figure 4. The correlation between M/L_V and M_V for Local Group dSph galaxies with good kinematic data. The dashed line shows a model in which each galaxy has a dark halo mass of $2.5 \times 10^7 M_\odot$ plus a luminous component with $M/L_V = 5$. (From Mateo 1997).

10 The Tully-Fisher Law

Simple centrifugal equilibrium arguments for a self-gravitating disk give a relation between the luminosity and rotational velocity known as the Tully-Fisher law:

$$L \propto V^4/[I_\circ (M/L)^2]$$

where L is the luminosity of the galaxy, V is its rotational velocity and the central surface brightness $I_\circ$ and M/L are roughly constant from galaxy to galaxy for spirals of normal surface brightness.

Observationally, the exponent of V in the Tully-Fisher law depends on the measured wavelength of the luminosity: it varies from about 3.2 at B to about 4.5

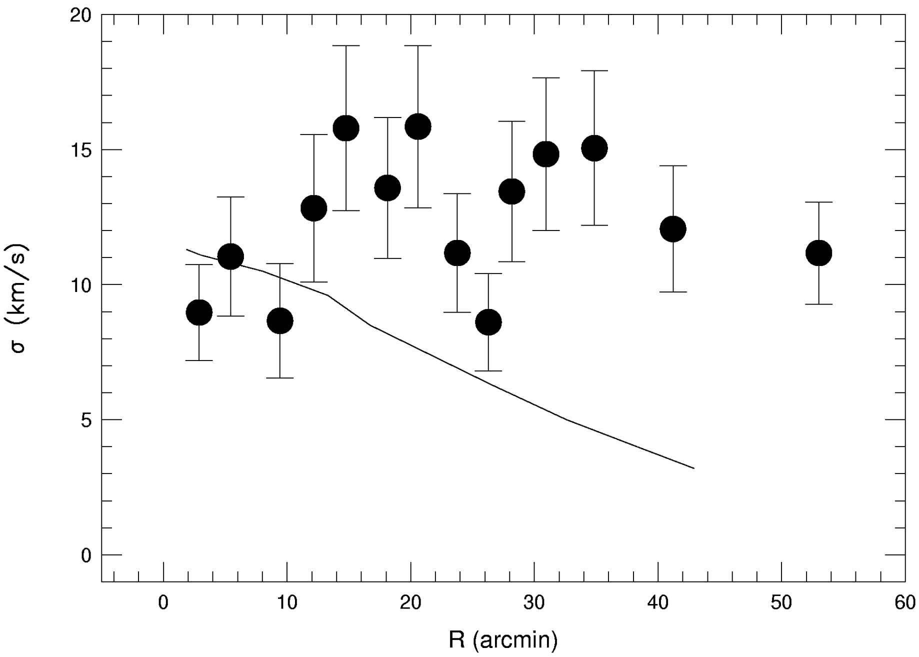

Figure 5. The radial variation of velocity dispersion in the Fornax dSph galaxy, from Mateo (1997). The curve shows the velocity dispersion expected if the mass were distributed like the light.

at H. This probably reflects a weak dependence of $I_\circ$ and M/L on L, analogous to the tilt of the fundamental plane for elliptical galaxies. Figure 6 shows how the observed slope varies, and also how the scatter in the Tully-Fisher law becomes smaller as the wavelength increases, due to the reduced effect of dust and star forming regions on the luminosity.

The zero point of the Tully-Fisher law needs explaining. For example, in the I-band, the Tully-Fisher law is

$$M_I = -10.00(\log W_{50} - 2.5) - 21.32$$

(Sakai et al 2000). Here W_{50} is the HI profile width at half peak height corrected for inclination, which is a measure of the rotational velocity. This equation states that a galaxy with $M_I = 21.32$ has a velocity width of 316 km s^{-1}, not 500 km s^{-1}. For a self-gravitating disk alone, *e.g.* an exponential disk, the zero point depends on the product $I_\circ(M/L)^2$. M/L is determined by the stellar population. The central surface density $\Sigma_\circ = I_\circ(M/L)$ depends on the mass M and angular momentum J for the disk: simple arguments show that $\Sigma_\circ = M^7/J^4$. The $J(M)$ relation is defined by the dynamics of galaxy formation and evolution. It determines the zero point of the Tully-Fisher law. This is a current problem in understanding galaxy formation (see §5): simulations show that too much angular momentum is lost from the baryons to the dark halo during the galaxy formation process. Because of the

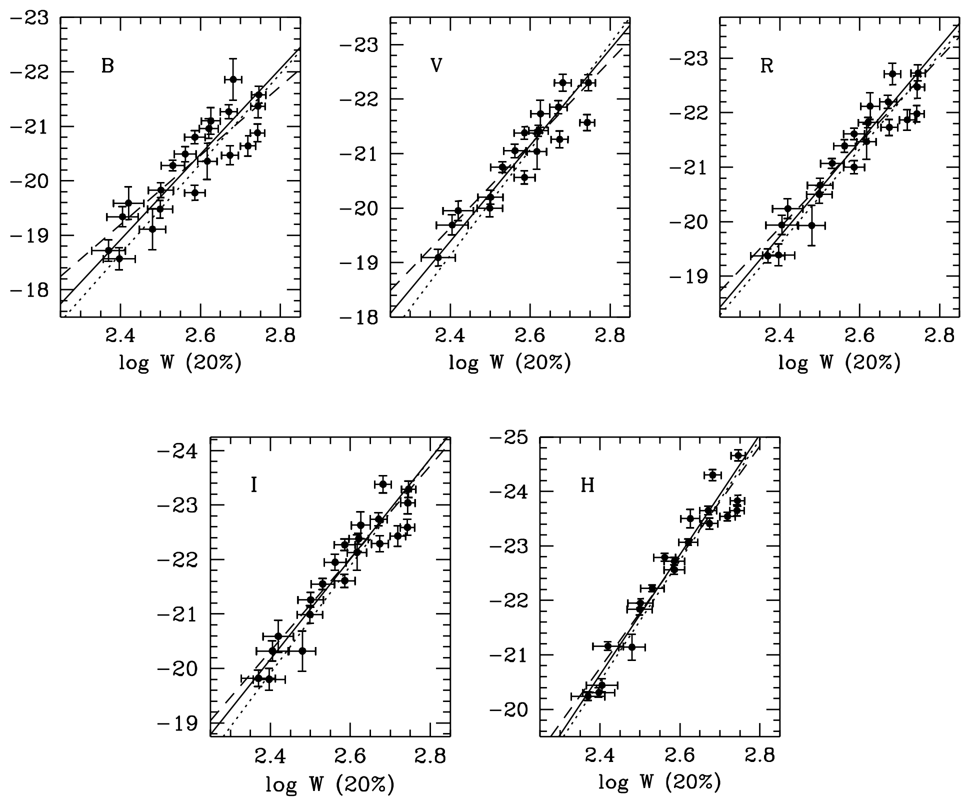

Figure 6. The observed Tully-Fisher law: note how the slope and the scatter change with wavelength (from Sakai *et al.* 2000).

conspiracy for disks of normal surface brightness (ie the approximate equality of the rotation curve contributions from disk and halo, as seen in Figure 2), this argument is not much changed by the presence of the dark halo.

Now consider low surface brightness (LSB) disks. Here the gravitational field is believed to be dominated by the dark halo everywhere. Yet the Tully-Fisher law for LSB galaxies is almost identical in slope and zero point to the Tully-Fisher law for the high surface brightness galaxies (Zwaan et al 1995). In the LSB galaxies, we believe that the dark halo determines W_{50}, while the baryons determine the absolute magnitude. We then infer that the baryon mass is related to the halo dynamics. Why should this be ?

The reason may be found in the scaling laws for dark halos, ie the relationship between parameters for the dark halos, like the central density $\rho_\circ$ and the core radius r_c, and the absolute magnitude of the galaxy. Kormendy & Freeman (2003, to be published) derived values for $\rho_\circ$ and r_c for a sample of galaxies with absolute magnitudes M_B ranging from -8 to -23. They found that the central density *decreases* with increasing luminosity, by about 3 orders of magnitude, while the core radius increases by about the same amount. In the mean, the product $\rho_\circ r_c$ is approximately constant for the dark halos. This means that the *surface density* of the halos is approximately constant, which is equivalent to a Faber-Jackson law for halos

$$M_{\rm halo} \propto \sigma^4 \simeq V_{\rm halo}^4/4$$

where $V_{\rm halo}$ is the rotational velocity in the gravitational field of the halo. Then, if the ratio of baryon mass to dark mass is constant from galaxy to galaxy, a Tully-Fisher law between the baryon mass and the halo rotational velocity $V_{\rm halo}$ would follow.

Why should the dark halos follow a Faber-Jackson law ? Fall (2002) describes how the index k of the mass-velocity relation $M_{\rm halo} \propto V_{\rm halo}^k$ for the dark halos depends on the initial spectrum of density perturbations, the comological parameters and the range of masses considered. A slope of 4 corresponds to an effective index $n \simeq -2$ of the CDM spectrum on galactic scales.

Some very gas-rich galaxies are under-luminous for the HI line widths. For example, for NGC 2915 and DDO 154 the order-of-magnitude of the ratios of dark matter mass to gas mass to stellar mass are 100 : 10 : 1. These two galaxies lie 2 to 3 magnitudes below the Tully-Fisher relation. However, if we notionally convert the gas into stars with a M/L ratio of about unity, these galaxies rise to the standard Tully-Fisher relation. This shows again how the Tully-Fisher law is about the relationship of *total baryon content* to the circular velocity of the dark halos (see Freeman 1999, McGaugh et al 2000).

11 How Much Galactic Dark Matter is There ?

Current estimates of the density of (stars + cold gas) and of the total baryon density from big bang nucleosynthesis arguments are $\Omega_{\rm stars+cold\ gas} = 0.0042$ and $\Omega_{\rm BBNS} = 0.04$, so the luminous mass in galaxies is only about 10% of the baryon mass. The rest of the baryons are believed to be hot gas, probably in groups of galaxies. See Fukugita *et al.* (1998), Table 3.

The current estimate of the total matter density of the universe $\Omega_{\rm matter}$ is about 0.27. Recent weak lensing studies indicate that the dark matter within the virial radii of halos is about 37 % of the total matter density of the universe; *i.e.* $\Omega_{\rm dark\ halos} \simeq 0.11$ (Hoekstra *et al.* 2003). If this is correct, then the typical ratio of dark matter to baryonic matter *within galaxies* is $0.11/0.0042 \simeq 25$. This is consistent with the independently derived lower limit of about 20 for our own Galaxy: see §6.1.

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